Computer algebra independent integration tests

5-Inverse-trig-functions/5.3-Inverse-tangent/5.3.5-u-a+b-arctan-c+d-x-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [70]. This is test number [151].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (70)	% 0.00 (0)
Mathematica	% 95.71 (67)	% 4.29 (3)
Maple	% 98.57 (69)	% 1.43 (1)
Maxima	% 52.86 (37)	% 47.14 (33)
Fricas	% 40.00 (28)	% 60.00 (42)
Sympy	% 32.86 (23)	% 67.14 (47)
Giac	% 8.57 (6)	% 91.43 (64)
Mupad	% 42.86 (30)	% 57.14 (40)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

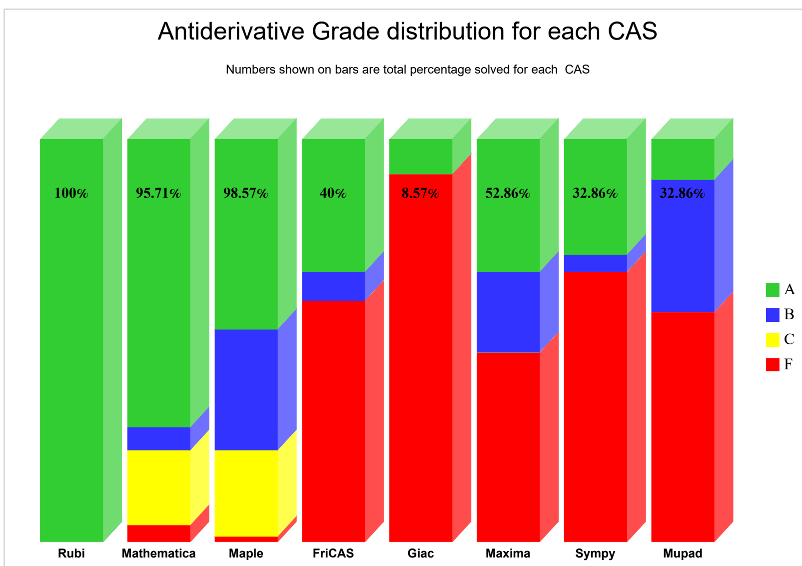
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

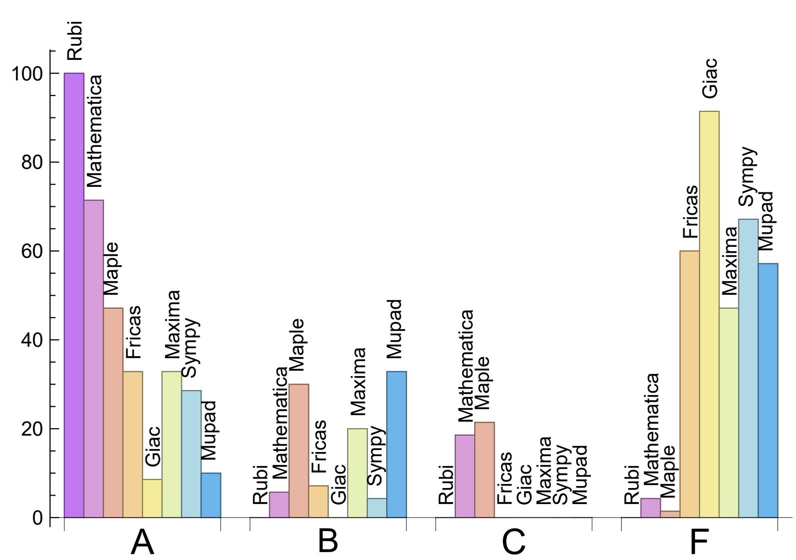
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	71.43	5.71	18.57	4.29
Maple	47.14	30.00	21.43	1.43
Maxima	32.86	20.00	0.00	47.14
Fricas	32.86	7.14	0.00	60.00
Sympy	28.57	4.29	0.00	67.14
Giac	8.57	0.00	0.00	91.43
Mupad	10.00	32.86	0.00	57.14

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	3	100.00 %	0.00 %	0.00 %
Maple	1	100.00 %	0.00 %	0.00 %
Maxima	33	81.82 %	12.12 %	6.06 %
Fricas	42	97.62 %	0.00 %	2.38 %
Sympy	47	59.57 %	40.43 %	0.00 %
Giac	64	64.06 %	35.94 %	0.00 %
Mupad	40	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

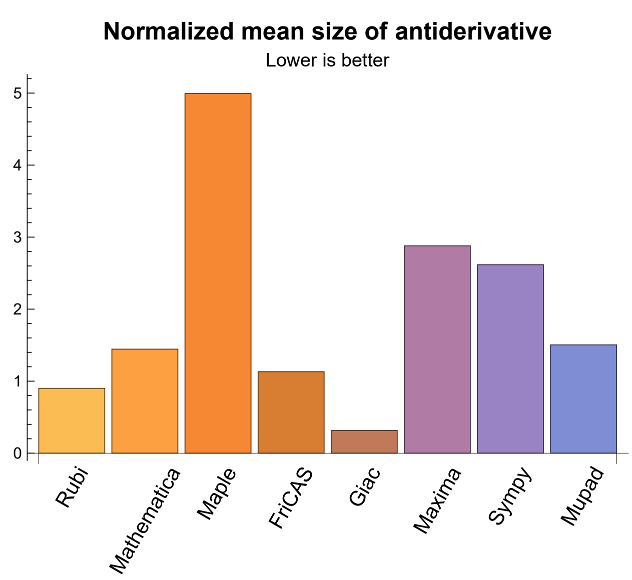
1.3 Performance

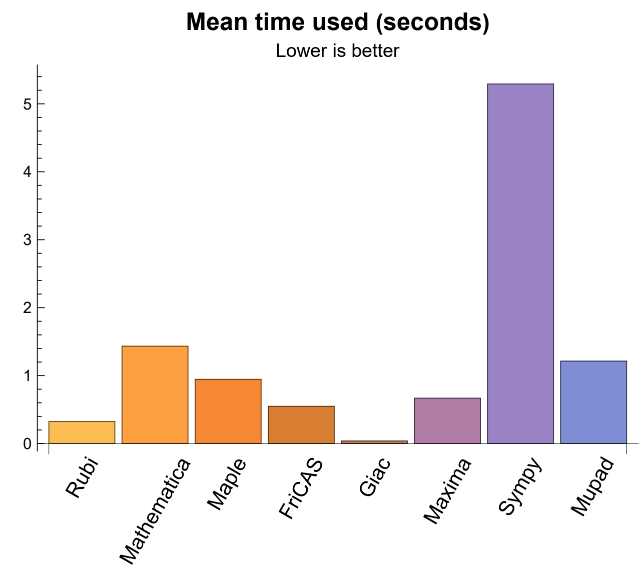
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.32	228.04	0.90	159.50	1.00
Mathematica	1.43	266.84	1.44	163.00	1.00
Maple	0.95	1882.19	4.99	225.00	1.83
Maxima	0.67	1001.03	2.88	123.00	1.28
Fricas	0.55	132.11	1.13	81.00	1.15
Sympy	5.29	262.48	2.62	177.00	2.43
Giac	0.04	11.17	0.31	0.00	0.00
Mupad	1.21	171.53	1.50	102.50	1.35

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{23, 42, 43, 65, 66, 69, 70}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {65, 66, 69, 70}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {8, 11, 13, 15, 16, 18, 19, 20, 28, 31, 32, 33, 35, 36, 37, 38, 48, 54, 55, 56, 57, 62, 63, 64, 65, 66, 67, 68, 69, 70}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
```

```
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))
```

```
    except Exception as ee:
           leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

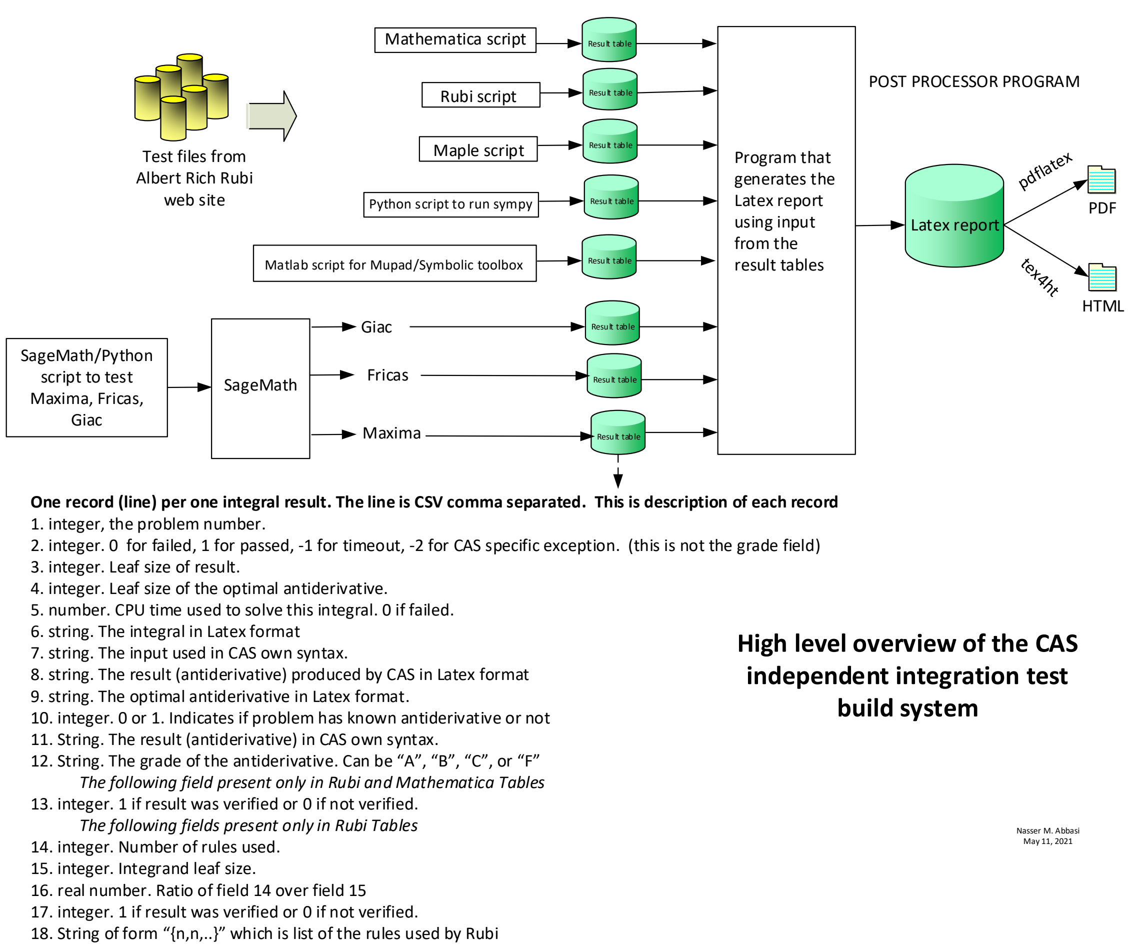
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 27, 28, 32, 33, 35, 37, 38, 41, 42, 43, 47, 48, 52, 53, 54, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70 }

B grade: { 31, 36, 55, 56 }

C grade: { 6, 24, 25, 26, 29, 30, 44, 45, 46, 49, 50, 51, 57 }

F grade: { 34, 39, 40 }

2.1.3 Maple

A grade: { 5, 6, 12, 14, 23, 25, 26, 27, 28, 29, 30, 33, 35, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 54, 55, 63, 64, 65, 66, 67, 68, 69, 70 }

B grade: { 1, 2, 3, 4, 7, 8, 9, 11, 13, 16, 19, 21, 22, 24, 31, 32, 38, 53, 60, 61, 62 }

C grade: { 10, 15, 17, 18, 20, 34, 36, 37, 39, 40, 52, 56, 57, 58, 59 }

F grade: { 41 }

2.1.4 Maxima

A grade: { 5, 24, 25, 26, 27, 29, 30, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 55, 60, 65, 66, 69, 70 }

B grade: { 1, 2, 3, 6, 7, 9, 12, 14, 21, 22, 53, 54, 56, 61 }

C grade: { }

F grade: { 4, 8, 10, 11, 13, 15, 16, 17, 18, 19, 20, 23, 28, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 52, 57, 58, 59, 62, 63, 64, 67, 68 }

2.1.5 FriCAS

A grade: { 3, 5, 6, 9, 12, 24, 25, 26, 27, 29, 42, 43, 44, 45, 46, 47, 49, 50, 51, 65, 66, 69, 70 }

B grade: { 1, 2, 7, 14, 30 }

C grade: { }

F grade: { 4, 8, 10, 11, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 28, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 67, 68 }

2.1.6 Sympy

A grade: { 1, 2, 3, 5, 6, 7, 9, 12, 23, 24, 25, 26, 27, 44, 45, 46, 47, 65, 66, 69 }

B grade: { 49, 50, 51 }

C grade: { }

F grade: { 4, 8, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 67, 68, 70 }

2.1.7 Giac

A grade: { 27, 42, 43, 47, 65, 66 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 67, 68, 69, 70 }

2.1.8 Mupad

A grade: { 23, 42, 43, 65, 66, 69, 70 }

B grade: { 1, 2, 3, 5, 6, 7, 9, 12, 14, 21, 24, 25, 26, 27, 29, 30, 44, 45, 46, 47, 49, 50, 51 }

C grade: { }

F grade: { 4, 8, 10, 11, 13, 15, 16, 17, 18, 19, 20, 22, 28, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 67, 68 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	56	225	370	151	231	0	371
normalized size	1	1.00	0.78	3.12	5.14	2.10	3.21	0.00	5.15
time (sec)	N/A	0.251	0.022	0.038	0.419	0.511	4.669	0.000	0.654

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	54	161	238	129	182	0	144
normalized size	1	1.00	0.81	2.40	3.55	1.93	2.72	0.00	2.15
time (sec)	N/A	0.055	0.023	0.040	0.421	0.656	3.929	0.000	1.170

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	40	92	120	60	95	0	73
normalized size	1	1.00	0.83	1.92	2.50	1.25	1.98	0.00	1.52
time (sec)	N/A	0.030	0.016	0.038	0.420	0.577	2.014	0.000	1.454

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	52	132	0	0	0	0	-1
normalized size	1	1.00	0.83	2.10	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.058	0.018	0.058	0.000	0.539	0.000	0.000	0.000

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	50	73	92	75	284	0	88
normalized size	1	1.00	0.82	1.20	1.51	1.23	4.66	0.00	1.44
time (sec)	N/A	0.047	0.027	0.044	0.316	0.512	6.886	0.000	0.665

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	51	71	120	70	314	0	103
normalized size	1	1.00	0.81	1.13	1.90	1.11	4.98	0.00	1.63
time (sec)	N/A	0.045	0.022	0.043	0.424	0.565	12.389	0.000	0.754

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	216	543	597	337	583	0	633
normalized size	1	1.00	1.38	3.46	3.80	2.15	3.71	0.00	4.03
time (sec)	N/A	0.221	0.120	0.057	1.802	0.647	11.420	0.000	3.263

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	163	593	0	0	0	0	-1
normalized size	1	1.00	0.89	3.24	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.221	0.431	0.203	0.000	0.671	0.000	0.000	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	107	220	218	150	240	0	216
normalized size	1	1.00	1.13	2.32	2.29	1.58	2.53	0.00	2.27
time (sec)	N/A	0.119	0.076	0.056	1.488	0.721	4.303	0.000	1.611

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	170	1433	0	0	0	0	-1
normalized size	1	1.00	0.93	7.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.339	0.079	0.775	0.000	0.415	0.000	0.000	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	135	471	0	0	0	0	-1
normalized size	1	1.00	1.13	3.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.187	0.229	0.172	0.000	0.452	0.000	0.000	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	194	182	268	209	1144	0	232
normalized size	1	1.00	1.66	1.56	2.29	1.79	9.78	0.00	1.98
time (sec)	N/A	0.153	0.132	0.057	0.538	0.436	21.956	0.000	2.857

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	163	547	0	0	0	0	-1
normalized size	1	1.00	0.84	2.82	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.257	0.750	0.143	0.000	0.472	0.000	0.000	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	245	242	534	448	0	0	438
normalized size	1	1.00	1.44	1.42	3.14	2.64	0.00	0.00	2.58
time (sec)	N/A	0.227	0.351	0.065	0.511	0.475	0.000	0.000	3.645

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	349	3242	0	0	0	0	-1
normalized size	1	1.00	1.29	11.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.439	0.766	2.247	0.000	0.469	0.000	0.000	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	196	567	0	0	0	0	-1
normalized size	1	1.00	1.20	3.46	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.243	0.522	0.145	0.000	0.455	0.000	0.000	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	252	2894	0	0	0	0	-1
normalized size	1	1.00	0.90	10.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.458	0.153	0.280	0.000	0.463	0.000	0.000	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	263	2696	0	0	0	0	-1
normalized size	1	1.00	1.61	16.54	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.300	0.690	0.442	0.000	0.459	0.000	0.000	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	225	631	0	0	0	0	-1
normalized size	1	1.00	1.25	3.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.319	0.307	0.159	0.000	0.445	0.000	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	360	7083	0	0	0	0	-1
normalized size	1	1.00	1.25	24.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.500	1.225	1.626	0.000	0.444	0.000	0.000	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	68	44	0	0	0	25
normalized size	1	1.00	1.00	2.19	1.42	0.00	0.00	0.00	0.81
time (sec)	N/A	0.038	0.004	0.062	0.456	0.461	0.000	0.000	0.081

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	34	98	123	0	0	0	-1
normalized size	1	1.00	0.83	2.39	3.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.045	0.008	0.061	0.486	0.494	0.000	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	6.110	3.232	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	157	494	346	317	654	0	787
normalized size	1	1.00	0.67	2.12	1.48	1.36	2.81	0.00	3.38
time (sec)	N/A	0.381	0.298	0.047	0.428	0.476	25.363	0.000	1.019

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	118	283	220	199	376	0	411
normalized size	1	1.00	0.76	1.83	1.42	1.28	2.43	0.00	2.65
time (sec)	N/A	0.190	0.160	0.054	0.427	0.468	10.655	0.000	0.782

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	163	146	116	103	177	0	136
normalized size	1	1.00	1.68	1.51	1.20	1.06	1.82	0.00	1.40
time (sec)	N/A	0.112	0.065	0.059	0.420	0.456	4.545	0.000	1.804

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	49	42	36	48	51	36	49
normalized size	1	1.00	1.29	1.11	0.95	1.26	1.34	0.95	1.29
time (sec)	N/A	0.018	0.016	0.036	0.317	0.430	0.366	0.106	1.103

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	160	224	0	0	0	0	-1
normalized size	1	1.00	0.99	1.38	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.150	0.122	0.099	0.000	0.441	0.000	0.000	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	121	205	177	190	0	0	127
normalized size	1	1.00	0.80	1.36	1.17	1.26	0.00	0.00	0.84
time (sec)	N/A	0.121	0.206	0.049	0.417	0.695	0.000	0.000	1.829

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	175	438	409	682	0	0	399
normalized size	1	1.00	0.77	1.93	1.80	3.00	0.00	0.00	1.76
time (sec)	N/A	0.303	0.795	0.051	0.427	2.129	0.000	0.000	7.534

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	801	1622	0	0	0	0	-1
normalized size	1	1.00	2.10	4.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.574	4.230	0.146	0.000	0.419	0.000	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	264	748	0	0	0	0	-1
normalized size	1	1.00	1.19	3.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.373	0.455	0.135	0.000	0.427	0.000	0.000	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	109	180	0	0	0	0	-1
normalized size	1	1.00	1.07	1.76	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.095	0.333	0.000	0.437	0.000	0.000	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	0	2149	0	0	0	0	-1
normalized size	1	1.00	0.00	8.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.165	6.510	2.312	0.000	0.403	0.000	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	568	568	419	1087	0	0	0	0	-1
normalized size	1	1.00	0.74	1.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.351	7.492	0.148	0.000	0.471	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	564	564	1844	6682	0	0	0	0	-1
normalized size	1	1.00	3.27	11.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.937	9.978	10.281	0.000	0.452	0.000	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	592	16362	0	0	0	0	-1
normalized size	1	1.00	1.76	48.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.632	0.819	2.493	0.000	0.433	0.000	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	266	359	0	0	0	0	-1
normalized size	1	1.00	1.86	2.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.214	0.154	0.421	0.000	0.437	0.000	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	0	4389	0	0	0	0	-1
normalized size	1	1.00	0.00	11.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.203	8.947	0.691	0.000	0.545	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1233	1233	0	4764	0	0	0	0	-1
normalized size	1	1.00	0.00	3.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.311	17.431	1.798	0.000	0.474	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	162	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.246	0.393	1.958	0.000	0.430	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.058	4.995	1.479	0.000	0.411	0.000	0.000	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.056	0.507	1.434	0.000	0.422	0.000	0.000	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	95	132	104	87	155	0	133
normalized size	1	1.00	0.90	1.25	0.98	0.82	1.46	0.00	1.25
time (sec)	N/A	0.111	0.077	0.040	0.409	0.414	1.580	0.000	0.594

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	114	95	85	66	117	0	102
normalized size	1	1.00	1.44	1.20	1.08	0.84	1.48	0.00	1.29
time (sec)	N/A	0.092	0.057	0.043	0.412	0.411	1.154	0.000	0.860

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	90	66	68	52	78	0	61
normalized size	1	1.00	1.50	1.10	1.13	0.87	1.30	0.00	1.02
time (sec)	N/A	0.055	0.036	0.038	0.410	0.407	0.643	0.000	0.970

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	39	36	31	39	46	31	42
normalized size	1	1.00	1.18	1.09	0.94	1.18	1.39	0.94	1.27
time (sec)	N/A	0.011	0.016	0.035	0.309	0.400	0.393	0.126	0.446

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	171	103	134	0	0	0	-1
normalized size	1	1.00	1.42	0.86	1.12	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.009	0.058	0.470	0.393	0.000	0.000	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	67	63	77	57	168	0	63
normalized size	1	1.00	1.08	1.02	1.24	0.92	2.71	0.00	1.02
time (sec)	N/A	0.039	0.062	0.046	0.411	0.414	1.886	0.000	1.042

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	92	105	112	95	382	0	232
normalized size	1	1.00	0.96	1.09	1.17	0.99	3.98	0.00	2.42
time (sec)	N/A	0.083	0.105	0.047	0.409	0.498	2.920	0.000	1.220

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	128	162	165	135	760	0	288
normalized size	1	1.00	0.99	1.26	1.28	1.05	5.89	0.00	2.23
time (sec)	N/A	0.114	0.149	0.048	0.415	0.449	4.656	0.000	1.048

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	863	863	701	631	0	0	0	0	-1
normalized size	1	1.00	0.81	0.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.207	0.913	1.312	0.000	0.628	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	543	543	409	2192	8520	0	0	0	-1
normalized size	1	1.00	0.75	4.04	15.69	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.607	0.362	1.286	5.931	0.440	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	231	198	284	0	0	0	-1
normalized size	1	1.00	1.52	1.30	1.87	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.023	0.062	0.530	0.443	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	771	317	284	0	0	0	-1
normalized size	1	1.00	3.16	1.30	1.16	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.239	11.805	0.067	0.552	0.458	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	668	668	1536	53434	8518	0	0	0	-1
normalized size	1	1.00	2.30	79.99	12.75	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.855	25.833	2.698	1.349	0.411	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	933	933	933	682	0	0	0	0	-1
normalized size	1	1.00	1.00	0.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.367	7.384	1.235	0.000	0.530	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	673	673	604	344	0	0	0	0	-1
normalized size	1	1.00	0.90	0.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.898	0.596	0.333	0.000	0.477	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	770	770	666	377	0	0	0	0	-1
normalized size	1	1.00	0.86	0.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.982	0.827	0.314	0.000	0.585	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	283	833	328	0	0	0	-1
normalized size	1	1.00	1.03	3.04	1.20	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.271	0.060	1.056	0.546	0.430	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	543	543	409	2192	14300	0	0	0	-1
normalized size	1	1.00	0.75	4.04	26.34	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.600	0.400	1.244	2.840	0.427	0.000	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	367	367	443	4743	0	0	0	0	-1
normalized size	1	1.00	1.21	12.92	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.673	0.463	1.622	0.000	0.462	0.000	0.000	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	97	143	0	0	0	0	-1
normalized size	1	1.00	0.73	1.08	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.117	0.684	0.000	0.439	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	125	176	0	0	0	0	-1
normalized size	1	1.00	0.58	0.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.162	0.069	0.927	0.000	0.436	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	163	0	0	0	0	0	-1
normalized size	1	0.00	7.09	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.038	0.422	1.473	0.000	0.424	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	165	0	0	0	0	0	-1
normalized size	1	0.00	6.60	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.051	0.100	1.376	0.000	0.448	0.000	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	145	187	0	0	0	0	-1
normalized size	1	1.00	0.78	1.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.216	0.724	3.160	0.000	0.452	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	189	222	0	0	0	0	-1
normalized size	1	1.00	0.67	0.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.331	0.162	4.530	0.000	0.470	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	181	0	0	0	0	0	-1
normalized size	1	0.00	6.03	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.131	1.577	4.682	0.000	0.441	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	225	0	0	0	0	0	-1
normalized size	1	0.00	7.03	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.179	0.800	4.702	0.000	0.457	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [35] had the largest ratio of [1.250]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	5	1.00	21	0.238
2	A	6	5	1.00	21	0.238
3	A	5	5	1.00	19	0.263
4	A	5	4	1.00	21	0.190
5	A	7	7	1.00	21	0.333
6	A	5	5	1.00	21	0.238
7	A	13	9	1.00	23	0.391
8	A	11	10	1.00	23	0.435
9	A	8	7	1.00	21	0.333
10	A	8	7	1.00	23	0.304
11	A	6	6	1.00	23	0.261
12	A	10	9	1.00	23	0.391
13	A	10	9	1.00	23	0.391
14	A	15	10	1.00	23	0.435
15	A	14	11	1.00	23	0.478
16	A	10	10	1.00	21	0.476
17	A	10	8	1.00	23	0.348
18	A	7	8	1.00	23	0.348
19	A	9	8	1.00	23	0.348
20	A	16	13	1.00	23	0.565
21	A	5	4	1.00	12	0.333
22	A	5	4	1.00	19	0.210
23	A	0	0	0.00	0	0.000
24	A	7	6	1.00	18	0.333
25	A	7	6	1.00	18	0.333
26	A	7	6	1.00	16	0.375
27	A	4	3	1.00	10	0.300
28	A	5	5	1.00	18	0.278
29	A	8	8	1.00	18	0.444
30	A	9	8	1.00	18	0.444
31	A	16	13	1.00	20	0.650
32	A	13	10	1.00	18	0.556
33	A	6	6	1.00	12	0.500
34	A	2	2	1.00	20	0.100
35	A	25	25	1.00	20	1.250

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
36	A	21	14	1.00	20	0.700
37	A	15	11	1.00	18	0.611
38	A	6	7	1.00	12	0.583
39	A	2	2	1.00	20	0.100
40	A	35	22	1.00	20	1.100
41	A	6	4	1.00	18	0.222
42	A	0	0	0.00	0	0.000
43	A	0	0	0.00	0	0.000
44	A	7	6	1.00	10	0.600
45	A	7	6	1.00	10	0.600
46	A	7	6	1.00	8	0.750
47	A	3	3	1.00	6	0.500
48	A	5	5	1.00	10	0.500
49	A	7	7	1.00	10	0.700
50	A	8	7	1.00	10	0.700
51	A	8	7	1.00	10	0.700
52	A	23	5	1.00	16	0.312
53	A	17	5	1.00	16	0.312
54	A	5	5	1.00	14	0.357
55	A	15	7	1.00	16	0.438
56	A	25	7	1.00	16	0.438
57	A	31	7	1.00	16	0.438
58	A	31	13	1.00	18	0.722
59	A	37	16	1.00	18	0.889
60	A	17	5	1.00	14	0.357
61	A	17	5	1.00	16	0.312
62	A	12	8	1.00	19	0.421
63	A	2	2	1.00	28	0.071
64	A	3	3	1.00	33	0.091
65	A	0	0	0.00	0	0.000
66	A	0	0	0.00	0	0.000
67	A	4	4	1.00	35	0.114
68	A	5	5	1.00	40	0.125
69	A	0	0	0.00	0	0.000
70	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

3.1 $\int (ce+dex)^3 \left(a+b\tan^{-1}(c+dx)\right) dx$

Optimal. Leaf size=72

$$\frac{e^3(c+dx)^4\left(a+b\tan^{-1}(c+dx)\right)}{4d}-\frac{be^3(c+dx)^3}{12d}-\frac{be^3\tan^{-1}(c+dx)}{4d}+\frac{1}{4}be^3x$$

```
[Out] 1/4*b*e^3*x-1/12*b*e^3*(d*x+c)^3/d-1/4*b*e^3*arctan(d*x+c)/d+1/4*e^3*(d*x+c
)^4*(a+b*arctan(d*x+c))/d
```

Rubi [A] time = 0.25, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5043, 12, 4852, 302, 203}

$$\frac{e^3(c+dx)^4\left(a+b\tan^{-1}(c+dx)\right)}{4d}-\frac{be^3(c+dx)^3}{12d}-\frac{be^3\tan^{-1}(c+dx)}{4d}+\frac{1}{4}be^3x$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] (b*e^3*x)/4 - (b*e^3*(c + d*x)^3)/(12*d) - (b*e^3*ArcTan[c + d*x])/(4*d) +
(e^3*(c + d*x)^4*(a + b*ArcTan[c + d*x]))/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
```

```
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\tan^{-1}(c+dx)\right) dx = \frac{\text{Subst}\left(\int e^3x^3 \left(a+b\tan^{-1}(x)\right) dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int x^3 \left(a+b\tan^{-1}(x)\right) dx, x, c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4\left(a+b\tan^{-1}(c+dx)\right)}{4d} - \frac{\left(be^3\right)\text{Subst}\left(\int \frac{x^4}{1+x^2}\,dx, x, c+dx\right)}{4d}$$

$$= \frac{e^3(c+dx)^4\left(a+b\tan^{-1}(c+dx)\right)}{4d} - \frac{\left(be^3\right)\text{Subst}\left(\int \left(-1+x^2+\frac{1}{1+x^2}\right) dx,\right.}{4d}$$

$$= \frac{1}{4}be^3x - \frac{be^3(c+dx)^3}{12d} + \frac{e^3(c+dx)^4\left(a+b\tan^{-1}(c+dx)\right)}{4d} - \frac{\left(be^3\right)\text{Subst}}{}$$

$$= \frac{1}{4}be^3x - \frac{be^3(c+dx)^3}{12d} - \frac{be^3\tan^{-1}(c+dx)}{4d} + \frac{e^3(c+dx)^4\left(a+b\tan^{-1}(c+\right.}{4d}$$

Mathematica [A] time = 0.02, size = 56, normalized size = 0.78

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4\left(a+b\tan^{-1}(c+dx)\right)-\frac{1}{4}b\left(\frac{1}{3}(c+dx)^3+\tan^{-1}(c+dx)-dx\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] (e^3*(-1/4*(b*(-(d*x) + (c + d*x)^3/3 + ArcTan[c + d*x])) + ((c + d*x)^4*(a
 + b*ArcTan[c + d*x]))/4))/d
```

fricas [B] time = 0.51, size = 151, normalized size = 2.10

$$\frac{3\,ad^4e^3x^4+(12\,ac-b)d^3e^3x^3+3\left(6\,ac^2-bc\right)d^2e^3x^2+3\left(4\,ac^3-bc^2+b\right)de^3x+3\left(bd^4e^3x^4+4\,bcd^3e^3x^3+6\,bc^2d^2\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arctan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*d^4*e^3*x^4 + (12*a*c - b)*d^3*e^3*x^3 + 3*(6*a*c^2 - b*c)*d^2*e^
3*x^2 + 3*(4*a*c^3 - b*c^2 + b)*d*e^3*x + 3*(b*d^4*e^3*x^4 + 4*b*c*d^3*e^3*
x^3 + 6*b*c^2*d^2*e^3*x^2 + 4*b*c^3*d*e^3*x + (b*c^4 - b)*e^3)*arctan(d*x +
 c))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.04, size = 225, normalized size = 3.12

$$\frac{d^3x^4a\,e^3}{4}+d^2x^3ac\,e^3+\frac{3d\,x^2a\,c^2e^3}{2}+xa\,c^3e^3+\frac{a\,c^4e^3}{4d}+\frac{d^3\arctan\left(dx+c\right)x^4b\,e^3}{4}+d^2\arctan\left(dx+c\right)x^3bc\,e^3+\frac{3d\,\mathrm{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arctan(d*x+c)),x)
```

```
[Out] 1/4*d^3*x^4*a*e^3+d^2*x^3*a*c*e^3+3/2*d*x^2*a*c^2*e^3+x*a*c^3*e^3+1/4/d*a*c
^4*e^3+1/4*d^3*arctan(d*x+c)*x^4*b*e^3+d^2*arctan(d*x+c)*x^3*b*c*e^3+3/2*d*
arctan(d*x+c)*x^2*b*c^2*e^3+arctan(d*x+c)*x*b*c^3*e^3+1/4/d*arctan(d*x+c)*b
*c^4*e^3-1/12*d^2*x^3*b*e^3-1/4*d*x^2*b*c*e^3-1/4*x*b*c^2*e^3-1/12/d*b*c^3*
e^3+1/4*b*e^3*x+1/4/d*b*c*e^3-1/4*b*e^3*arctan(d*x+c)/d
```

maxima [B] time = 0.42, size = 370, normalized size = 5.14

$$\frac{1}{4}\,ad^3e^3x^4+acd^2e^3x^3+\frac{3}{2}\,ac^2de^3x^2+\frac{3}{2}\left(x^2\arctan\left(dx+c\right)-d\left(\frac{x}{d^2}+\frac{\left(c^2-1\right)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3}-\frac{c\log\left(d^2x^2+2\,cd\right.}{d^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arctan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*a*d^3*e^3*x^4 + a*c*d^2*e^3*x^3 + 3/2*a*c^2*d*e^3*x^2 + 3/2*(x^2*arctan
(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^
2 + 2*c*d*x + c^2 + 1)/d^3))*b*c^2*d*e^3 + 1/2*(2*x^3*arctan(d*x + c) - d*(
(d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*c^2 -
1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*b*c*d^2*e^3 + 1/12*(3*x^4*arctan(
d*x + c) - d*((d^2*x^3 - 3*c*d*x^2 + 3*(3*c^2 - 1)*x)/d^4 + 3*(c^4 - 6*c^2
+ 1)*arctan((d^2*x + c*d)/d)/d^5 - 6*(c^3 - c)*log(d^2*x^2 + 2*c*d*x + c^2
+ 1)/d^5))*b*d^3*e^3 + a*c^3*e^3*x + 1/2*(2*(d*x + c)*arctan(d*x + c) - log
((d*x + c)^2 + 1))*b*c^3*e^3/d
```

mupad [B] time = 0.65, size = 371, normalized size = 5.15

$$\mathrm{atan}\left(c+d\,x\right)\,\left(b\,c^3\,e^3\,x+\frac{3\,b\,c^2\,d\,e^3\,x^2}{2}+b\,c\,d^2\,e^3\,x^3+\frac{b\,d^3\,e^3\,x^4}{4}\right)-x^3\,\left(\frac{d^2\,e^3\,\left(b-20\,a\,c\right)}{12}+\frac{2\,a\,c\,d^2\,e^3}{3}\right)+x^2\,\left(\frac{c\,\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*atan(c + d*x)),x)
```

```
[Out] atan(c + d*x)*((b*d^3*e^3*x^4)/4 + b*c^3*e^3*x + (3*b*c^2*d*e^3*x^2)/2 + b*
c*d^2*e^3*x^3) - x^3*((d^2*e^3*(b - 20*a*c))/12 + (2*a*c*d^2*e^3)/3) + x^2*
((c*((d^2*e^3*(b - 20*a*c))/4 + 2*a*c*d^2*e^3))/d + (d*e^3*(a - b*c + 10*a*
c^2))/2 - (a*d*e^3*(4*c^2 + 4))/8) + x*((c*e^3*(6*a - 3*b*c + 20*a*c^2))/2
+ ((4*c^2 + 4)*((d^2*e^3*(b - 20*a*c))/4 + 2*a*c*d^2*e^3))/(4*d^2) - (2*c*(
(2*c*((d^2*e^3*(b - 20*a*c))/4 + 2*a*c*d^2*e^3))/d + d*e^3*(a - b*c + 10*a*
c^2) - (a*d*e^3*(4*c^2 + 4))/4))/d) + (a*d^3*e^3*x^4)/4 - (b*e^3*atan(((b*c
```

```
*e^3*(c^2 + 1)*(c - 1)*(c + 1))/4 + (b*d*e^3*x*(c^2 + 1)*(c - 1)*(c + 1))/4
)/((b*e^3)/4 - (b*c^4*e^3)/4))*(c^2 + 1)*(c - 1)*(c + 1))/(4*d)
```

sympy [A] time = 4.67, size = 231, normalized size = 3.21

$$\begin{cases} ac^3e^3x + \frac{3ac^2de^3x^2}{2} + acd^2e^3x^3 + \frac{ad^3e^3x^4}{4} + \frac{bc^4e^3\operatorname{atan}(c+dx)}{4d} + bc^3e^3x\operatorname{atan}(c+dx) + \frac{3bc^2de^3x^2\operatorname{atan}(c+dx)}{2} - \frac{bc^2e^3x}{4} + bcd^2 \\ c^3e^3x\,(a + b\operatorname{atan}(c)) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*atan(d*x+c)),x)
```

```
[Out] Piecewise((a*c**3*e**3*x + 3*a*c**2*d*e**3*x**2/2 + a*c*d**2*e**3*x**3 + a*
d**3*e**3*x**4/4 + b*c**4*e**3*atan(c + d*x)/(4*d) + b*c**3*e**3*x*atan(c +
 d*x) + 3*b*c**2*d*e**3*x**2*atan(c + d*x)/2 - b*c**2*e**3*x/4 + b*c*d**2*e
**3*x**3*atan(c + d*x) - b*c*d*e**3*x**2/4 + b*d**3*e**3*x**4*atan(c + d*x)
/4 - b*d**2*e**3*x**3/12 + b*e**3*x/4 - b*e**3*atan(c + d*x)/(4*d), Ne(d, 0
)), (c**3*e**3*x*(a + b*atan(c)), True))
```

3.2 $\int (ce+dex)^2 \left(a+b\tan^{-1}(c+dx)\right) \, dx$

Optimal. Leaf size=67

$$\frac{e^2(c+dx)^3\left(a+b\tan^{-1}(c+dx)\right)}{3d}-\frac{be^2(c+dx)^2}{6d}+\frac{be^2\log\left((c+dx)^2+1\right)}{6d}$$

```
[Out] -1/6*b*e^2*(d*x+c)^2/d+1/3*e^2*(d*x+c)^3*(a+b*arctan(d*x+c))/d+1/6*b*e^2*ln
(1+(d*x+c)^2)/d
```

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5043, 12, 4852, 266, 43}

$$\frac{e^2(c+dx)^3\left(a+b\tan^{-1}(c+dx)\right)}{3d}-\frac{be^2(c+dx)^2}{6d}+\frac{be^2\log\left((c+dx)^2+1\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] -(b*e^2*(c + d*x)^2)/(6*d) + (e^2*(c + d*x)^3*(a + b*ArcTan[c + d*x]))/(3*d
) + (b*e^2*Log[1 + (c + d*x)^2])/(6*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (ce+dex)^2 \left(a+b\tan^{-1}(c+dx)\right) dx = \frac{\text{Subst}\left(\int e^2x^2 \left(a+b\tan^{-1}(x)\right) dx, x, c+dx\right)}{d}$$
$$= \frac{e^2\,\text{Subst}\left(\int x^2 \left(a+b\tan^{-1}(x)\right) dx, x, c+dx\right)}{d}$$
$$= \frac{e^2(c+dx)^3 \left(a+b\tan^{-1}(c+dx)\right)}{3d} - \frac{\left(be^2\right)\text{Subst}\left(\int \frac{x^3}{1+x^2} dx, x, c+dx\right)}{3d}$$
$$= \frac{e^2(c+dx)^3 \left(a+b\tan^{-1}(c+dx)\right)}{3d} - \frac{\left(be^2\right)\text{Subst}\left(\int \frac{x}{1+x} dx, x, (c+dx)^2\right)}{6d}$$
$$= \frac{e^2(c+dx)^3 \left(a+b\tan^{-1}(c+dx)\right)}{3d} - \frac{\left(be^2\right)\text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right) dx, x, (c+\right.}{6d}$$
$$= -\frac{be^2(c+dx)^2}{6d} + \frac{e^2(c+dx)^3 \left(a+b\tan^{-1}(c+dx)\right)}{3d} + \frac{be^2\log\left(1+(c+d\right.}{6d}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 0.81

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3\left(a+b\tan^{-1}(c+dx)\right)-\frac{1}{6}b\left((c+dx)^2-\log\left((c+dx)^2+1\right)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] (e^2*(((c + d*x)^3*(a + b*ArcTan[c + d*x]))/3 - (b*((c + d*x)^2 - Log[1 + (
c + d*x)^2]))/6))/d
```

fricas [B] time = 0.66, size = 129, normalized size = 1.93

$$\frac{2\,ad^3e^2x^3+(6\,ac-b)d^2e^2x^2+2\left(3\,ac^2-bc\right)de^2x+be^2\log\left(d^2x^2+2\,cdx+c^2+1\right)+2\left(bd^3e^2x^3+3\,bcd^2e^2x^2+3\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*d^3*e^2*x^3 + (6*a*c - b)*d^2*e^2*x^2 + 2*(3*a*c^2 - b*c)*d*e^2*x
+ b*e^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*(b*d^3*e^2*x^3 + 3*b*c*d^2*e^2
*x^2 + 3*b*c^2*d*e^2*x + b*c^3*e^2)*arctan(d*x + c))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.04, size = 161, normalized size = 2.40

$$\frac{d^2x^3a\,e^2}{3}+d\,x^2ac\,e^2+xa\,c^2e^2+\frac{a\,c^3e^2}{3d}+\frac{d^2\arctan\left(dx+c\right)x^3b\,e^2}{3}+d\arctan\left(dx+c\right)x^2bc\,e^2+\arctan\left(dx+c\right)xb\,c^2e^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arctan(d*x+c)),x)
```

```
[Out] 1/3*d^2*x^3*a*e^2+d*x^2*a*c*e^2+x*a*c^2*e^2+1/3/d*a*c^3*e^2+1/3*d^2*arctan(
d*x+c)*x^3*b*e^2+d*arctan(d*x+c)*x^2*b*c*e^2+arctan(d*x+c)*x*b*c^2*e^2+1/3/
d*arctan(d*x+c)*b*c^3*e^2-1/6*d*x^2*b*e^2-1/3*x*b*c*e^2-1/6/d*b*c^2*e^2+1/6
*b*e^2*ln(1+(d*x+c)^2)/d
```

maxima [B] time = 0.42, size = 238, normalized size = 3.55

$$\frac{1}{3}ad^2e^2x^3+acde^2x^2+\left(x^2\arctan\left(dx+c\right)-d\left(\frac{x}{d^2}+\frac{\left(c^2-1\right)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3}-\frac{c\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^3}\right)\right)bc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*a*d^2*e^2*x^3 + a*c*d*e^2*x^2 + (x^2*arctan(d*x + c) - d*(x/d^2 + (c^2
 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))
*b*c*d*e^2 + 1/6*(2*x^3*arctan(d*x + c) - d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 -
 3*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2
 + 1)/d^4))*b*d^2*e^2 + a*c^2*e^2*x + 1/2*(2*(d*x + c)*arctan(d*x + c) - lo
g((d*x + c)^2 + 1))*b*c^2*e^2/d
```

mupad [B] time = 1.17, size = 144, normalized size = 2.15

$$\frac{a\,d^2\,e^2\,x^3}{3}-\frac{b\,c\,e^2\,x}{3}+\frac{b\,e^2\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)}{6\,d}+a\,c^2\,e^2\,x-\frac{b\,d\,e^2\,x^2}{6}+b\,c^2\,e^2\,x\,\mathrm{atan}\left(c+d\,x\right)+a\,c\,d\,e^2\,x^2+\frac{b\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*atan(c + d*x)),x)
```

```
[Out] (a*d^2*e^2*x^3)/3 - (b*c*e^2*x)/3 + (b*e^2*log(c^2 + d^2*x^2 + 2*c*d*x + 1)
)/(6*d) + a*c^2*e^2*x - (b*d*e^2*x^2)/6 + b*c^2*e^2*x*atan(c + d*x) + a*c*d
*e^2*x^2 + (b*c^3*e^2*atan(c + d*x))/(3*d) + (b*d^2*e^2*x^3*atan(c + d*x))/
3 + b*c*d*e^2*x^2*atan(c + d*x)
```

sympy [A] time = 3.93, size = 182, normalized size = 2.72

$$\begin{cases} ac^2e^2x+acde^2x^2+\frac{ad^2e^2x^3}{3}+\frac{bc^3e^2\,\mathrm{atan}\,(c+dx)}{3d}+bc^2e^2x\,\mathrm{atan}\,(c+dx)+bcde^2x^2\,\mathrm{atan}\,(c+dx)-\frac{bce^2x}{3}+\frac{bd^2e^2x^3\,\mathrm{atan}}{3} \\ c^2e^2x\left(a+b\,\mathrm{atan}\,(c)\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*atan(d*x+c)),x)
```

```
[Out] Piecewise((a*c**2*e**2*x + a*c*d*e**2*x**2 + a*d**2*e**2*x**3/3 + b*c**3*e*
*2*atan(c + d*x)/(3*d) + b*c**2*e**2*x*atan(c + d*x) + b*c*d*e**2*x**2*atan
(c + d*x) - b*c*e**2*x/3 + b*d**2*e**2*x**3*atan(c + d*x)/3 - b*d*e**2*x**2
/6 + b*e**2*log(c/d + x - I/d)/(3*d) - I*b*e**2*atan(c + d*x)/(3*d), Ne(d,
0)), (c**2*e**2*x*(a + b*atan(c)), True))
```

3.3 $\int (ce+dex)\left(a+b\tan^{-1}(c+dx)\right)\,dx$

Optimal. Leaf size=48

$$\frac{e(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)}{2d}+\frac{be\tan^{-1}(c+dx)}{2d}-\frac{bex}{2}$$

```
[Out] -1/2*b*e*x+1/2*b*e*arctan(d*x+c)/d+1/2*e*(d*x+c)^2*(a+b*arctan(d*x+c))/d
```

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5043, 12, 4852, 321, 203}

$$\frac{e(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)}{2d}+\frac{be\tan^{-1}(c+dx)}{2d}-\frac{bex}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] -(b*e*x)/2 + (b*e*ArcTan[c + d*x])/(2*d) + (e*(c + d*x)^2*(a + b*ArcTan[c +
 d*x]))/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (ce+dex)\left(a+b\tan^{-1}(c+dx)\right)dx = \frac{\text{Subst}\left(\int ex\left(a+b\tan^{-1}(x)\right)dx,x,c+dx\right)}{d}$$
$$= \frac{e\,\text{Subst}\left(\int x\left(a+b\tan^{-1}(x)\right)dx,x,c+dx\right)}{d}$$
$$= \frac{e(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)}{2d} - \frac{(be)\,\text{Subst}\left(\int \frac{x^2}{1+x^2}dx,x,c+dx\right)}{2d}$$
$$= -\frac{1}{2}bex + \frac{e(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)}{2d} + \frac{(be)\,\text{Subst}\left(\int \frac{1}{1+x^2}dx,x,c\right.}{2d}$$
$$= -\frac{1}{2}bex + \frac{be\tan^{-1}(c+dx)}{2d} + \frac{e(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)}{2d}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.83

$$\frac{e\left((c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)+b\left(\tan^{-1}(c+dx)-dx\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcTan[c + d*x]),x]
[Out] (e*(b*(-(d*x) + ArcTan[c + d*x]) + (c + d*x)^2*(a + b*ArcTan[c + d*x])))/(2
*d)
```

fricas [A] time = 0.58, size = 60, normalized size = 1.25

$$\frac{ad^2ex^2 + (2\,ac-b)dex + \left(bd^2ex^2 + 2\,bcdex + \left(bc^2+b\right)e\right)\arctan\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c)),x, algorithm="fricas")
[Out] 1/2*(a*d^2*e*x^2 + (2*a*c - b)*d*e*x + (b*d^2*e*x^2 + 2*b*c*d*e*x + (b*c^2
+ b)*e)*arctan(d*x + c))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c)),x, algorithm="giac")
[Out] sage0*x
```

maple [B] time = 0.04, size = 92, normalized size = 1.92

$$\frac{aed\,x^2}{2}+xace+\frac{a\,c^2e}{2d}+\frac{d\arctan\left(dx+c\right)x^2be}{2}+\arctan\left(dx+c\right)xbce+\frac{\arctan\left(dx+c\right)b\,c^2e}{2d}-\frac{bex}{2}-\frac{bce}{2d}+\frac{be\arctan}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arctan(d*x+c)),x)
[Out] 1/2*a*e*d*x^2+x*a*c*e+1/2/d*a*c^2*e+1/2*d*arctan(d*x+c)*x^2*b*e+arctan(d*x+
c)*x*b*c*e+1/2/d*arctan(d*x+c)*b*c^2*e-1/2*b*e*x-1/2/d*b*c*e+1/2*b*e*arctan
(d*x+c)/d
```

maxima [B] time = 0.42, size = 120, normalized size = 2.50

$$\frac{1}{2}\,adex^2+\frac{1}{2}\left(x^2\arctan\left(dx+c\right)-d\left(\frac{x}{d^2}+\frac{\left(c^2-1\right)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3}-\frac{c\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^3}\right)\right)bde+acex+\frac{(2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*e*x^2 + 1/2*(x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2
*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*b*d*e + a*c*e*x
 + 1/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*b*c*e/d
```

mupad [B] time = 1.45, size = 73, normalized size = 1.52

$$a\,c\,e\,x-\frac{b\,e\,x}{2}+\frac{b\,e\,\mathrm{atan}\left(c+d\,x\right)}{2\,d}+\frac{a\,d\,e\,x^2}{2}+\frac{b\,c^2\,e\,\mathrm{atan}\left(c+d\,x\right)}{2\,d}+b\,c\,e\,x\,\mathrm{atan}\left(c+d\,x\right)+\frac{b\,d\,e\,x^2\,\mathrm{atan}\left(c+d\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*atan(c + d*x)),x)
```

```
[Out] a*c*e*x - (b*e*x)/2 + (b*e*atan(c + d*x))/(2*d) + (a*d*e*x^2)/2 + (b*c^2*e*
atan(c + d*x))/(2*d) + b*c*e*x*atan(c + d*x) + (b*d*e*x^2*atan(c + d*x))/2
```

sympy [A] time = 2.01, size = 95, normalized size = 1.98

$$\begin{cases} acex + \frac{adex^2}{2} + \frac{bc^2e\,\mathrm{atan}\,(c+dx)}{2d} + bcex\,\mathrm{atan}\,(c+dx) + \frac{bdex^2\,\mathrm{atan}\,(c+dx)}{2} - \frac{bex}{2} + \frac{be\,\mathrm{atan}\,(c+dx)}{2d} & \text{for } d \neq 0 \\ cex\,(a + b\,\mathrm{atan}\,(c)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*atan(d*x+c)),x)
```

```
[Out] Piecewise((a*c*e*x + a*d*e*x**2/2 + b*c**2*e*atan(c + d*x)/(2*d) + b*c*e*x*
atan(c + d*x) + b*d*e*x**2*atan(c + d*x)/2 - b*e*x/2 + b*e*atan(c + d*x)/(2
*d), Ne(d, 0)), (c*e*x*(a + b*atan(c)), True))
```

3.4 $\int \frac{a+b\tan^{-1}(c+dx)}{ce+dex}\,dx$

Optimal. Leaf size=63

$$\frac{a\log(c+dx)}{de}+\frac{ib\text{Li}_2(-i(c+dx))}{2de}-\frac{ib\text{Li}_2(i(c+dx))}{2de}$$

```
[Out] a*ln(d*x+c)/d/e+1/2*I*b*polylog(2,-I*(d*x+c))/d/e-1/2*I*b*polylog(2,I*(d*x+
c))/d/e
```

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5043, 12, 4848, 2391}

$$\frac{ib\text{PolyLog}(2,-i(c+dx))}{2de}-\frac{ib\text{PolyLog}(2,i(c+dx))}{2de}+\frac{a\log(c+dx)}{de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])/(c*e + d*e*x),x]
```

```
[Out] (a*Log[c + d*x])/(d*e) + ((I/2)*b*PolyLog[2, (-I)*(c + d*x)])/(d*e) - ((I/2
)*b*PolyLog[2, I*(c + d*x)])/(d*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan^{-1}(c+dx)}{ce+dex}\,dx &= \frac{\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{ex}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x}\,dx,x,c+dx\right)}{de}\\ &= \frac{a\log(c+dx)}{de}+\frac{(ib)\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,c+dx\right)}{2de}-\frac{(ib)\,\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx,x,c+dx\right)}{2de}\\ &= \frac{a\log(c+dx)}{de}+\frac{ib\text{Li}_2(-i(c+dx))}{2de}-\frac{ib\text{Li}_2(i(c+dx))}{2de}\end{aligned}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 0.83

$$\frac{a\log(c+dx)+\frac{1}{2}ib\text{Li}_2(-i(c+dx))-\frac{1}{2}ib\text{Li}_2(i(c+dx))}{de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])/(c*e + d*e*x),x]
```

```
[Out] (a*Log[c + d*x] + (I/2)*b*PolyLog[2, (-I)*(c + d*x)] - (I/2)*b*PolyLog[2, I
*(c + d*x)])/(d*e)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\arctan(dx+c)+a}{dex+ce},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(d*x + c) + a)/(d*e*x + c*e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.06, size = 132, normalized size = 2.10

$$\frac{a\ln(dx+c)}{de}+\frac{b\ln(dx+c)\arctan(dx+c)}{de}+\frac{ib\ln(dx+c)\ln(1+i(dx+c))}{2de}-\frac{ib\ln(dx+c)\ln(1-i(dx+c))}{2de}+\frac{ib\,\mathrm{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))/(d*e*x+c*e),x)
```

```
[Out] a*ln(d*x+c)/d/e+1/d*b/e*ln(d*x+c)*arctan(d*x+c)+1/2*I/d*b/e*ln(d*x+c)*ln(1+
I*(d*x+c))-1/2*I/d*b/e*ln(d*x+c)*ln(1-I*(d*x+c))+1/2*I/d*b/e*dilog(1+I*(d*x
+c))-1/2*I/d*b/e*dilog(1-I*(d*x+c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$2b\int\frac{\arctan(dx+c)}{2(dex+ce)}\,dx+\frac{a\log(dex+ce)}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] 2*b*integrate(1/2*arctan(d*x + c)/(d*e*x + c*e), x) + a*log(d*e*x + c*e)/(d
*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{a+b\,\mathrm{atan}(c+d\,x)}{c\,e+d\,e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))/(c*e + d*e*x),x)
```

```
[Out] int((a + b*atan(c + d*x))/(c*e + d*e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c+dx}\,dx + \int \frac{b\,\mathrm{atan}\,(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))/(d*e*x+c*e),x)
```

```
[Out] (Integral(a/(c + d*x), x) + Integral(b*atan(c + d*x)/(c + d*x), x))/e
```

3.5 $\int \frac{a+b\tan^{-1}(c+dx)}{(ce+dex)^2}\,dx$

Optimal. Leaf size=61

$$-\frac{a+b\tan^{-1}(c+dx)}{de^2(c+dx)}+\frac{b\log(c+dx)}{de^2}-\frac{b\log\left((c+dx)^2+1\right)}{2de^2}$$

```
[Out] (-a-b*arctan(d*x+c))/d/e^2/(d*x+c)+b*ln(d*x+c)/d/e^2-1/2*b*ln(1+(d*x+c)^2)/
d/e^2
```

Rubi [A] time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5043, 12, 4852, 266, 36, 29, 31}

$$-\frac{a+b\tan^{-1}(c+dx)}{de^2(c+dx)}+\frac{b\log(c+dx)}{de^2}-\frac{b\log\left((c+dx)^2+1\right)}{2de^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])/(c*e + d*e*x)^2,x]
```

```
[Out] -((a + b*ArcTan[c + d*x])/(d*e^2*(c + d*x))) + (b*Log[c + d*x])/(d*e^2) - (
b*Log[1 + (c + d*x)^2])/(2*d*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tan^{-1}(c+dx)}{(ce+dex)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{e^2x^2}\,dx, x, c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x^2}\,dx, x, c+dx\right)}{de^2}\\
&= -\frac{a+b\tan^{-1}(c+dx)}{de^2(c+dx)} + \frac{b\,\text{Subst}\left(\int \frac{1}{x(1+x^2)}\,dx, x, c+dx\right)}{de^2}\\
&= -\frac{a+b\tan^{-1}(c+dx)}{de^2(c+dx)} + \frac{b\,\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, (c+dx)^2\right)}{2de^2}\\
&= -\frac{a+b\tan^{-1}(c+dx)}{de^2(c+dx)} + \frac{b\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, (c+dx)^2\right)}{2de^2} - \frac{b\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, (c+\right.}{2de^2}\\
&= -\frac{a+b\tan^{-1}(c+dx)}{de^2(c+dx)} + \frac{b\log(c+dx)}{de^2} - \frac{b\log\left(1+(c+dx)^2\right)}{2de^2}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.82

$$\frac{-\frac{a+b\tan^{-1}(c+dx)}{c+dx} + b\log(c+dx) - \frac{1}{2}b\log\left((c+dx)^2+1\right)}{de^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])/(c*e + d*e*x)^2,x]
```

```
[Out] (-((a + b*ArcTan[c + d*x])/(c + d*x)) + b*Log[c + d*x] - (b*Log[1 + (c + d*
x)^2])/2)/(d*e^2)
```

fricas [A] time = 0.51, size = 75, normalized size = 1.23

$$-\frac{2\,b\arctan\left(dx+c\right) + \left(bdx+bc\right)\log\left(d^2x^2+2\,cdx+c^2+1\right) - 2\left(bdx+bc\right)\log\left(dx+c\right) + 2\,a}{2\left(d^2e^2x+cde^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*arctan(d*x + c) + (b*d*x + b*c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)
- 2*(b*d*x + b*c)*log(d*x + c) + 2*a)/(d^2*e^2*x + c*d*e^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.04, size = 73, normalized size = 1.20

$$-\frac{a}{d\,e^2\,(dx+c)}-\frac{b\arctan(dx+c)}{d\,e^2\,(dx+c)}+\frac{b\ln(dx+c)}{d\,e^2}-\frac{b\ln\left(1+(dx+c)^2\right)}{2d\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))/(d*e*x+c*e)^2,x)
```

```
[Out] -1/d*a/e^2/(d*x+c)-1/d*b/e^2/(d*x+c)*arctan(d*x+c)+b*ln(d*x+c)/d/e^2-1/2*b*
ln(1+(d*x+c)^2)/d/e^2
```

maxima [A] time = 0.32, size = 92, normalized size = 1.51

$$-\frac{1}{2}\left(d\left(\frac{\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^2e^2}-\frac{2\,\log(dx+c)}{d^2e^2}\right)+\frac{2\,\arctan(dx+c)}{d^2e^2x+cde^2}\right)b-\frac{a}{d^2e^2x+cde^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(d*(log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*e^2) - 2*log(d*x + c)/(d^2*e
^2)) + 2*arctan(d*x + c)/(d^2*e^2*x + c*d*e^2))*b - a/(d^2*e^2*x + c*d*e^2)
```

mupad [B] time = 0.67, size = 88, normalized size = 1.44

$$\frac{b\,\ln(c+d\,x)}{d\,e^2}-\frac{b\,\mathrm{atan}(c+d\,x)}{x\,d^2\,e^2+c\,d\,e^2}-\frac{b\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)}{2\,d\,e^2}-\frac{a}{x\,d^2\,e^2+c\,d\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))/(c*e + d*e*x)^2,x)
```

```
[Out] (b*log(c + d*x))/(d*e^2) - (b*atan(c + d*x))/(d^2*e^2*x + c*d*e^2) - (b*log
(c^2 + d^2*x^2 + 2*c*d*x + 1))/(2*d*e^2) - a/(d^2*e^2*x + c*d*e^2)
```

sympy [A] time = 6.89, size = 284, normalized size = 4.66

$$\begin{cases} \frac{\tilde{\infty}a}{e^2x} \\ \frac{x(a+b\,\mathrm{atan}(c))}{c^2e^2} \\ \frac{-\frac{a}{x}+bd\log(x)-\frac{bd\log\left(x^2+\frac{1}{d^2}\right)}{2}-\frac{b\,\mathrm{atan}(dx)}{x}}{d^2e^2} \\ \frac{a}{-cde^2-d^2e^2x}-\frac{bc\log\left(\frac{c}{d}+x\right)}{-cde^2-d^2e^2x}+\frac{bc\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{-cde^2-d^2e^2x}-\frac{ibc\,\mathrm{atan}(c+dx)}{-cde^2-d^2e^2x}-\frac{bdx\log\left(\frac{c}{d}+x\right)}{-cde^2-d^2e^2x}+\frac{bdx\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{-cde^2-d^2e^2x}-\frac{ibdx\,\mathrm{atan}(c+dx)}{-cde^2-d^2e^2x}+\frac{b\,\mathrm{atan}(c+dx)}{-cde^2-d^2e^2x} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))/(d*e*x+c*e)**2,x)
```

```
[Out] Piecewise((zoo*a/(e**2*x), Eq(c, 0) & Eq(d, 0)), (x*(a + b*atan(c))/(c**2*e
**2), Eq(d, 0)), ((-a/x + b*d*log(x) - b*d*log(x**2 + d**(-2))/2 - b*atan(d
*x)/x)/(d**2*e**2), Eq(c, 0)), (a/(-c*d*e**2 - d**2*e**2*x) - b*c*log(c/d +
 x)/(-c*d*e**2 - d**2*e**2*x) + b*c*log(c/d + x - I/d)/(-c*d*e**2 - d**2*e*
*2*x) - I*b*c*atan(c + d*x)/(-c*d*e**2 - d**2*e**2*x) - b*d*x*log(c/d + x)/
(-c*d*e**2 - d**2*e**2*x) + b*d*x*log(c/d + x - I/d)/(-c*d*e**2 - d**2*e**2
*x) - I*b*d*x*atan(c + d*x)/(-c*d*e**2 - d**2*e**2*x) + b*atan(c + d*x)/(-c
*d*e**2 - d**2*e**2*x), True))
```

3.6 $\int \frac{a+b\tan^{-1}(c+dx)}{(ce+dex)^3}\,dx$

Optimal. Leaf size=63

$$-\frac{a+b\tan^{-1}(c+dx)}{2de^3(c+dx)^2}-\frac{b}{2de^3(c+dx)}-\frac{b\tan^{-1}(c+dx)}{2de^3}$$

```
[Out] -1/2*b/d/e^3/(d*x+c)-1/2*b*arctan(d*x+c)/d/e^3+1/2*(-a-b*arctan(d*x+c))/d/e
^3/(d*x+c)^2
```

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5043, 12, 4852, 325, 203}

$$-\frac{a+b\tan^{-1}(c+dx)}{2de^3(c+dx)^2}-\frac{b}{2de^3(c+dx)}-\frac{b\tan^{-1}(c+dx)}{2de^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])/(c*e + d*e*x)^3,x]
```

```
[Out] -b/(2*d*e^3*(c + d*x)) - (b*ArcTan[c + d*x])/(2*d*e^3) - (a + b*ArcTan[c +
d*x])/(2*d*e^3*(c + d*x)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\tan^{-1}(c+dx)}{(ce+dex)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{e^3x^3}\,dx,x,c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x^3}\,dx,x,c+dx\right)}{de^3}\\
&= -\frac{a+b\tan^{-1}(c+dx)}{2de^3(c+dx)^2}+\frac{b\,\text{Subst}\left(\int \frac{1}{x^2(1+x^2)}\,dx,x,c+dx\right)}{2de^3}\\
&= -\frac{b}{2de^3(c+dx)}-\frac{a+b\tan^{-1}(c+dx)}{2de^3(c+dx)^2}-\frac{b\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,c+dx\right)}{2de^3}\\
&= -\frac{b}{2de^3(c+dx)}-\frac{b\tan^{-1}(c+dx)}{2de^3}-\frac{a+b\tan^{-1}(c+dx)}{2de^3(c+dx)^2}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 51, normalized size = 0.81

$$
-\frac{a+b(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-(c+dx)^2\right)+b\tan^{-1}(c+dx)}{2de^3(c+dx)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])/(c*e + d*e*x)^3,x]
```

```
[Out] -1/2*(a + b*ArcTan[c + d*x] + b*(c + d*x)*Hypergeometric2F1[-1/2, 1, 1/2, -
(c + d*x)^2])/(d*e^3*(c + d*x)^2)
```

fricas [A] time = 0.57, size = 70, normalized size = 1.11

$$
-\frac{bdx+bc+\left(bd^2x^2+2\,bcdx+bc^2+b\right)\arctan\left(dx+c\right)+a}{2\left(d^3e^3x^2+2\,cd^2e^3x+c^2de^3\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b*d*x + b*c + (b*d^2*x^2 + 2*b*c*d*x + b*c^2 + b)*arctan(d*x + c) + a
)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
sage_0x
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.04, size = 71, normalized size = 1.13

$$
-\frac{a}{2d\,e^3\left(dx+c\right)^2}-\frac{b\arctan\left(dx+c\right)}{2d\,e^3\left(dx+c\right)^2}-\frac{b}{2d\,e^3\left(dx+c\right)}-\frac{b\arctan\left(dx+c\right)}{2d\,e^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))/(d*e*x+c*e)^3,x)
```

```
[Out] -1/2/d*a/e^3/(d*x+c)^2-1/2/d*b/e^3/(d*x+c)^2*arctan(d*x+c)-1/2*b/d/e^3/(d*x
+c)-1/2*b*arctan(d*x+c)/d/e^3
```

maxima [B] time = 0.42, size = 120, normalized size = 1.90

$$-\frac{1}{2}\left(d\left(\frac{1}{d^3e^3x+cd^2e^3}+\frac{\arctan\left(\frac{d^2x+cd}{d}\right)}{d^2e^3}\right)+\frac{\arctan\left(dx+c\right)}{d^3e^3x^2+2\,cd^2e^3x+c^2de^3}\right)b-\frac{a}{2\left(d^3e^3x^2+2\,cd^2e^3x+c^2de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(d*(1/(d^3*e^3*x + c*d^2*e^3) + arctan((d^2*x + c*d)/d)/(d^2*e^3)) + a
rctan(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3))*b - 1/2*a/(d^3*e^
3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

mupad [B] time = 0.75, size = 103, normalized size = 1.63

$$-\frac{\frac{a+b\,c}{d}+b\,x}{2\,c^2\,e^3+4\,c\,d\,e^3\,x+2\,d^2\,e^3\,x^2}-\frac{b\,\mathrm{atan}\left(\frac{b\,c+b\,d\,x}{b}\right)}{2\,d\,e^3}-\frac{b\,\mathrm{atan}\left(c+d\,x\right)}{2\,d^3\,e^3\,\left(x^2+\frac{c^2}{d^2}+\frac{2\,c\,x}{d}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))/(c*e + d*e*x)^3,x)
```

```
[Out] - ((a + b*c)/d + b*x)/(2*c^2*e^3 + 2*d^2*e^3*x^2 + 4*c*d*e^3*x) - (b*atan((
b*c + b*d*x)/b))/(2*d*e^3) - (b*atan(c + d*x))/(2*d^3*e^3*(x^2 + c^2/d^2 +
(2*c*x)/d))
```

sympy [A] time = 12.39, size = 314, normalized size = 4.98

$$\begin{cases} -\frac{a}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2}-\frac{bc^2\operatorname{atan}(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2}-\frac{2bcdx\operatorname{atan}(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2}-\frac{bc}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2}-\frac{bd^2x^2\operatorname{atan}(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x} \\ \frac{x(a+b\operatorname{atan}(c))}{c^3e^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))/(d*e*x+c*e)**3,x)
```

```
[Out] Piecewise((-a/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b*c**2
*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*b*c
*d*x*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b
*c/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b*d**2*x**2*atan(
c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b*d*x/(2*c*
*2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b*atan(c + d*x)/(2*c**2*d
*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2), Ne(d, 0)), (x*(a + b*atan(c))/
(c**3*e**3), True))
```

3.7 $\int (ce + dex)^3 \left(a + b\tan^{-1}(c + dx)\right)^2 dx$

Optimal. Leaf size=157

$$\frac{e^3(c+dx)^4\left(a+b\tan^{-1}(c+dx)\right)^2}{4d}-\frac{be^3(c+dx)^3\left(a+b\tan^{-1}(c+dx)\right)}{6d}-\frac{e^3\left(a+b\tan^{-1}(c+dx)\right)^2}{4d}+\frac{1}{2}abe^3x+\frac{b^2e^3(c}{1}$$

```
[Out] 1/2*a*b*e^3*x+1/12*b^2*e^3*(d*x+c)^2/d+1/2*b^2*e^3*(d*x+c)*arctan(d*x+c)/d-
1/6*b*e^3*(d*x+c)^3*(a+b*arctan(d*x+c))/d-1/4*e^3*(a+b*arctan(d*x+c))^2/d+1
/4*e^3*(d*x+c)^4*(a+b*arctan(d*x+c))^2/d-1/3*b^2*e^3*ln(1+(d*x+c)^2)/d
```

Rubi [A] time = 0.22, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5043, 12, 4852, 4916, 266, 43, 4846, 260, 4884}

$$\frac{e^3(c+dx)^4\left(a+b\tan^{-1}(c+dx)\right)^2}{4d}-\frac{be^3(c+dx)^3\left(a+b\tan^{-1}(c+dx)\right)}{6d}-\frac{e^3\left(a+b\tan^{-1}(c+dx)\right)^2}{4d}+\frac{1}{2}abe^3x+\frac{b^2e^3(c}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (a*b*e^3*x)/2 + (b^2*e^3*(c + d*x)^2)/(12*d) + (b^2*e^3*(c + d*x)*ArcTan[c
+ d*x])/(2*d) - (b*e^3*(c + d*x)^3*(a + b*ArcTan[c + d*x]))/(6*d) - (e^3*(a
 + b*ArcTan[c + d*x])^2)/(4*d) + (e^3*(c + d*x)^4*(a + b*ArcTan[c + d*x])^2
)/(4*d) - (b^2*e^3*Log[1 + (c + d*x)^2])/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
```

```
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\tan^{-1}(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int e^3x^3 \left(a+b\tan^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int x^3 \left(a+b\tan^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4\left(a+b\tan^{-1}(c+dx)\right)^2}{4d} - \frac{\left(be^3\right)\text{Subst}\left(\int \frac{x^4\left(a+b\tan^{-1}(x)\right)}{1+x^2} d\right.}{2d}$$

$$= \frac{e^3(c+dx)^4\left(a+b\tan^{-1}(c+dx)\right)^2}{4d} - \frac{\left(be^3\right)\text{Subst}\left(\int x^2\left(a+b\tan^{-1}(\right.\right.}{2d}$$

$$= -\frac{be^3(c+dx)^3\left(a+b\tan^{-1}(c+dx)\right)}{6d} + \frac{e^3(c+dx)^4\left(a+b\tan^{-1}(c+\right.}{4d}$$

$$= \frac{1}{2}abe^3x - \frac{be^3(c+dx)^3\left(a+b\tan^{-1}(c+dx)\right)}{6d} - \frac{e^3\left(a+b\tan^{-1}(c+d\right.}{4d}$$

$$= \frac{1}{2}abe^3x + \frac{b^2e^3(c+dx)\tan^{-1}(c+dx)}{2d} - \frac{be^3(c+dx)^3\left(a+b\tan^{-1}(c+\right.}{6d}$$

$$= \frac{1}{2}abe^3x + \frac{b^2e^3(c+dx)^2}{12d} + \frac{b^2e^3(c+dx)\tan^{-1}(c+dx)}{2d} - \frac{be^3(c+dx)^3}{}$$

Mathematica [A] time = 0.12, size = 216, normalized size = 1.38

$$e^3\left((c+dx)\left(3a^2(c+dx)^3-2ab\left(c^2+2cdx+d^2x^2-3\right)+b^2(c+dx)\right)+2b\tan^{-1}(c+dx)\left(3a\left(c^4+4c^3dx+6c^2\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (e^3*((c + d*x)*(b^2*(c + d*x) + 3*a^2*(c + d*x)^3 - 2*a*b*(-3 + c^2 + 2*c*
d*x + d^2*x^2)) + 2*b*(-(b*(-3*c + c^3 - 3*d*x + 3*c^2*d*x + 3*c*d^2*x^2 +
d^3*x^3)) + 3*a*(-1 + c^4 + 4*c^3*d*x + 6*c^2*d^2*x^2 + 4*c*d^3*x^3 + d^4*x
^4))*ArcTan[c + d*x] + 3*b^2*(-1 + c^4 + 4*c^3*d*x + 6*c^2*d^2*x^2 + 4*c*d^
3*x^3 + d^4*x^4)*ArcTan[c + d*x]^2 - 4*b^2*Log[1 + (c + d*x)^2]))/(12*d)
```

fricas [B] time = 0.65, size = 337, normalized size = 2.15

$$\frac{3\,a^2d^4e^3x^4 + 2\left(6\,a^2c - ab\right)d^3e^3x^3 + \left(18\,a^2c^2 - 6\,abc + b^2\right)d^2e^3x^2 + 2\left(6\,a^2c^3 - 3\,abc^2 + b^2c + 3\,ab\right)de^3x - 4\,b^2e^3\,\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arctan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^2*d^4*e^3*x^4 + 2*(6*a^2*c - a*b)*d^3*e^3*x^3 + (18*a^2*c^2 - 6*a
*b*c + b^2)*d^2*e^3*x^2 + 2*(6*a^2*c^3 - 3*a*b*c^2 + b^2*c + 3*a*b)*d*e^3*x
 - 4*b^2*e^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 3*(b^2*d^4*e^3*x^4 + 4*b^2*
c*d^3*e^3*x^3 + 6*b^2*c^2*d^2*e^3*x^2 + 4*b^2*c^3*d*e^3*x + (b^2*c^4 - b^2)
*e^3)*arctan(d*x + c)^2 + 2*(3*a*b*d^4*e^3*x^4 + (12*a*b*c - b^2)*d^3*e^3*x
^3 + 3*(6*a*b*c^2 - b^2*c)*d^2*e^3*x^2 + 3*(4*a*b*c^3 - b^2*c^2 + b^2)*d*e^
3*x + (3*a*b*c^4 - b^2*c^3 + 3*b^2*c - 3*a*b)*e^3)*arctan(d*x + c))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arctan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.06, size = 543, normalized size = 3.46

$$\frac{\arctan\left(dx+c\right)b^2c\,e^3}{2d} - \frac{e^3ab\arctan\left(dx+c\right)}{2d} - \frac{\arctan\left(dx+c\right)x\,b^2c^2e^3}{2} - \frac{xab\,c^2e^3}{2} - \frac{d^2x^3ab\,e^3}{6} + \frac{3d\,x^2a^2c^2e^3}{2} + d^2x^3a^2c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arctan(d*x+c))^2,x)
```

```
[Out] 1/2/d*arctan(d*x+c)*b^2*c*e^3-1/2/d*e^3*a*b*arctan(d*x+c)-1/2*arctan(d*x+c)
*x*b^2*c^2*e^3-1/2*x*a*b*c^2*e^3-1/6*d^2*x^3*a*b*e^3+3/2*d*x^2*a^2*c^2*e^3+
d^2*x^3*a^2*c*e^3+1/4/d*arctan(d*x+c)^2*b^2*c^4*e^3-1/6/d*arctan(d*x+c)*b^2
*c^3*e^3+1/4*d^3*arctan(d*x+c)^2*x^4*b^2*e^3-1/6*d^2*arctan(d*x+c)*x^3*b^2*
e^3-1/6/d*a*b*c^3*e^3+1/2/d*a*b*c*e^3+x*a^2*c^3*e^3+1/12*d*x^2*b^2*e^3+1/2*
arctan(d*x+c)*x*b^2*e^3-1/4/d*e^3*b^2*arctan(d*x+c)^2+1/4*d^3*x^4*a^2*e^3+1
/4/d*a^2*c^4*e^3+1/6*x*b^2*c*e^3+1/12/d*b^2*c^2*e^3+1/2*a*b*e^3*x-1/3*b^2*e
^3*ln(1+(d*x+c)^2)/d+1/2*d^3*arctan(d*x+c)*x^4*a*b*e^3-1/2*d*x^2*a*b*c*e^3+
d^2*arctan(d*x+c)^2*x^3*b^2*c*e^3+1/2/d*arctan(d*x+c)*a*b*c^4*e^3+3/2*d*arc
tan(d*x+c)^2*x^2*b^2*c^2*e^3-1/2*d*arctan(d*x+c)*x^2*b^2*c*e^3+2*arctan(d*x
+c)*x*a*b*c^3*e^3+2*d^2*arctan(d*x+c)*x^3*a*b*c*e^3+3*d*arctan(d*x+c)*x^2*a
*b*c^2*e^3+arctan(d*x+c)^2*x*b^2*c^3*e^3
```

maxima [B] time = 1.80, size = 597, normalized size = 3.80

$$\frac{1}{4}\,a^2d^3e^3x^4 + a^2cd^2e^3x^3 + \frac{3}{2}\,a^2c^2de^3x^2 + 3\left(x^2\arctan\left(dx+c\right) - d\left(\frac{x}{d^2} + \frac{\left(c^2-1\right)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3} - \frac{c\log\left(d^2x^2 + 2\,cd\right.}{d^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arctan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*d^3*e^3*x^4 + a^2*c*d^2*e^3*x^3 + 3/2*a^2*c^2*d*e^3*x^2 + 3*(x^2*ar
ctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^
2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a*b*c^2*d*e^3 + (2*x^3*arctan(d*x + c) - d
*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*c^2
- 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*a*b*c*d^2*e^3 + 1/6*(3*x^4*arct
an(d*x + c) - d*((d^2*x^3 - 3*c*d*x^2 + 3*(3*c^2 - 1)*x)/d^4 + 3*(c^4 - 6*c
^2 + 1)*arctan((d^2*x + c*d)/d)/d^5 - 6*(c^3 - c)*log(d^2*x^2 + 2*c*d*x + c
^2 + 1)/d^5))*a*b*d^3*e^3 + a^2*c^3*e^3*x + (2*(d*x + c)*arctan(d*x + c) -
log((d*x + c)^2 + 1))*a*b*c^3*e^3/d + 1/12*(b^2*d^2*e^3*x^2 + 2*b^2*c*d*e^3
*x - 4*b^2*e^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 3*(b^2*d^4*e^3*x^4 + 4*b^
2*c*d^3*e^3*x^3 + 6*b^2*c^2*d^2*e^3*x^2 + 4*b^2*c^3*d*e^3*x + (b^2*c^4 - b^
2)*e^3)*arctan(d*x + c)^2 - 2*(b^2*d^3*e^3*x^3 + 3*b^2*c*d^2*e^3*x^2 + 3*(b
^2*c^2 - b^2)*d*e^3*x + (b^2*c^3 - 3*b^2*c)*e^3)*arctan(d*x + c))/d
```

mupad [B] time = 3.26, size = 633, normalized size = 4.03

$$x\left(\frac{c\,e^3\,\left(20\,a^2\,c^2+6\,a^2-6\,a\,b\,c+b^2\right)}{2}+\frac{\left(6\,c^2+6\right)\,\left(2\,a^2\,c\,d^2\,e^3+\frac{a\,d^2\,e^3\,(b-10\,a\,c)}{2}\right)}{6\,d^2}-\frac{2\,c\,\left(\frac{2\,c\,\left(2\,a^2\,c\,d^2\,e^3+\frac{a\,d^2\,e^3\,(b-10\,a\,c)}{2}\right.}{d}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*atan(c + d*x))^2,x)
```

```
[Out] x*((c*e^3*(6*a^2 + b^2 + 20*a^2*c^2 - 6*a*b*c))/2 + ((6*c^2 + 6)*(2*a^2*c*d
^2*e^3 + (a*d^2*e^3*(b - 10*a*c))/2))/(6*d^2) - (2*c*((2*c*(2*a^2*c*d^2*e^3
 + (a*d^2*e^3*(b - 10*a*c))/2))/d + (d*e^3*(6*a^2 + b^2 + 60*a^2*c^2 - 12*a
*b*c))/6 - (a^2*d*e^3*(6*c^2 + 6))/6))/d) + x^2*((c*(2*a^2*c*d^2*e^3 + (a*d
^2*e^3*(b - 10*a*c))/2))/d + (d*e^3*(6*a^2 + b^2 + 60*a^2*c^2 - 12*a*b*c))/
12 - (a^2*d*e^3*(6*c^2 + 6))/12) - x^3*((2*a^2*c*d^2*e^3)/3 + (a*d^2*e^3*(b
 - 10*a*c))/6) + atan(c + d*x)^2*(b^2*c^3*e^3*x - (b^2*e^3 - b^2*c^4*e^3)/(
4*d) + (b^2*d^3*e^3*x^4)/4 + (3*b^2*c^2*d*e^3*x^2)/2 + b^2*c*d^2*e^3*x^3) -
 d^2*atan(c + d*x)*(x^3*((b^2*e^3)/6 - 2*a*b*c*e^3) - (x*(b^2*e^3 - b^2*c^2
*e^3 + 4*a*b*c^3*e^3))/(2*d^2) + (x^2*(b^2*c*e^3 - 6*a*b*c^2*e^3))/(2*d) -
(a*b*d*e^3*x^4)/2) + (a^2*d^3*e^3*x^4)/4 - (b^2*e^3*log(c^2 + d^2*x^2 + 2*c
*d*x + 1))/(3*d) + (b*e^3*atan(((b*c*e^3*(3*a - 3*b*c - 3*a*c^4 + b*c^3))/6
 + (b*d*e^3*x*(3*a - 3*b*c - 3*a*c^4 + b*c^3))/6)/((b^2*c*e^3)/2 - (b^2*c^3
*e^3)/6 - (a*b*e^3)/2 + (a*b*c^4*e^3)/2))*(3*a - 3*b*c - 3*a*c^4 + b*c^3))/
(6*d)
```

sympy [A] time = 11.42, size = 583, normalized size = 3.71

$$\begin{cases} a^2c^3e^3x+\frac{3a^2c^2de^3x^2}{2}+a^2cd^2e^3x^3+\frac{a^2d^3e^3x^4}{4}+\frac{abc^4e^3\operatorname{atan}(c+dx)}{2d}+2abc^3e^3x\operatorname{atan}(c+dx)+3abc^2de^3x^2\operatorname{atan}(c+ \\ c^3e^3x\left(a+b\operatorname{atan}(c)\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*atan(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c**3*e**3*x + 3*a**2*c**2*d*e**3*x**2/2 + a**2*c*d**2*e**3*
x**3 + a**2*d**3*e**3*x**4/4 + a*b*c**4*e**3*atan(c + d*x)/(2*d) + 2*a*b*c*
*3*e**3*x*atan(c + d*x) + 3*a*b*c**2*d*e**3*x**2*atan(c + d*x) - a*b*c**2*e
```

```
**3*x/2 + 2*a*b*c*d**2*e**3*x**3*atan(c + d*x) - a*b*c*d*e**3*x**2/2 + a*b*
d**3*e**3*x**4*atan(c + d*x)/2 - a*b*d**2*e**3*x**3/6 + a*b*e**3*x/2 - a*b*
e**3*atan(c + d*x)/(2*d) + b**2*c**4*e**3*atan(c + d*x)**2/(4*d) + b**2*c**
3*e**3*x*atan(c + d*x)**2 - b**2*c**3*e**3*atan(c + d*x)/(6*d) + 3*b**2*c**
2*d*e**3*x**2*atan(c + d*x)**2/2 - b**2*c**2*e**3*x*atan(c + d*x)/2 + b**2*
c*d**2*e**3*x**3*atan(c + d*x)**2 - b**2*c*d*e**3*x**2*atan(c + d*x)/2 + b*
*2*c*e**3*x/6 + b**2*c*e**3*atan(c + d*x)/(2*d) + b**2*d**3*e**3*x**4*atan(
c + d*x)**2/4 - b**2*d**2*e**3*x**3*atan(c + d*x)/6 + b**2*d*e**3*x**2/12 +
 b**2*e**3*x*atan(c + d*x)/2 - 2*b**2*e**3*log(c/d + x - I/d)/(3*d) - b**2*
e**3*atan(c + d*x)**2/(4*d) + 2*I*b**2*e**3*atan(c + d*x)/(3*d), Ne(d, 0)),
 (c**3*e**3*x*(a + b*atan(c))**2, True))
```

3.8 $\int (ce+dex)^2 \left(a+b\tan^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=183

$$\frac{e^2(c+dx)^3\left(a+b\tan^{-1}(c+dx)\right)^2}{3d}-\frac{be^2(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)}{3d}-\frac{ie^2\left(a+b\tan^{-1}(c+dx)\right)^2}{3d}-\frac{2be^2\log\left(\frac{}{1}\right.}{}$$

```
[Out] 1/3*b^2*e^2*x-1/3*b^2*e^2*arctan(d*x+c)/d-1/3*b*e^2*(d*x+c)^2*(a+b*arctan(d
*x+c))/d-1/3*I*e^2*(a+b*arctan(d*x+c))^2/d+1/3*e^2*(d*x+c)^3*(a+b*arctan(d*
x+c))^2/d-2/3*b*e^2*(a+b*arctan(d*x+c))*ln(2/(1+I*(d*x+c)))/d-1/3*I*b^2*e^2
*polylog(2,1-2/(1+I*(d*x+c)))/d
```

Rubi [A] time = 0.22, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5043, 12, 4852, 4916, 321, 203, 4920, 4854, 2402, 2315}

$$-\frac{ib^2e^2\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{3d}+\frac{e^2(c+dx)^3\left(a+b\tan^{-1}(c+dx)\right)^2}{3d}-\frac{be^2(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)}{3d}-ie^2\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (b^2*e^2*x)/3 - (b^2*e^2*ArcTan[c + d*x])/(3*d) - (b*e^2*(c + d*x)^2*(a + b
*ArcTan[c + d*x]))/(3*d) - ((I/3)*e^2*(a + b*ArcTan[c + d*x])^2)/d + (e^2*(
c + d*x)^3*(a + b*ArcTan[c + d*x])^2)/(3*d) - (2*b*e^2*(a + b*ArcTan[c + d*
x])*Log[2/(1 + I*(c + d*x))])/(3*d) - ((I/3)*b^2*e^2*PolyLog[2, 1 - 2/(1 +
I*(c + d*x))])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (ce+dex)^2 \left(a+b\tan^{-1}(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int e^2x^2\left(a+b\tan^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$
$$= \frac{e^2\,\text{Subst}\left(\int x^2\left(a+b\tan^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$
$$= \frac{e^2(c+dx)^3\left(a+b\tan^{-1}(c+dx)\right)^2}{3d} - \frac{\left(2be^2\right)\text{Subst}\left(\int \frac{x^3\left(a+b\tan^{-1}(x)\right)}{1+x^2}\right.}{3d}$$
$$= \frac{e^2(c+dx)^3\left(a+b\tan^{-1}(c+dx)\right)^2}{3d} - \frac{\left(2be^2\right)\text{Subst}\left(\int x\left(a+b\tan^{-1}\right.\right.}{3d}$$
$$= -\frac{be^2(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)}{3d} - \frac{ie^2\left(a+b\tan^{-1}(c+dx)\right)^2}{3d} + $$
$$= \frac{1}{3}b^2e^2x - \frac{be^2(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)}{3d} - \frac{ie^2\left(a+b\tan^{-1}(c+\right.}{3d}$$
$$= \frac{1}{3}b^2e^2x - \frac{b^2e^2\tan^{-1}(c+dx)}{3d} - \frac{be^2(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)}{3d} -$$
$$= \frac{1}{3}b^2e^2x - \frac{b^2e^2\tan^{-1}(c+dx)}{3d} - \frac{be^2(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)}{3d} -$$

Mathematica [A] time = 0.43, size = 163, normalized size = 0.89

$$\frac{e^2\left(a^2(c+dx)^3+ab\left(-(c+dx)^2+\log\left((c+dx)^2+1\right)+2(c+dx)^3\tan^{-1}(c+dx)\right)+b^2\left(i\text{Li}_2\left(-e^{2i\tan^{-1}(c+dx)}\right)+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (e^2*(a^2*(c + d*x)^3 + a*b*(-(c + d*x)^2 + 2*(c + d*x)^3*ArcTan[c + d*x] +
 Log[1 + (c + d*x)^2]) + b^2*(c + d*x - ArcTan[c + d*x] - (c + d*x)^2*ArcTa
n[c + d*x] + I*ArcTan[c + d*x]^2 + (c + d*x)^3*ArcTan[c + d*x]^2 - 2*ArcTan
[c + d*x]*Log[1 + E^((2*I)*ArcTan[c + d*x])] + I*PolyLog[2, -E^((2*I)*ArcTa
n[c + d*x])])))/(3*d)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(a^2d^2e^2x^2+2\,a^2cde^2x+a^2c^2e^2+\left(b^2d^2e^2x^2+2\,b^2cde^2x+b^2c^2e^2\right)\arctan\left(dx+c\right)^2+2\left(abd^2e^2x^2+2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*d^2*e^2*x^2 + 2*a^2*c*d*e^2*x + a^2*c^2*e^2 + (b^2*d^2*e^2*x^2
 + 2*b^2*c*d*e^2*x + b^2*c^2*e^2)*arctan(d*x + c)^2 + 2*(a*b*d^2*e^2*x^2 +
2*a*b*c*d*e^2*x + a*b*c^2*e^2)*arctan(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.20, size = 593, normalized size = 3.24

$$\frac{a^2c^3e^2}{3d}+\frac{b^2c\,e^2}{3d}+\arctan(dx+c)^2\,x\,b^2c^2e^2-\frac{d\,x^2ab\,e^2}{3}+d\,x^2a^2c\,e^2+\frac{\arctan(dx+c)^2\,b^2c^3e^2}{3d}-\frac{\arctan(dx+c)\,b^2c^2e^2}{3d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^2,x)
```

```
[Out] 1/3/d*a^2*c^3*e^2+1/3/d*b^2*c*e^2-1/12*I/d*e^2*b^2*ln(d*x+c-I)^2+1/6*I/d*e^
2*b^2*dilog(1/2*I*(d*x+c-I))+arctan(d*x+c)^2*x*b^2*c^2*e^2-1/6*I/d*e^2*b^2*
dilog(-1/2*I*(I+d*x+c))-1/3*d*x^2*a*b*e^2+d*x^2*a^2*c*e^2+1/3/d*arctan(d*x+
c)^2*b^2*c^3*e^2-1/3/d*arctan(d*x+c)*b^2*c^2*e^2+1/3*d^2*arctan(d*x+c)^2*x^
3*b^2*e^2-1/3*d*arctan(d*x+c)*x^2*b^2*e^2+1/3/d*e^2*a*b*ln(1+(d*x+c)^2)+1/3
*d^2*x^3*a^2*e^2+x*a^2*c^2*e^2+1/12*I/d*e^2*b^2*ln(I+d*x+c)^2+1/3/d*e^2*b^2
*arctan(d*x+c)*ln(1+(d*x+c)^2)-2/3*arctan(d*x+c)*x*b^2*c*e^2-2/3*x*a*b*c*e^
2-1/3/d*a*b*c^2*e^2-1/3*b^2*e^2*arctan(d*x+c)/d-1/6*I/d*e^2*b^2*ln(d*x+c-I)
*ln(-1/2*I*(I+d*x+c))-1/6*I/d*e^2*b^2*ln(I+d*x+c)*ln(1+(d*x+c)^2)+1/6*I/d*e
^2*b^2*ln(d*x+c-I)*ln(1+(d*x+c)^2)+1/6*I/d*e^2*b^2*ln(I+d*x+c)*ln(1/2*I*(d*
x+c-I))+2/3/d*arctan(d*x+c)*a*b*c^3*e^2+2/3*d^2*arctan(d*x+c)*x^3*a*b*e^2+d
*arctan(d*x+c)^2*x^2*b^2*c*e^2+2*arctan(d*x+c)*x*a*b*c^2*e^2+2*d*arctan(d*x
+c)*x^2*a*b*c*e^2+1/3*b^2*e^2*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 3/4*b^2*c^4*e^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 1/4*(3*arctan
(d*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*b^2*c^
4*e^2 + 1/3*a^2*d^2*e^2*x^3 + 36*b^2*d^4*e^2*integrate(1/48*x^4*arctan(d*x
+ c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*d^4*e^2*integrate(1/48*x^4
*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 144
*b^2*c*d^3*e^2*integrate(1/48*x^3*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^
2 + 1), x) + 4*b^2*d^4*e^2*integrate(1/48*x^4*log(d^2*x^2 + 2*c*d*x + c^2 +
 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^2*c*d^3*e^2*integrate(1/48*x^3
*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 216
*b^2*c^2*d^2*e^2*integrate(1/48*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x +
c^2 + 1), x) + 16*b^2*c*d^3*e^2*integrate(1/48*x^3*log(d^2*x^2 + 2*c*d*x +
c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 18*b^2*c^2*d^2*e^2*integrate(1
/48*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x
) + 144*b^2*c^3*d*e^2*integrate(1/48*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x
 + c^2 + 1), x) + 24*b^2*c^2*d^2*e^2*integrate(1/48*x^2*log(d^2*x^2 + 2*c*d
*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^2*c^3*d*e^2*integrat
e(1/48*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1),
x) + 12*b^2*c^3*d*e^2*integrate(1/48*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^
2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*c^4*e^2*integrate(1/48*log(d^2*x^2 +
 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + a^2*c*d*e^2*x^2 +
 3/4*b^2*c^2*e^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 8*b^2*d^3*e^
2*integrate(1/48*x^3*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 24
*b^2*c*d^2*e^2*integrate(1/48*x^2*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2
+ 1), x) - 24*b^2*c^2*d*e^2*integrate(1/48*x*arctan(d*x + c)/(d^2*x^2 + 2*c
*d*x + c^2 + 1), x) - 1/4*(3*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^2/d -
arctan((d^2*x + c*d)/d)^3/d)*b^2*c^2*e^2 + 2*(x^2*arctan(d*x + c) - d*(x/d^
```

```
2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 +
 1)/d^3))*a*b*c*d*e^2 + 1/3*(2*x^3*arctan(d*x + c) - d*((d*x^2 - 4*c*x)/d^3
 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*
c*d*x + c^2 + 1)/d^4))*a*b*d^2*e^2 + a^2*c^2*e^2*x + 36*b^2*d^2*e^2*integra
te(1/48*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*d^2
*e^2*integrate(1/48*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d
*x + c^2 + 1), x) + 72*b^2*c*d*e^2*integrate(1/48*x*arctan(d*x + c)^2/(d^2*
x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^2*c*d*e^2*integrate(1/48*x*log(d^2*x^2 +
 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*c^2*e^2*int
egrate(1/48*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1
), x) + (2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*a*b*c^2*e^2/d
+ 1/12*(b^2*d^2*e^2*x^3 + 3*b^2*c*d*e^2*x^2 + 3*b^2*c^2*e^2*x)*arctan(d*x +
 c)^2 - 1/48*(b^2*d^2*e^2*x^3 + 3*b^2*c*d*e^2*x^2 + 3*b^2*c^2*e^2*x)*log(d^
2*x^2 + 2*c*d*x + c^2 + 1)^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^2\,(a+b\,\mathrm{atan}\,(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*atan(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^2*(a + b*atan(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^2\left(\int a^2c^2\,dx+\int a^2d^2x^2\,dx+\int b^2c^2\,\mathrm{atan}^2\,(c+dx)\,dx+\int 2abc^2\,\mathrm{atan}\,(c+dx)\,dx+\int 2a^2cdx\,dx+\int b^2d^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*atan(d*x+c))**2,x)
```

```
[Out] e**2*(Integral(a**2*c**2, x) + Integral(a**2*d**2*x**2, x) + Integral(b**2*
c**2*atan(c + d*x)**2, x) + Integral(2*a*b*c**2*atan(c + d*x), x) + Integra
l(2*a**2*c*d*x, x) + Integral(b**2*d**2*x**2*atan(c + d*x)**2, x) + Integra
l(2*a*b*d**2*x**2*atan(c + d*x), x) + Integral(2*b**2*c*d*x*atan(c + d*x)**
2, x) + Integral(4*a*b*c*d*x*atan(c + d*x), x))
```

3.9 $\int (ce+dex)\left(a+b\tan^{-1}(c+dx)\right)^2\,dx$

Optimal. Leaf size=95

$$\frac{e(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}+\frac{e\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}-abex+\frac{b^2e\log\left((c+dx)^2+1\right)}{2d}-\frac{b^2e(c+dx)\tan^{-1}(c+$$

```
[Out] -a*b*e*x-b^2*e*(d*x+c)*arctan(d*x+c)/d+1/2*e*(a+b*arctan(d*x+c))^2/d+1/2*e*
(d*x+c)^2*(a+b*arctan(d*x+c))^2/d+1/2*b^2*e*ln(1+(d*x+c)^2)/d
```

Rubi [A] time = 0.12, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5043, 12, 4852, 4916, 4846, 260, 4884}

$$\frac{e(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}+\frac{e\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}-abex+\frac{b^2e\log\left((c+dx)^2+1\right)}{2d}-\frac{b^2e(c+dx)\tan^{-1}(c+$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] -(a*b*e*x) - (b^2*e*(c + d*x)*ArcTan[c + d*x])/d + (e*(a + b*ArcTan[c + d*x
])^2)/(2*d) + (e*(c + d*x)^2*(a + b*ArcTan[c + d*x])^2)/(2*d) + (b^2*e*Log[
1 + (c + d*x)^2])/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
```

```
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (ce+dex)\left(a+b\tan^{-1}(c+dx)\right)^2\,dx &= \frac{\text{Subst}\left(\int ex\left(a+b\tan^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}\\ &= \frac{e\,\text{Subst}\left(\int x\left(a+b\tan^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}\\ &= \frac{e(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}-\frac{(be)\,\text{Subst}\left(\int \frac{x^2\left(a+b\tan^{-1}(x)\right)}{1+x^2}\,dx,x\right.}{d}\\ &= \frac{e(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}-\frac{(be)\,\text{Subst}\left(\int \left(a+b\tan^{-1}(x)\right)\,dx\right.}{d}\\ &= -abex+\frac{e\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}+\frac{e(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}\\ &= -abex-\frac{b^2e(c+dx)\tan^{-1}(c+dx)}{d}+\frac{e\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}+\frac{e(c+}{}\\ &= -abex-\frac{b^2e(c+dx)\tan^{-1}(c+dx)}{d}+\frac{e\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}+\frac{e(c+}{}\end{aligned}$$

Mathematica [A] time = 0.08, size = 107, normalized size = 1.13

$$\frac{e\left(2b\tan^{-1}(c+dx)\left(a\left(c^2+2cdx+d^2x^2+1\right)-b(c+dx)\right)+a(c+dx)(ac+adx-2b)+b^2\left(c^2+2cdx+d^2x^2+\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (e*(a*(c + d*x)*(-2*b + a*c + a*d*x) + 2*b*(-(b*(c + d*x)) + a*(1 + c^2 + 2
*c*d*x + d^2*x^2))*ArcTan[c + d*x] + b^2*(1 + c^2 + 2*c*d*x + d^2*x^2)*ArcT
an[c + d*x]^2 + b^2*Log[1 + (c + d*x)^2]))/(2*d)
```

fricas [A] time = 0.72, size = 150, normalized size = 1.58

$$\frac{a^2d^2ex^2+2\left(a^2c-ab\right)dex+b^2e\log\left(d^2x^2+2\,cdx+c^2+1\right)+\left(b^2d^2ex^2+2\,b^2cdex+\left(b^2c^2+b^2\right)e\right)\arctan\left(dx\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*d^2*e*x^2 + 2*(a^2*c - a*b)*d*e*x + b^2*e*log(d^2*x^2 + 2*c*d*x +
c^2 + 1) + (b^2*d^2*e*x^2 + 2*b^2*c*d*e*x + (b^2*c^2 + b^2)*e)*arctan(d*x +
```

```
c)^2 + 2*(a*b*d^2*e*x^2 + (2*a*b*c - b^2)*d*e*x + (a*b*c^2 - b^2*c + a*b)*
e)*arctan(d*x + c))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.06, size = 220, normalized size = 2.32

$$\frac{a^2ex^2d}{2}+xa^2ce+\frac{a^2c^2e}{2d}+\frac{d\arctan(dx+c)^2x^2b^2e}{2}+\arctan(dx+c)^2xb^2ce+\frac{\arctan(dx+c)^2b^2c^2e}{2d}+\frac{eb^2\arctan(dx}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arctan(d*x+c))^2,x)
```

```
[Out] 1/2*a^2*e*x^2*d+x*a^2*c*e+1/2/d*a^2*c^2*e+1/2*d*arctan(d*x+c)^2*x^2*b^2*e+a
rctan(d*x+c)^2*x*b^2*c*e+1/2/d*arctan(d*x+c)^2*b^2*c^2*e+1/2/d*e*b^2*arctan
(d*x+c)^2-arctan(d*x+c)*x*b^2*e-1/d*arctan(d*x+c)*b^2*c*e+1/2*b^2*e*ln(1+(d
*x+c)^2)/d+d*arctan(d*x+c)*x^2*a*b*e+2*arctan(d*x+c)*x*a*b*c*e+1/d*arctan(d
*x+c)*a*b*c^2*e+1/d*e*a*b*arctan(d*x+c)-a*b*e*x-1/d*a*b*c*e
```

maxima [B] time = 1.49, size = 218, normalized size = 2.29

$$\frac{1}{2}a^2dex^2+\left(x^2\arctan(dx+c)-d\left(\frac{x}{d^2}+\frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3}-\frac{c\log\left(d^2x^2+2cdx+c^2+1\right)}{d^3}\right)\right)abde+a^2cex+\frac{(2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*d*e*x^2 + (x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*x
 + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a*b*d*e + a^2*c*e
*x + (2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*a*b*c*e/d + 1/2*(
b^2*e*log(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^2*d^2*e*x^2 + 2*b^2*c*d*e*x + (
b^2*c^2 + b^2)*e)*arctan(d*x + c)^2 - 2*(b^2*d*e*x + b^2*c*e)*arctan(d*x +
c))/d
```

mupad [B] time = 1.61, size = 216, normalized size = 2.27

$$\mathrm{atan}(c+d\,x)^2\left(\frac{e\,b^2\,c^2+e\,b^2}{2\,d}+b^2\,c\,e\,x+\frac{b^2\,d\,e\,x^2}{2}\right)-x\,\left(a\,e\,(b-3\,a\,c)+2\,a^2\,c\,e\right)-d^2\,\mathrm{atan}(c+d\,x)\left(\frac{x\,\left(b^2\,e-2\,a\,b}{d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*atan(c + d*x))^2,x)
```

```
[Out] atan(c + d*x)^2*((b^2*e + b^2*c^2*e)/(2*d) + b^2*c*e*x + (b^2*d*e*x^2)/2) -
 x*(a*e*(b - 3*a*c) + 2*a^2*c*e) - d^2*atan(c + d*x)*((x*(b^2*e - 2*a*b*c*e
))/d^2 - (a*b*e*x^2)/d) + (b^2*e*log(c^2 + d^2*x^2 + 2*c*d*x + 1))/(2*d) +
(a^2*d*e*x^2)/2 + (b*e*atan((b*c*e*(a - b*c + a*c^2) + b*d*e*x*(a - b*c + a
*c^2))/(a*b*e - b^2*c*e + a*b*c^2*e))*(a - b*c + a*c^2))/d
```

sympy [A] time = 4.30, size = 240, normalized size = 2.53

$$\begin{cases} a^2cex + \frac{a^2dex^2}{2} + \frac{abc^2e\,\mathrm{atan}\,(c+dx)}{d} + 2abcex\,\mathrm{atan}\,(c+dx) + abdex^2\,\mathrm{atan}\,(c+dx) - abex + \frac{abe\,\mathrm{atan}\,(c+dx)}{d} + \frac{b^2c^2e\,\mathrm{ata}}{2} \\ cex\,(a + b\,\mathrm{atan}\,(c))^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*atan(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c*e*x + a**2*d*e*x**2/2 + a*b*c**2*e*atan(c + d*x)/d + 2*a*
b*c*e*x*atan(c + d*x) + a*b*d*e*x**2*atan(c + d*x) - a*b*e*x + a*b*e*atan(c
 + d*x)/d + b**2*c**2*e*atan(c + d*x)**2/(2*d) + b**2*c*e*x*atan(c + d*x)**
2 - b**2*c*e*atan(c + d*x)/d + b**2*d*e*x**2*atan(c + d*x)**2/2 - b**2*e*x*
atan(c + d*x) + b**2*e*log(c/d + x - I/d)/d + b**2*e*atan(c + d*x)**2/(2*d)
 - I*b**2*e*atan(c + d*x)/d, Ne(d, 0)), (c*e*x*(a + b*atan(c))**2, True))
```

3.10 $\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{ce+dex}\,dx$

Optimal. Leaf size=183

$$-\frac{ib\text{Li}_2\left(1-\frac{2}{i(c+dx)+1}\right)\left(a+b\tan^{-1}(c+dx)\right)}{de}+\frac{ib\text{Li}_2\left(\frac{2}{i(c+dx)+1}-1\right)\left(a+b\tan^{-1}(c+dx)\right)}{de}+\frac{2\tanh^{-1}\left(1-\frac{2}{1+i(c+dx)}\right)(}{de}$$

```
[Out] -2*(a+b*arctan(d*x+c))^2*arctanh(-1+2/(1+I*(d*x+c)))/d/e-I*b*(a+b*arctan(d*
x+c))*polylog(2,1-2/(1+I*(d*x+c)))/d/e+I*b*(a+b*arctan(d*x+c))*polylog(2,-1
+2/(1+I*(d*x+c)))/d/e-1/2*b^2*polylog(3,1-2/(1+I*(d*x+c)))/d/e+1/2*b^2*poly
log(3,-1+2/(1+I*(d*x+c)))/d/e
```

Rubi [A] time = 0.34, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5043, 12, 4850, 4988, 4884, 4994, 6610}

$$-\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{de}+\frac{ib\text{PolyLog}\left(2,-1+\frac{2}{1+i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{de}-\frac{b^2\text{Poly}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x),x]
```

```
[Out] (2*(a + b*ArcTan[c + d*x])^2*ArcTanh[1 - 2/(1 + I*(c + d*x))])/(d*e) - (I*b
*(a + b*ArcTan[c + d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/(d*e) + (I*b*
(a + b*ArcTan[c + d*x])*PolyLog[2, -1 + 2/(1 + I*(c + d*x))])/(d*e) - (b^2*
PolyLog[3, 1 - 2/(1 + I*(c + d*x))])/(2*d*e) + (b^2*PolyLog[3, -1 + 2/(1 +
I*(c + d*x))])/(2*d*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
```

```
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{ce+dex}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{ex}\,dx,x,c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{x}\,dx,x,c+dx\right)}{de}$$

$$= \frac{2\left(a+b\tan^{-1}(c+dx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} - \frac{(4b)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)\tanh}{1+x^2}\right.}{de}$$

$$= \frac{2\left(a+b\tan^{-1}(c+dx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} - \frac{(2b)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)\log(}{1+x^2}\right.}{de}$$

$$= \frac{2\left(a+b\tan^{-1}(c+dx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} - \frac{ib\left(a+b\tan^{-1}(c+dx)\right)\text{Li}_2\left(1\right.}{de}$$

$$= \frac{2\left(a+b\tan^{-1}(c+dx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} - \frac{ib\left(a+b\tan^{-1}(c+dx)\right)\text{Li}_2\left(1\right.}{de}$$

Mathematica [A] time = 0.08, size = 170, normalized size = 0.93

$$\frac{2ib\text{Li}_2\left(-\frac{c+dx+i}{c+dx-i}\right)\left(a+b\tan^{-1}(c+dx)\right) - 2ib\text{Li}_2\left(\frac{c+dx+i}{c+dx-i}\right)\left(a+b\tan^{-1}(c+dx)\right) + 4\tanh^{-1}\left(\frac{c+dx+i}{c+dx-i}\right)\left(a+b\tan^{-1}\right.}{2de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x),x]
```

```
[Out] (4*(a + b*ArcTan[c + d*x])^2*ArcTanh[(I + c + d*x)/(-I + c + d*x)] + (2*I)*
b*(a + b*ArcTan[c + d*x])*PolyLog[2, -((I + c + d*x)/(-I + c + d*x))] - (2*
I)*b*(a + b*ArcTan[c + d*x])*PolyLog[2, (I + c + d*x)/(-I + c + d*x)] + b^2
*PolyLog[3, -((I + c + d*x)/(-I + c + d*x))] - b^2*PolyLog[3, (I + c + d*x)
/(-I + c + d*x)])/(2*d*e)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\arctan\left(dx+c\right)^2 + 2\,ab\arctan\left(dx+c\right) + a^2}{dex+ce}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2)/(d*e*x + c*e
), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.78, size = 1433, normalized size = 7.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^2/(d*e*x+c*e),x)
```

```
[Out] -2*I/d*b^2/e*arctan(d*x+c)*polylog(2,(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))-2*I
/d*b^2/e*arctan(d*x+c)*polylog(2,-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+I/d*a*
b/e*dilog(1+I*(d*x+c))+2/d*a*b/e*ln(d*x+c)*arctan(d*x+c)-I/d*a*b/e*dilog(1-
I*(d*x+c))+1/2*I/d*b^2/e*Pi*arctan(d*x+c)^2+I/d*b^2/e*arctan(d*x+c)*polylog
(2,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))-I/d*a*b/e*ln(d*x+c)*ln(1-I*(d*x+c))+I/d*
a*b/e*ln(d*x+c)*ln(1+I*(d*x+c))+1/2*I/d*b^2/e*Pi*csgn(I*((1+I*(d*x+c))^2/(1
+(d*x+c)^2)-1))*csgn(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csgn(I*((1+I*(d*x
+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*arctan(d*x+c)^2+
1/2*I/d*b^2/e*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/
(1+(d*x+c)^2)+1))^3*arctan(d*x+c)^2+1/2*I/d*b^2/e*Pi*csgn(((1+I*(d*x+c))^2/
(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^3*arctan(d*x+c)^2-1/2*I
/d*b^2/e*Pi*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x
+c)^2)+1))^2*arctan(d*x+c)^2-1/2*I/d*b^2/e*Pi*csgn(I/((1+I*(d*x+c))^2/(1+(d
*x+c)^2)+1))*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(
d*x+c)^2)+1))^2*arctan(d*x+c)^2-1/2*I/d*b^2/e*Pi*csgn(I*((1+I*(d*x+c))^2/(1
+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csgn(((1+I*(d*x+c))^2/(1+
(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*arctan(d*x+c)^2+1/2*I/d*
b^2/e*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+
c)^2)+1))*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c
)^2)+1))*arctan(d*x+c)^2-1/2*I/d*b^2/e*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c
)^2)-1))*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+
c)^2)+1))^2*arctan(d*x+c)^2+2/d*b^2/e*polylog(3,-(1+I*(d*x+c))/(1+(d*x+c)^2
)^(1/2))+2/d*b^2/e*polylog(3,(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+1/d*a^2/e*l
n(d*x+c)-1/2/d*b^2/e*polylog(3,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))+1/d*b^2/e*ar
ctan(d*x+c)^2*ln(1+(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+1/d*b^2/e*arctan(d*x+
c)^2*ln(1-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+1/d*b^2/e*ln(d*x+c)*arctan(d*x
+c)^2-1/d*b^2/e*arctan(d*x+c)^2*ln((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2 \log(dex + ce)}{de} + \int \frac{12\, b^2 \arctan(dx+c)^2 + b^2 \log\left(d^2x^2 + 2\,cdx + c^2 + 1\right)^2 + 32\,ab \arctan(dx+c)}{16\,(dex + ce)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] a^2*log(d*e*x + c*e)/(d*e) + integrate(1/16*(12*b^2*arctan(d*x + c)^2 + b^2
*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 32*a*b*arctan(d*x + c))/(d*e*x + c*e)
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atan}\,(c+d\,x))^2}{c\,e+d\,e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))^2/(c*e + d*e*x),x)
```

```
[Out] int((a + b*atan(c + d*x))^2/(c*e + d*e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c+dx}\,dx + \int \frac{b^2\,\mathrm{atan}^2\,(c+dx)}{c+dx}\,dx + \int \frac{2ab\,\mathrm{atan}\,(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**2/(d*e*x+c*e),x)
```

```
[Out] (Integral(a**2/(c + d*x), x) + Integral(b**2*atan(c + d*x)**2/(c + d*x), x)
 + Integral(2*a*b*atan(c + d*x)/(c + d*x), x))/e
```

3.11 $\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{(ce+dex)^2}\,dx$

Optimal. Leaf size=119

$$-\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{de^2(c+dx)}-\frac{i\left(a+b\tan^{-1}(c+dx)\right)^2}{de^2}+\frac{2b\log\left(2-\frac{2}{1-i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{de^2}-\frac{ib^2\mathrm{Li}_2\left(\frac{2}{1-i(c+dx)}-\right.}{de^2}$$

```
[Out] -I*(a+b*arctan(d*x+c))^2/d/e^2-(a+b*arctan(d*x+c))^2/d/e^2/(d*x+c)+2*b*(a+b
*arctan(d*x+c))*ln(2-2/(1-I*(d*x+c)))/d/e^2-I*b^2*polylog(2,-1+2/(1-I*(d*x+
c)))/d/e^2
```

Rubi [A] time = 0.19, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5043, 12, 4852, 4924, 4868, 2447}

$$-\frac{ib^2\mathrm{PolyLog}\left(2,-1+\frac{2}{1-i(c+dx)}\right)}{de^2}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{de^2(c+dx)}-\frac{i\left(a+b\tan^{-1}(c+dx)\right)^2}{de^2}+\frac{2b\log\left(2-\frac{2}{1-i(c+dx)}\right)\left(a+b\,\mathrm{ta}\right.}{de^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^2,x]
```

```
[Out] ((-I)*(a + b*ArcTan[c + d*x])^2)/(d*e^2) - (a + b*ArcTan[c + d*x])^2/(d*e^2
*(c + d*x)) + (2*b*(a + b*ArcTan[c + d*x])*Log[2 - 2/(1 - I*(c + d*x))])/(d
*e^2) - (I*b^2*PolyLog[2, -1 + 2/(1 - I*(c + d*x))])/(d*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
```

```
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{(ce+dex)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{e^2x^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{x^2}\,dx, x, c+dx\right)}{de^2}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{de^2(c+dx)} + \frac{(2b)\,\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x\left(1+x^2\right)}\,dx, x, c+dx\right)}{de^2}$$

$$= -\frac{i\left(a+b\tan^{-1}(c+dx)\right)^2}{de^2} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{de^2(c+dx)} + \frac{(2ib)\,\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x(i+x)}\right.}{de^2}$$

$$= -\frac{i\left(a+b\tan^{-1}(c+dx)\right)^2}{de^2} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{de^2(c+dx)} + \frac{2b\left(a+b\tan^{-1}(c+dx)\right)\text{l}}{de^2}$$

$$= -\frac{i\left(a+b\tan^{-1}(c+dx)\right)^2}{de^2} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{de^2(c+dx)} + \frac{2b\left(a+b\tan^{-1}(c+dx)\right)\text{l}}{de^2}$$

Mathematica [A] time = 0.23, size = 135, normalized size = 1.13

$$\frac{a\left(2b(c+dx)\log\left(\frac{c+dx}{\sqrt{(c+dx)^2+1}}\right)-a\right)+2b\tan^{-1}(c+dx)\left(-a+b(c+dx)\log\left(1-e^{2i\tan^{-1}(c+dx)}\right)\right)-ib^2(c+dx)\text{Li}_2}{de^2(c+dx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^2,x]
```

```
[Out] ((-I)*b^2*(-I + c + d*x)*ArcTan[c + d*x]^2 + 2*b*ArcTan[c + d*x]*(-a + b*(c
 + d*x)*Log[1 - E^((2*I)*ArcTan[c + d*x])]) + a*(-a + 2*b*(c + d*x)*Log[(c
+ d*x)/Sqrt[1 + (c + d*x)^2]]) - I*b^2*(c + d*x)*PolyLog[2, E^((2*I)*ArcTan
[c + d*x])])/(d*e^2*(c + d*x))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\arctan\left(dx+c\right)^2+2\,ab\arctan\left(dx+c\right)+a^2}{d^2e^2x^2+2\,cde^2x+c^2e^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2)/(d^2*e^2*x^2
 + 2*c*d*e^2*x + c^2*e^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.17, size = 471, normalized size = 3.96

$$-\frac{a^2}{d\,e^2\,(dx+c)}-\frac{b^2\arctan\left(dx+c\right)^2}{d\,e^2\,(dx+c)}+\frac{2b^2\ln\left(dx+c\right)\arctan\left(dx+c\right)}{d\,e^2}-\frac{b^2\arctan\left(dx+c\right)\ln\left(1+(dx+c)^2\right)}{d\,e^2}-\frac{ib^2\ln(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^2,x)
```

```
[Out] -1/d*a^2/e^2/(d*x+c)-1/d*b^2/e^2/(d*x+c)*arctan(d*x+c)^2+2/d*b^2/e^2*ln(d*x
+c)*arctan(d*x+c)-1/d*b^2/e^2*arctan(d*x+c)*ln(1+(d*x+c)^2)-1/2*I/d*b^2/e^2
*ln(d*x+c-I)*ln(1+(d*x+c)^2)-1/4*I/d*b^2/e^2*ln(I+d*x+c)^2+1/2*I/d*b^2/e^2*
dilog(-1/2*I*(I+d*x+c))+1/2*I/d*b^2/e^2*ln(d*x+c-I)*ln(-1/2*I*(I+d*x+c))-I/
d*b^2/e^2*dilog(1-I*(d*x+c))+I/d*b^2/e^2*dilog(1+I*(d*x+c))-I/d*b^2/e^2*ln(
d*x+c)*ln(1-I*(d*x+c))+I/d*b^2/e^2*ln(d*x+c)*ln(1+I*(d*x+c))+1/4*I/d*b^2/e^
2*ln(d*x+c-I)^2-1/2*I/d*b^2/e^2*ln(I+d*x+c)*ln(1/2*I*(d*x+c-I))-1/2*I/d*b^2
/e^2*dilog(1/2*I*(d*x+c-I))+1/2*I/d*b^2/e^2*ln(I+d*x+c)*ln(1+(d*x+c)^2)-2/d
*a*b/e^2/(d*x+c)*arctan(d*x+c)+2/d*a*b/e^2*ln(d*x+c)-1/d*a*b/e^2*ln(1+(d*x+
c)^2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{atan}\left(c+d\,x\right)\right)^2}{\left(c\,e+d\,e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))^2/(c*e + d*e*x)^2,x)
```

```
[Out] int((a + b*atan(c + d*x))^2/(c*e + d*e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^2+2cdx+d^2x^2}\,dx+\int \frac{b^2\operatorname{atan}^2{(c+dx)}}{c^2+2cdx+d^2x^2}\,dx+\int \frac{2ab\operatorname{atan}{(c+dx)}}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**2/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**2*atan(c + d*
x)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(2*a*b*atan(c + d*x)/(c**2
 + 2*c*d*x + d**2*x**2), x))/e**2
```

3.12 $\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{(ce+dex)^3}\,dx$

Optimal. Leaf size=117

$$-\frac{b\left(a+b\tan^{-1}(c+dx)\right)}{de^3(c+dx)}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3}+\frac{b^2\log(c+dx)}{de^3}-\frac{b^2\log\left((c+dx)^2+\right.}{2de^3}$$

```
[Out] -b*(a+b*arctan(d*x+c))/d/e^3/(d*x+c)-1/2*(a+b*arctan(d*x+c))^2/d/e^3-1/2*(a
+b*arctan(d*x+c))^2/d/e^3/(d*x+c)^2+b^2*ln(d*x+c)/d/e^3-1/2*b^2*ln(1+(d*x+c
)^2)/d/e^3
```

Rubi [A] time = 0.15, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5043, 12, 4852, 4918, 266, 36, 29, 31, 4884}

$$-\frac{b\left(a+b\tan^{-1}(c+dx)\right)}{de^3(c+dx)}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3}+\frac{b^2\log(c+dx)}{de^3}-\frac{b^2\log\left((c+dx)^2+\right.}{2de^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^3,x]
```

```
[Out] -((b*(a + b*ArcTan[c + d*x]))/(d*e^3*(c + d*x))) - (a + b*ArcTan[c + d*x])^
2/(2*d*e^3) - (a + b*ArcTan[c + d*x])^2/(2*d*e^3*(c + d*x)^2) + (b^2*Log[c
+ d*x])/(d*e^3) - (b^2*Log[1 + (c + d*x)^2])/(2*d*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
```

```
erQ[m]) && NeQ[m, -1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{(ce+dex)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{e^3x^3}\,dx, x, c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{x^3}\,dx, x, c+dx\right)}{de^3}\\
&= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2}+\frac{b\,\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x^2\left(1+x^2\right)}\,dx, x, c+dx\right)}{de^3}\\
&= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2}+\frac{b\,\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x^2}\,dx, x, c+dx\right)}{de^3}-\frac{b\,\text{Subst}\left(\int \frac{a+b\,\text{ta}}{1+}\right.}{}\\
&= -\frac{b\left(a+b\tan^{-1}(c+dx)\right)}{de^3(c+dx)}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2}+\frac{b^2\,\text{S}}{}\\
&= -\frac{b\left(a+b\tan^{-1}(c+dx)\right)}{de^3(c+dx)}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2}+\frac{b^2\,\text{S}}{}\\
&= -\frac{b\left(a+b\tan^{-1}(c+dx)\right)}{de^3(c+dx)}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2}+\frac{b^2\,\text{S}}{}\\
&= -\frac{b\left(a+b\tan^{-1}(c+dx)\right)}{de^3(c+dx)}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2}+\frac{b^2\,\text{l}}{}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 194, normalized size = 1.66

$$-\frac{a^2+2b\tan^{-1}(c+dx)\left(a\left(c^2+2cdx+d^2x^2+1\right)+b(c+dx)\right)+2abc+2abdx+b^2c^2\log\left(c^2+2cdx+d^2x^2+1\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^3,x]
```

```
[Out] -1/2*(a^2 + 2*a*b*c + 2*a*b*d*x + 2*b*(b*(c + d*x) + a*(1 + c^2 + 2*c*d*x +
 d^2*x^2))*ArcTan[c + d*x] + b^2*(1 + c^2 + 2*c*d*x + d^2*x^2)*ArcTan[c + d
*x]^2 - 2*b^2*(c + d*x)^2*Log[c + d*x] + b^2*c^2*Log[1 + c^2 + 2*c*d*x + d^
2*x^2] + 2*b^2*c*d*x*Log[1 + c^2 + 2*c*d*x + d^2*x^2] + b^2*d^2*x^2*Log[1 +
 c^2 + 2*c*d*x + d^2*x^2])/(d*e^3*(c + d*x)^2)
```

fricas [A] time = 0.44, size = 209, normalized size = 1.79

$$-\frac{2\,abdx+2\,abc+\left(b^2d^2x^2+2\,b^2cdx+b^2c^2+b^2\right)\arctan\left(dx+c\right)^2+a^2+2\left(abd^2x^2+abc^2+b^2c+\left(2\,abc+b^2\right.\right.}{2\left(d^3e^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*d*x + 2*a*b*c + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + b^2)*arc
tan(d*x + c)^2 + a^2 + 2*(a*b*d^2*x^2 + a*b*c^2 + b^2*c + (2*a*b*c + b^2)*d
*x + a*b)*arctan(d*x + c) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(d^2*x
^2 + 2*c*d*x + c^2 + 1) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(d*x +
 c))/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.06, size = 182, normalized size = 1.56

$$-\frac{a^2}{2d\,e^3\left(dx+c\right)^2}-\frac{b^2\arctan\left(dx+c\right)^2}{2d\,e^3\left(dx+c\right)^2}-\frac{b^2\arctan\left(dx+c\right)}{d\,e^3\left(dx+c\right)}-\frac{b^2\arctan\left(dx+c\right)^2}{2d\,e^3}+\frac{b^2\ln\left(dx+c\right)}{d\,e^3}-\frac{b^2\ln\left(1+\left(dx+\right.\right.}{2d\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^3,x)
```

```
[Out] -1/2/d*a^2/e^3/(d*x+c)^2-1/2/d*b^2/e^3/(d*x+c)^2*arctan(d*x+c)^2-1/d*b^2/e^
3*arctan(d*x+c)/(d*x+c)-1/2/d*b^2/e^3*arctan(d*x+c)^2+b^2*ln(d*x+c)/d/e^3-1
/2*b^2*ln(1+(d*x+c)^2)/d/e^3-1/d*a*b/e^3/(d*x+c)^2*arctan(d*x+c)-1/d*a*b/e^
3/(d*x+c)-1/d*a*b/e^3*arctan(d*x+c)
```

maxima [B] time = 0.54, size = 268, normalized size = 2.29

$$-\left(d\left(\frac{1}{d^3e^3x+cd^2e^3}+\frac{\arctan\left(\frac{d^2x+cd}{d}\right)}{d^2e^3}\right)+\frac{\arctan\left(dx+c\right)}{d^3e^3x^2+2\,cd^2e^3x+c^2de^3}\right)ab-\frac{1}{2}\left(2\,d\left(\frac{1}{d^3e^3x+cd^2e^3}+\frac{\arctan\left(\frac{d^2x+c\right.}{d}\right.}{d^2e^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] -(d*(1/(d^3*e^3*x + c*d^2*e^3) + arctan((d^2*x + c*d)/d)/(d^2*e^3)) + arcta
n(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3))*a*b - 1/2*(2*d*(1/(d^
3*e^3*x + c*d^2*e^3) + arctan((d^2*x + c*d)/d)/(d^2*e^3))*arctan(d*x + c) -
 (arctan(d*x + c)^2 - log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*log(d*x + c))/(d
```

```
*e^3))*b^2 - 1/2*b^2*arctan(d*x + c)^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d
*e^3) - 1/2*a^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

mupad [B] time = 2.86, size = 232, normalized size = 1.98

$$\frac{b^2\ln(c+d\,x)}{d\,e^3}-\frac{\frac{a^2+2\,b\,c\,a}{2\,d}+a\,b\,x}{c^2\,e^3+2\,c\,d\,e^3\,x+d^2\,e^3\,x^2}-\frac{\mathrm{atan}(c+d\,x)\left(\frac{b^2\,c}{d^3\,e^3}+\frac{b^2\,x}{d^2\,e^3}+\frac{a\,b}{d^3\,e^3}\right)}{x^2+\frac{c^2}{d^2}+\frac{2\,c\,x}{d}}-\mathrm{atan}(c+d\,x)^2\left(\frac{b^2}{2\,d\,e^3}+\frac{}{2\,d^3\,e^3\,(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))^2/(c*e + d*e*x)^3,x)
```

```
[Out] (b^2*log(c + d*x))/(d*e^3) - ((a^2 + 2*a*b*c)/(2*d) + a*b*x)/(c^2*e^3 + d^2
*e^3*x^2 + 2*c*d*e^3*x) - (atan(c + d*x)*((b^2*c)/(d^3*e^3) + (b^2*x)/(d^2*
e^3) + (a*b)/(d^3*e^3)))/(x^2 + c^2/d^2 + (2*c*x)/d) - atan(c + d*x)^2*(b^2
/(2*d*e^3) + b^2/(2*d^3*e^3*(x^2 + c^2/d^2 + (2*c*x)/d))) + (log(c + d*x -
1i)*((a*b*1i)/2 - b^2/2))/(d*e^3) - (log(c + d*x + 1i)*((a*b*1i)/2 + b^2/2)
)/(d*e^3)
```

sympy [A] time = 21.96, size = 1144, normalized size = 9.78

$$\begin{cases} \frac{a^2}{-2c^2de^3-4cd^2e^3x-2d^3e^3x^2}+\frac{2abc^2\,\mathrm{atan}(c+dx)}{-2c^2de^3-4cd^2e^3x-2d^3e^3x^2}+\frac{4abcdx\,\mathrm{atan}(c+dx)}{-2c^2de^3-4cd^2e^3x-2d^3e^3x^2}+\frac{2abc}{-2c^2de^3-4cd^2e^3x-2d^3e^3x^2}+\frac{2abd^2x^2\,\mathrm{atan}(c+dx)}{-2c^2de^3-4cd^2e^3x-2d^3e^3} \\ \frac{x(a+b\,\mathrm{atan}(c))^2}{c^3e^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**2/(d*e*x+c*e)**3,x)
```

```
[Out] Piecewise((a**2/(-2*c**2*d*e**3 - 4*c*d**2*e**3*x - 2*d**3*e**3*x**2) + 2*a
*b*c**2*atan(c + d*x)/(-2*c**2*d*e**3 - 4*c*d**2*e**3*x - 2*d**3*e**3*x**2)
 + 4*a*b*c*d*x*atan(c + d*x)/(-2*c**2*d*e**3 - 4*c*d**2*e**3*x - 2*d**3*e**
3*x**2) + 2*a*b*c/(-2*c**2*d*e**3 - 4*c*d**2*e**3*x - 2*d**3*e**3*x**2) + 2
*a*b*d**2*x**2*atan(c + d*x)/(-2*c**2*d*e**3 - 4*c*d**2*e**3*x - 2*d**3*e**
3*x**2) + 2*a*b*d*x/(-2*c**2*d*e**3 - 4*c*d**2*e**3*x - 2*d**3*e**3*x**2) +
 2*a*b*atan(c + d*x)/(-2*c**2*d*e**3 - 4*c*d**2*e**3*x - 2*d**3*e**3*x**2)
- 2*b**2*c**2*log(c/d + x)/(-2*c**2*d*e**3 - 4*c*d**2*e**3*x - 2*d**3*e**3*
x**2) + 2*b**2*c**2*log(c/d + x - I/d)/(-2*c**2*d*e**3 - 4*c*d**2*e**3*x -
2*d**3*e**3*x**2) + b**2*c**2*atan(c + d*x)**2/(-2*c**2*d*e**3 - 4*c*d**2*e
**3*x - 2*d**3*e**3*x**2) - 2*I*b**2*c**2*atan(c + d*x)/(-2*c**2*d*e**3 - 4
*c*d**2*e**3*x - 2*d**3*e**3*x**2) - 4*b**2*c*d*x*log(c/d + x)/(-2*c**2*d*e
**3 - 4*c*d**2*e**3*x - 2*d**3*e**3*x**2) + 4*b**2*c*d*x*log(c/d + x - I/d)
/(-2*c**2*d*e**3 - 4*c*d**2*e**3*x - 2*d**3*e**3*x**2) + 2*b**2*c*d*x*atan(
c + d*x)**2/(-2*c**2*d*e**3 - 4*c*d**2*e**3*x - 2*d**3*e**3*x**2) - 4*I*b**
2*c*d*x*atan(c + d*x)/(-2*c**2*d*e**3 - 4*c*d**2*e**3*x - 2*d**3*e**3*x**2)
 + 2*b**2*c*atan(c + d*x)/(-2*c**2*d*e**3 - 4*c*d**2*e**3*x - 2*d**3*e**3*x
**2) - 2*b**2*d**2*x**2*log(c/d + x)/(-2*c**2*d*e**3 - 4*c*d**2*e**3*x - 2*
d**3*e**3*x**2) + 2*b**2*d**2*x**2*log(c/d + x - I/d)/(-2*c**2*d*e**3 - 4*c
*d**2*e**3*x - 2*d**3*e**3*x**2) + b**2*d**2*x**2*atan(c + d*x)**2/(-2*c**2
*d*e**3 - 4*c*d**2*e**3*x - 2*d**3*e**3*x**2) - 2*I*b**2*d**2*x**2*atan(c +
 d*x)/(-2*c**2*d*e**3 - 4*c*d**2*e**3*x - 2*d**3*e**3*x**2) + 2*b**2*d*x*at
an(c + d*x)/(-2*c**2*d*e**3 - 4*c*d**2*e**3*x - 2*d**3*e**3*x**2) + b**2*at
an(c + d*x)**2/(-2*c**2*d*e**3 - 4*c*d**2*e**3*x - 2*d**3*e**3*x**2), Ne(d,
 0)), (x*(a + b*atan(c))**2/(c**3*e**3), True))
```

3.13 $\int \frac{(a+b\tan^{-1}(c+dx))^2}{(ce+dex)^4}\,dx$

Optimal. Leaf size=194

$$-\frac{b\left(a+b\tan^{-1}(c+dx)\right)}{3de^4(c+dx)^2}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{3de^4(c+dx)^3}+\frac{i\left(a+b\tan^{-1}(c+dx)\right)^2}{3de^4}-\frac{2b\log\left(2-\frac{2}{1-i(c+dx)}\right)\left(a+b\tan^{-1}(\right.}{3de^4}$$

```
[Out] -1/3*b^2/d/e^4/(d*x+c)-1/3*b^2*arctan(d*x+c)/d/e^4-1/3*b*(a+b*arctan(d*x+c)
)/d/e^4/(d*x+c)^2+1/3*I*(a+b*arctan(d*x+c))^2/d/e^4-1/3*(a+b*arctan(d*x+c))
^2/d/e^4/(d*x+c)^3-2/3*b*(a+b*arctan(d*x+c))*ln(2-2/(1-I*(d*x+c)))/d/e^4+1/
3*I*b^2*polylog(2,-1+2/(1-I*(d*x+c)))/d/e^4
```

Rubi [A] time = 0.26, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5043, 12, 4852, 4918, 325, 203, 4924, 4868, 2447}

$$\frac{ib^2\text{PolyLog}\left(2,-1+\frac{2}{1-i(c+dx)}\right)}{3de^4}-\frac{b\left(a+b\tan^{-1}(c+dx)\right)}{3de^4(c+dx)^2}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{3de^4(c+dx)^3}+\frac{i\left(a+b\tan^{-1}(c+dx)\right)^2}{3de^4}-\frac{2b\,\mathrm{l}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^4,x]
```

```
[Out] -b^2/(3*d*e^4*(c + d*x)) - (b^2*ArcTan[c + d*x])/(3*d*e^4) - (b*(a + b*ArcT
an[c + d*x]))/(3*d*e^4*(c + d*x)^2) + ((I/3)*(a + b*ArcTan[c + d*x])^2)/(d*
e^4) - (a + b*ArcTan[c + d*x])^2/(3*d*e^4*(c + d*x)^3) - (2*b*(a + b*ArcTan
[c + d*x])*Log[2 - 2/(1 - I*(c + d*x))])/(3*d*e^4) + ((I/3)*b^2*PolyLog[2,
-1 + 2/(1 - I*(c + d*x))])/(d*e^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{(ce+dex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{e^4x^4}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{x^4}\,dx, x, c+dx\right)}{de^4}$$
$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{3de^4(c+dx)^3} + \frac{(2b)\,\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x^3(1+x^2)}\,dx, x, c+dx\right)}{3de^4}$$
$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{3de^4(c+dx)^3} + \frac{(2b)\,\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x^3}\,dx, x, c+dx\right)}{3de^4} - (2b)\,\text{Subst}$$
$$= -\frac{b\left(a+b\tan^{-1}(c+dx)\right)}{3de^4(c+dx)^2} + \frac{i\left(a+b\tan^{-1}(c+dx)\right)^2}{3de^4} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{3de^4(c+dx)^3} -$$
$$= -\frac{b^2}{3de^4(c+dx)} - \frac{b\left(a+b\tan^{-1}(c+dx)\right)}{3de^4(c+dx)^2} + \frac{i\left(a+b\tan^{-1}(c+dx)\right)^2}{3de^4} - \frac{\left(a+b\tan\right.}{3de^4}$$
$$= -\frac{b^2}{3de^4(c+dx)} - \frac{b^2\tan^{-1}(c+dx)}{3de^4} - \frac{b\left(a+b\tan^{-1}(c+dx)\right)}{3de^4(c+dx)^2} + \frac{i\left(a+b\tan^{-1}(c\right.}{3de^4}$$

Mathematica [A] time = 0.75, size = 163, normalized size = 0.84

$$-\frac{\frac{a^2}{(c+dx)^3} + \frac{ab}{(c+dx)^2} + 2ab\log\left(\frac{c+dx}{\sqrt{(c+dx)^2+1}}\right) + b\tan^{-1}(c+dx)\left(\frac{2a}{(c+dx)^3} + \frac{b}{(c+dx)^2} + 2b\log\left(1-e^{2i\tan^{-1}(c+dx)}\right) + b\right) +}{3de^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^4,x]
```

```
[Out] -1/3*(a*b + a^2/(c + d*x)^3 + (a*b)/(c + d*x)^2 + b^2/(c + d*x) + b^2*(-I +
 (c + d*x)^(-3))*ArcTan[c + d*x]^2 + b*ArcTan[c + d*x]*(b + (2*a)/(c + d*x)
^3 + b/(c + d*x)^2 + 2*b*Log[1 - E^((2*I)*ArcTan[c + d*x])]) + 2*a*b*Log[(c
 + d*x)/Sqrt[1 + (c + d*x)^2]] - I*b^2*PolyLog[2, E^((2*I)*ArcTan[c + d*x])
])/(d*e^4)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\arctan\left(dx+c\right)^2 + 2\,ab\arctan\left(dx+c\right) + a^2}{d^4e^4x^4 + 4\,cd^3e^4x^3 + 6\,c^2d^2e^4x^2 + 4\,c^3de^4x + c^4e^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2)/(d^4*e^4*x^4
 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.14, size = 547, normalized size = 2.82

$$-\frac{a^2}{3d\,e^4\,(dx+c)^3}-\frac{b^2\arctan(dx+c)^2}{3d\,e^4\,(dx+c)^3}-\frac{b^2\arctan(dx+c)}{3d\,e^4\,(dx+c)^2}-\frac{2b^2\ln(dx+c)\arctan(dx+c)}{3d\,e^4}+\frac{b^2\arctan(dx+c)\ln(}{3d\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^4,x)
```

```
[Out] -1/3/d*a^2/e^4/(d*x+c)^3-1/3/d*b^2/e^4/(d*x+c)^3*arctan(d*x+c)^2-1/3/d*b^2/
e^4*arctan(d*x+c)/(d*x+c)^2-2/3/d*b^2/e^4*ln(d*x+c)*arctan(d*x+c)+1/3/d*b^2
/e^4*arctan(d*x+c)*ln(1+(d*x+c)^2)+1/12*I/d*b^2/e^4*ln(I+d*x+c)^2-1/6*I/d*b
^2/e^4*ln(I+d*x+c)*ln(1+(d*x+c)^2)-1/12*I/d*b^2/e^4*ln(d*x+c-I)^2-1/3*I/d*b
^2/e^4*dilog(1+I*(d*x+c))+1/6*I/d*b^2/e^4*ln(I+d*x+c)*ln(1/2*I*(d*x+c-I))-1
/6*I/d*b^2/e^4*ln(d*x+c-I)*ln(-1/2*I*(I+d*x+c))+1/6*I/d*b^2/e^4*dilog(1/2*I
*(d*x+c-I))-1/6*I/d*b^2/e^4*dilog(-1/2*I*(I+d*x+c))-1/3*b^2/d/e^4/(d*x+c)-1
/3*b^2*arctan(d*x+c)/d/e^4-1/3*I/d*b^2/e^4*ln(d*x+c)*ln(1+I*(d*x+c))+1/3*I/
d*b^2/e^4*dilog(1-I*(d*x+c))+1/6*I/d*b^2/e^4*ln(d*x+c-I)*ln(1+(d*x+c)^2)+1/
3*I/d*b^2/e^4*ln(d*x+c)*ln(1-I*(d*x+c))-2/3/d*a*b/e^4/(d*x+c)^3*arctan(d*x+
c)-1/3/d*a*b/e^4/(d*x+c)^2-2/3/d*a*b/e^4*ln(d*x+c)+1/3/d*a*b/e^4*ln(1+(d*x+
c)^2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atan}(c+d\,x))^2}{(c\,e+d\,e\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))^2/(c*e + d*e*x)^4,x)
```

```
[Out] int((a + b*atan(c + d*x))^2/(c*e + d*e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int \frac{b^2\,\mathrm{atan}^2(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int \frac{2ab\,\mathrm{atan}(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**2/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4
*x**4), x) + Integral(b**2*atan(c + d*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*d**
2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(2*a*b*atan(c + d*x)/(c**
4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

3.14 $\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{(ce+dex)^5}\,dx$

Optimal. Leaf size=170

$$\frac{b\left(a+b\tan^{-1}(c+dx)\right)}{2de^5(c+dx)}-\frac{b\left(a+b\tan^{-1}(c+dx)\right)}{6de^5(c+dx)^3}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{4de^5(c+dx)^4}+\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{4de^5}-\frac{b^2}{12de^5(c+d}$$

```
[Out] -1/12*b^2/d/e^5/(d*x+c)^2-1/6*b*(a+b*arctan(d*x+c))/d/e^5/(d*x+c)^3+1/2*b*(
a+b*arctan(d*x+c))/d/e^5/(d*x+c)+1/4*(a+b*arctan(d*x+c))^2/d/e^5-1/4*(a+b*a
rctan(d*x+c))^2/d/e^5/(d*x+c)^4-2/3*b^2*ln(d*x+c)/d/e^5+1/3*b^2*ln(1+(d*x+c
)^2)/d/e^5
```

Rubi [A] time = 0.23, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {5043, 12, 4852, 4918, 266, 44, 36, 29, 31, 4884}

$$\frac{b\left(a+b\tan^{-1}(c+dx)\right)}{2de^5(c+dx)}-\frac{b\left(a+b\tan^{-1}(c+dx)\right)}{6de^5(c+dx)^3}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{4de^5(c+dx)^4}+\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{4de^5}-\frac{b^2}{12de^5(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^5,x]
```

```
[Out] -b^2/(12*d*e^5*(c + d*x)^2) - (b*(a + b*ArcTan[c + d*x]))/(6*d*e^5*(c + d*x
)^3) + (b*(a + b*ArcTan[c + d*x]))/(2*d*e^5*(c + d*x)) + (a + b*ArcTan[c +
d*x])^2/(4*d*e^5) - (a + b*ArcTan[c + d*x])^2/(4*d*e^5*(c + d*x)^4) - (2*b^
2*Log[c + d*x])/(3*d*e^5) + (b^2*Log[1 + (c + d*x)^2])/(3*d*e^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{(ce+dex)^5}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{e^5x^5}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{x^5}\,dx, x, c+dx\right)}{de^5}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{4de^5(c+dx)^4} + \frac{b\,\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x^4(1+x^2)}\,dx, x, c+dx\right)}{2de^5}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{4de^5(c+dx)^4} + \frac{b\,\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x^4}\,dx, x, c+dx\right)}{2de^5} - \frac{b\,\text{Subst}\left(\int \frac{a+}{}\right.}{}$$

$$= -\frac{b\left(a+b\tan^{-1}(c+dx)\right)}{6de^5(c+dx)^3} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{4de^5(c+dx)^4} - \frac{b\,\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x^2}\,dx,\right.}{2de^5}$$

$$= -\frac{b\left(a+b\tan^{-1}(c+dx)\right)}{6de^5(c+dx)^3} + \frac{b\left(a+b\tan^{-1}(c+dx)\right)}{2de^5(c+dx)} + \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{4de^5} -$$

$$= -\frac{b\left(a+b\tan^{-1}(c+dx)\right)}{6de^5(c+dx)^3} + \frac{b\left(a+b\tan^{-1}(c+dx)\right)}{2de^5(c+dx)} + \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{4de^5} -$$

$$= -\frac{b^2}{12de^5(c+dx)^2} - \frac{b\left(a+b\tan^{-1}(c+dx)\right)}{6de^5(c+dx)^3} + \frac{b\left(a+b\tan^{-1}(c+dx)\right)}{2de^5(c+dx)} + \frac{(a+b\,t}{}$$

$$= -\frac{b^2}{12de^5(c+dx)^2} - \frac{b\left(a+b\tan^{-1}(c+dx)\right)}{6de^5(c+dx)^3} + \frac{b\left(a+b\tan^{-1}(c+dx)\right)}{2de^5(c+dx)} + \frac{(a+b\,t}{}$$

Mathematica [A] time = 0.35, size = 245, normalized size = 1.44

$$-\frac{3a^2 - 2b\tan^{-1}(c+dx)\left(3a\left(c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4-1\right)+b\left(3c^3+9c^2dx+9cd^2x^2-c+3d^3x\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^5,x]
```

```
[Out] -1/12*(3*a^2 + 2*a*b*(c + d*x) + b^2*(c + d*x)^2 - 6*a*b*(c + d*x)^3 - 2*b*
(b*(-c + 3*c^3 - d*x + 9*c^2*d*x + 9*c*d^2*x^2 + 3*d^3*x^3) + 3*a*(-1 + c^4
 + 4*c^3*d*x + 6*c^2*d^2*x^2 + 4*c*d^3*x^3 + d^4*x^4))*ArcTan[c + d*x] - 3*
b^2*(-1 + c^4 + 4*c^3*d*x + 6*c^2*d^2*x^2 + 4*c*d^3*x^3 + d^4*x^4)*ArcTan[c
 + d*x]^2 + 8*b^2*(c + d*x)^4*Log[c + d*x] - 4*b^2*(c + d*x)^4*Log[1 + c^2
+ 2*c*d*x + d^2*x^2])/(d*e^5*(c + d*x)^4)
```

fricas [B] time = 0.47, size = 448, normalized size = 2.64

$$\frac{6\,abd^3x^3 + 6\,abc^3 + \left(18\,abc - b^2\right)d^2x^2 - b^2c^2 - 2\,abc + 2\left(9\,abc^2 - b^2c - ab\right)dx + 3\left(b^2d^4x^4 + 4\,b^2cd^3x^3 + 6\,b^2c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(6*a*b*d^3*x^3 + 6*a*b*c^3 + (18*a*b*c - b^2)*d^2*x^2 - b^2*c^2 - 2*a*
b*c + 2*(9*a*b*c^2 - b^2*c - a*b)*d*x + 3*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 +
```

```
6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 - b^2)*arctan(d*x + c)^2 - 3*a^
2 + 2*(3*a*b*d^4*x^4 + 3*(4*a*b*c + b^2)*d^3*x^3 + 3*a*b*c^4 + 3*b^2*c^3 +
9*(2*a*b*c^2 + b^2*c)*d^2*x^2 - b^2*c + (12*a*b*c^3 + 9*b^2*c^2 - b^2)*d*x
- 3*a*b)*arctan(d*x + c) + 4*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2
*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*log(d^2*x^2 + 2*c*d*x + c^2 + 1) - 8*(b^2*d
^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*log
(d*x + c))/(d^5*e^5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d^2*e
^5*x + c^4*d*e^5)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^5,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.06, size = 242, normalized size = 1.42

$$-\frac{a^2}{4d\,e^5\,(dx+c)^4}-\frac{b^2\arctan(dx+c)^2}{4d\,e^5\,(dx+c)^4}-\frac{b^2\arctan(dx+c)}{6d\,e^5\,(dx+c)^3}+\frac{b^2\arctan(dx+c)}{2d\,e^5\,(dx+c)}+\frac{b^2\arctan(dx+c)^2}{4d\,e^5}-\frac{b^2}{12d\,e^5\,(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^5,x)
```

```
[Out] -1/4/d*a^2/e^5/(d*x+c)^4-1/4/d*b^2/e^5/(d*x+c)^4*arctan(d*x+c)^2-1/6/d*b^2/
e^5*arctan(d*x+c)/(d*x+c)^3+1/2/d*b^2/e^5*arctan(d*x+c)/(d*x+c)+1/4/d*b^2/e
^5*arctan(d*x+c)^2-1/12*b^2/d/e^5/(d*x+c)^2-2/3*b^2*ln(d*x+c)/d/e^5+1/3*b^2
*ln(1+(d*x+c)^2)/d/e^5-1/2/d*a*b/e^5/(d*x+c)^4*arctan(d*x+c)-1/6/d*a*b/e^5/
(d*x+c)^3+1/2/d*a*b/e^5/(d*x+c)+1/2/d*a*b/e^5*arctan(d*x+c)
```

maxima [B] time = 0.51, size = 534, normalized size = 3.14

$$\frac{1}{6}\left(d\left(\frac{3\,d^2x^2+6\,cdx+3\,c^2-1}{d^5e^5x^3+3\,cd^4e^5x^2+3\,c^2d^3e^5x+c^3d^2e^5}+\frac{3\,\arctan\left(\frac{d^2x+cd}{d}\right)}{d^2e^5}\right)-\frac{3\,\arctan(dx+c)}{d^5e^5x^4+4\,cd^4e^5x^3+6\,c^2d^3e^5x^2+4\,c^3d^2e^5x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^5,x, algorithm="maxima")
```

```
[Out] 1/6*(d*((3*d^2*x^2 + 6*c*d*x + 3*c^2 - 1)/(d^5*e^5*x^3 + 3*c*d^4*e^5*x^2 +
3*c^2*d^3*e^5*x + c^3*d^2*e^5) + 3*arctan((d^2*x + c*d)/d)/(d^2*e^5)) - 3*a
rctan(d*x + c)/(d^5*e^5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d
^2*e^5*x + c^4*d*e^5))*a*b + 1/12*(2*d*((3*d^2*x^2 + 6*c*d*x + 3*c^2 - 1)/(
d^5*e^5*x^3 + 3*c*d^4*e^5*x^2 + 3*c^2*d^3*e^5*x + c^3*d^2*e^5) + 3*arctan((
d^2*x + c*d)/d)/(d^2*e^5))*arctan(d*x + c) - (3*(d^2*x^2 + 2*c*d*x + c^2)*a
rctan(d*x + c)^2 - 4*(d^2*x^2 + 2*c*d*x + c^2)*log(d^2*x^2 + 2*c*d*x + c^2
+ 1) + 8*(d^2*x^2 + 2*c*d*x + c^2)*log(d*x + c) + 1)*d^2/(d^5*e^5*x^2 + 2*c
*d^4*e^5*x + c^2*d^3*e^5))*b^2 - 1/4*b^2*arctan(d*x + c)^2/(d^5*e^5*x^4 + 4
*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d^2*e^5*x + c^4*d*e^5) - 1/4*a^2
/(d^5*e^5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d^2*e^5*x + c^4
*d*e^5)
```

mupad [B] time = 3.65, size = 438, normalized size = 2.58

$$\operatorname{atan}(c+d\,x)^2\left(\frac{b^2}{4\,d\,e^5}-\frac{b^2}{4\,d^3\,e^5\,\left(\frac{c^4}{d^2}+6\,c^2\,x^2+d^2\,x^4+\frac{4\,c^3\,x}{d}+4\,c\,d\,x^3\right)}\right)-\frac{x^2\,\left(\frac{b^2\,d}{2}-9\,a\,b\,c\,d\right)+x\,\left(b^2\,c-9\,a\,b\,c^2+\right.}{6\,c^4\,e^5+24\,c^3\,d\,e^5\,x+36\,c^2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))^2/(c*e + d*e*x)^5,x)
```

```
[Out] atan(c + d*x)^2*(b^2/(4*d*e^5) - b^2/(4*d^3*e^5*(c^4/d^2 + 6*c^2*x^2 + d^2*
x^4 + (4*c^3*x)/d + 4*c*d*x^3))) - (x^2*((b^2*d)/2 - 9*a*b*c*d) + x*(a*b +
b^2*c - 9*a*b*c^2) + (3*a^2 + b^2*c^2 + 2*a*b*c - 6*a*b*c^3)/(2*d) - 3*a*b*
d^2*x^3)/(6*c^4*e^5 + 6*d^4*e^5*x^4 + 24*c*d^3*e^5*x^3 + 36*c^2*d^2*e^5*x^2
 + 24*c^3*d*e^5*x) + (atan(c + d*x)*((b^2*x^3)/(2*e^5) - (a*b)/(2*d^3*e^5)
+ (b^2*c*((c^2 - 1)/(3*d^2) + (2*c^2)/(3*d^2)))/(2*d*e^5) + (b^2*x*(d*((c^2
 - 1)/(3*d^2) + (2*c^2)/(3*d^2)) + (2*c^2)/d))/(2*d*e^5) + (3*b^2*c*x^2)/(2
*d*e^5)))/(c^4/d^2 + 6*c^2*x^2 + d^2*x^4 + (4*c^3*x)/d + 4*c*d*x^3) - (2*b^
2*log(c + d*x))/(3*d*e^5) - (log(c + d*x - 1i)*((a*b*1i)/4 - b^2/3))/(d*e^5
) + (log(c + d*x + 1i)*((a*b*1i)/4 + b^2/3))/(d*e^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**2/(d*e*x+c*e)**5,x)
```

```
[Out] Timed out
```

3.15 $\int (ce+dex)^2 \left(a+b\tan^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=271

$$-\frac{ib^2e^2\text{Li}_2\left(1-\frac{2}{i(c+dx)+1}\right)\left(a+b\tan^{-1}(c+dx)\right)}{d}+ab^2e^2x-\frac{be^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}-\frac{be^2(c+dx)^2\left(a+b\tan^{-1}(c+\right.}{2d}$$

```
[Out] a*b^2*e^2*x+b^3*e^2*(d*x+c)*arctan(d*x+c)/d-1/2*b*e^2*(a+b*arctan(d*x+c))^2
/d-1/2*b*e^2*(d*x+c)^2*(a+b*arctan(d*x+c))^2/d-1/3*I*e^2*(a+b*arctan(d*x+c)
)^3/d+1/3*e^2*(d*x+c)^3*(a+b*arctan(d*x+c))^3/d-b*e^2*(a+b*arctan(d*x+c))^2
*ln(2/(1+I*(d*x+c)))/d-1/2*b^3*e^2*ln(1+(d*x+c)^2)/d-I*b^2*e^2*(a+b*arctan(
d*x+c))*polylog(2,1-2/(1+I*(d*x+c)))/d-1/2*b^3*e^2*polylog(3,1-2/(1+I*(d*x+
c)))/d
```

Rubi [A] time = 0.44, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {5043, 12, 4852, 4916, 4846, 260, 4884, 4920, 4854, 4994, 6610}

$$-\frac{ib^2e^2\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{d}-\frac{b^3e^2\text{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2d}+ab^2e^2x-\frac{be^2\left(a+b\tan^{-1}(\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] a*b^2*e^2*x + (b^3*e^2*(c + d*x)*ArcTan[c + d*x])/d - (b*e^2*(a + b*ArcTan[
c + d*x])^2)/(2*d) - (b*e^2*(c + d*x)^2*(a + b*ArcTan[c + d*x])^2)/(2*d) -
((I/3)*e^2*(a + b*ArcTan[c + d*x])^3)/d + (e^2*(c + d*x)^3*(a + b*ArcTan[c
+ d*x])^3)/(3*d) - (b*e^2*(a + b*ArcTan[c + d*x])^2*Log[2/(1 + I*(c + d*x))
])/d - (b^3*e^2*Log[1 + (c + d*x)^2])/(2*d) - (I*b^2*e^2*(a + b*ArcTan[c +
d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d - (b^3*e^2*PolyLog[3, 1 - 2/(1
 + I*(c + d*x))])/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (ce+dex)^2\left(a+b\tan^{-1}(c+dx)\right)^3\,dx=\frac{\text{Subst}\left(\int e^2x^2\left(a+b\tan^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$
$$=\frac{e^2\,\text{Subst}\left(\int x^2\left(a+b\tan^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$
$$=\frac{e^2(c+dx)^3\left(a+b\tan^{-1}(c+dx)\right)^3}{3d}-\frac{\left(be^2\right)\text{Subst}\left(\int\frac{x^3\left(a+b\tan^{-1}(x)\right)^2}{1+x^2}\,dx,\right.}{d}$$
$$=\frac{e^2(c+dx)^3\left(a+b\tan^{-1}(c+dx)\right)^3}{3d}-\frac{\left(be^2\right)\text{Subst}\left(\int x\left(a+b\tan^{-1}(x)\right)^2\right.}{d}$$
$$=-\frac{be^2(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}-\frac{ie^2\left(a+b\tan^{-1}(c+dx)\right)^3}{3d}+\frac{e^2(}{}$$
$$=-\frac{be^2(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}-\frac{ie^2\left(a+b\tan^{-1}(c+dx)\right)^3}{3d}+\frac{e^2(}{}$$
$$=ab^2e^2x-\frac{be^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}-\frac{be^2(c+dx)^2\left(a+b\tan^{-1}(c+dx\right.}{2d}$$
$$=ab^2e^2x+\frac{b^3e^2(c+dx)\tan^{-1}(c+dx)}{d}-\frac{be^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}-\frac{be^2}{}$$
$$=ab^2e^2x+\frac{b^3e^2(c+dx)\tan^{-1}(c+dx)}{d}-\frac{be^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}-\frac{be^2}{}$$

Mathematica [A] time = 0.77, size = 349, normalized size = 1.29

$$\frac{e^2\left(2a^3(c+dx)^3-3a^2b(c+dx)^2+3a^2b\log\left((c+dx)^2+1\right)+6a^2b(c+dx)^3\tan^{-1}(c+dx)+6ab^2\left(i\text{Li}_2\left(-e^{2i\tan^{-1}(c}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] (e^2*(-3*a^2*b*(c + d*x)^2 + 2*a^3*(c + d*x)^3 + 6*a^2*b*(c + d*x)^3*ArcTan
[c + d*x] + 3*a^2*b*Log[1 + (c + d*x)^2] + 6*a*b^2*(c + d*x - ArcTan[c + d*
x] - (c + d*x)^2*ArcTan[c + d*x] + I*ArcTan[c + d*x]^2 + (c + d*x)^3*ArcTan
[c + d*x]^2 - 2*ArcTan[c + d*x]*Log[1 + E^((2*I)*ArcTan[c + d*x])] + I*Poly
Log[2, -E^((2*I)*ArcTan[c + d*x])]) + b^3*(6*(c + d*x)*ArcTan[c + d*x] - 3*
(1 + (c + d*x)^2)*ArcTan[c + d*x]^2 + (2*I)*ArcTan[c + d*x]^3 - 2*(c + d*x)
*ArcTan[c + d*x]^3 + 2*(c + d*x)*(1 + (c + d*x)^2)*ArcTan[c + d*x]^3 - 6*Ar
cTan[c + d*x]^2*Log[1 + E^((2*I)*ArcTan[c + d*x])] + 6*Log[1/Sqrt[1 + (c +
d*x)^2]] + (6*I)*ArcTan[c + d*x]*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])] - 3
*PolyLog[3, -E^((2*I)*ArcTan[c + d*x])])))/(6*d)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(a^3d^2e^2x^2+2\,a^3cde^2x+a^3c^2e^2+\left(b^3d^2e^2x^2+2\,b^3cde^2x+b^3c^2e^2\right)\arctan\left(dx+c\right)^3+3\left(ab^2d^2e^2x^2+2\,ab\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*d^2*e^2*x^2 + 2*a^3*c*d*e^2*x + a^3*c^2*e^2 + (b^3*d^2*e^2*x^2
 + 2*b^3*c*d*e^2*x + b^3*c^2*e^2)*arctan(d*x + c)^3 + 3*(a*b^2*d^2*e^2*x^2
+ 2*a*b^2*c*d*e^2*x + a*b^2*c^2*e^2)*arctan(d*x + c)^2 + 3*(a^2*b*d^2*e^2*x
^2 + 2*a^2*b*c*d*e^2*x + a^2*b*c^2*e^2)*arctan(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [C] time = 2.25, size = 3242, normalized size = 11.96

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^3,x)
```

```
[Out] 1/4/d*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csgn(I*((1+I*(d*x+c))^2/(1+
(d*x+c)^2)+1)^2)^2*arctan(d*x+c)^2*Pi*b^3*c*e^2+1/4*I/d*e^2*b^3*csgn(I*((1+
I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^2*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)
)*arctan(d*x+c)^2*Pi-1/8*I/d*e^2*b^3*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+
1)^2)*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*arctan(d*x+c)^2*Pi-1/4*I/
d*e^2*b^3*csgn(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)*arctan(d*x+c)^2*csgn(
I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^2*Pi-1
/2*I/d*e^2*b^3*arctan(d*x+c)^2*csgn(I*(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))*cs
gn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2*Pi+1/4*I/d*e^2*b^3*csgn(I*(1+I*(d*x+c
))^4/(1+(d*x+c)^2)^2+2*I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+I)^2*csgn(I*(1+I*(d*
x+c))^2/(1+(d*x+c)^2)+I)*arctan(d*x+c)^2*Pi-1/4/d*csgn(I*(1+I*(d*x+c))^4/(1
+(d*x+c)^2)^2+2*I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+I)^2*csgn(I*(1+I*(d*x+c))^2
/(1+(d*x+c)^2)+I)*arctan(d*x+c)^2*Pi*b^3*c*e^2-1/4*I/d*e^2*b^3*arctan(d*x+c
)^2*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^
2)^2*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))*Pi-1/8*I/d*e^2*b^3*csgn(I*(1+I*(
d*x+c))^4/(1+(d*x+c)^2)^2+2*I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+I)*csgn(I*(1+I*
(d*x+c))^2/(1+(d*x+c)^2)+I)^2*arctan(d*x+c)^2*Pi+1/4*I/d*e^2*b^3*arctan(d*x
+c)^2*csgn(I*(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))^2*csgn(I*(1+I*(d*x+c))^2/(1
+(d*x+c)^2))*Pi-1/2*d*x^2*a^2*b*e^2+d*x^2*a^3*c*e^2+arctan(d*x+c)^3*x*b^3*c
^2*e^2-arctan(d*x+c)^2*x*b^3*c*e^2-1/d*e^2*b^3*arctan(d*x+c)^2*ln((1+I*(d*x
+c))/(1+(d*x+c)^2)^(1/2))+1/2/d*e^2*b^3*arctan(d*x+c)^2*ln(1+(d*x+c)^2)+1/d
*arctan(d*x+c)*b^3*c*e^2+1/3/d*arctan(d*x+c)^3*b^3*c^3*e^2-1/2/d*arctan(d*x
+c)^2*b^3*c^2*e^2-1/d*e^2*b^3*ln(2)*arctan(d*x+c)^2-1/d*e^2*a*b^2*arctan(d*
x+c)+1/3*d^2*arctan(d*x+c)^3*x^3*b^3*e^2-1/2*d*arctan(d*x+c)^2*x^2*b^3*e^2+
1/2/d*e^2*a^2*b*ln(1+(d*x+c)^2)+1/3*I/d*e^2*b^3*arctan(d*x+c)^3-I/d*e^2*b^3
*arctan(d*x+c)-d*arctan(d*x+c)*x^2*a*b^2*e^2+d^2*arctan(d*x+c)*x^3*a^2*b*e^
2+1/d*arctan(d*x+c)^2*a*b^2*c^3*e^2-1/d*arctan(d*x+c)*a*b^2*c^2*e^2+d^2*arc
tan(d*x+c)^2*x^3*a*b^2*e^2+d*arctan(d*x+c)^3*x^2*b^3*c*e^2+1/3*d^2*x^3*a^3*
e^2+1/d*e^2*b^3*ln((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)-1/2/d*e^2*b^3*arctan(d*
x+c)^2-1/2/d*e^2*b^3*polylog(3,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))+arctan(d*x+c
)*x*b^3*e^2+x*a^3*c^2*e^2+1/d*a*b^2*c*e^2-x*a^2*b*c*e^2+a*b^2*e^2*x-1/8*csg
n(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)
^2)+1)^2)*arctan(d*x+c)^2*Pi*x*b^3*e^2-1/4*csgn(I*(1+I*(d*x+c))^4/(1+(d*x+c
)^2)^2+2*I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+I)^2*csgn(I*(1+I*(d*x+c))^2/(1+(d*
x+c)^2)+I)*arctan(d*x+c)^2*Pi*x*b^3*e^2+1/8*csgn(I*(1+I*(d*x+c))^4/(1+(d*x+
c)^2)^2+2*I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+I)*csgn(I*(1+I*(d*x+c))^2/(1+(d*x
+c)^2)+I)^2*arctan(d*x+c)^2*Pi*x*b^3*e^2+3*d*arctan(d*x+c)^2*x^2*a*b^2*c*e^
```

```
2-1/8/d*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^3*arctan(d*x+c)^2*Pi*b^
3*c*e^2+1/8/d*csgn(I*(1+I*(d*x+c))^4/(1+(d*x+c)^2)^2+2*I*(1+I*(d*x+c))^2/(1
+(d*x+c)^2)+I)^3*arctan(d*x+c)^2*Pi*b^3*c*e^2-1/2*I/d*e^2*a*b^2*ln(I+d*x+c)
*ln(1+(d*x+c)^2)+1/2*I/d*e^2*a*b^2*ln(I+d*x+c)*ln(1/2*I*(d*x+c-I))-1/8*I/d*
e^2*b^3*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^3*arctan(d*x+c)^2*Pi-1/
8*I/d*e^2*b^3*csgn(I*(1+I*(d*x+c))^4/(1+(d*x+c)^2)^2+2*I*(1+I*(d*x+c))^2/(1
+(d*x+c)^2)+I)^3*arctan(d*x+c)^2*Pi+1/4*I/d*e^2*b^3*arctan(d*x+c)^2*csgn(I*
(1+I*(d*x+c))^2/(1+(d*x+c)^2)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^3*Pi+1/4
*I/d*e^2*b^3*arctan(d*x+c)^2*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))^3*Pi+3*d
*arctan(d*x+c)*x^2*a^2*b*c*e^2+1/2*I/d*e^2*a*b^2*ln(d*x+c-I)*ln(1+(d*x+c)^2
)-1/2*I/d*e^2*a*b^2*ln(d*x+c-I)*ln(-1/2*I*(I+d*x+c))+1/3/d*a^3*c^3*e^2+1/d*
arctan(d*x+c)*a^2*b*c^3*e^2+1/d*e^2*a*b^2*arctan(d*x+c)*ln(1+(d*x+c)^2)+I/d
*e^2*b^3*arctan(d*x+c)*polylog(2,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))-1/8*csgn(I
*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^3*arctan(d*x+c)^2*Pi*x*b^3*e^2+3*arct
an(d*x+c)^2*x*a*b^2*c^2*e^2+3*arctan(d*x+c)*x*a^2*b*c^2*e^2+1/8*csgn(I*(1+I
*(d*x+c))^4/(1+(d*x+c)^2)^2+2*I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+I)^3*arctan(d
*x+c)^2*Pi*x*b^3*e^2-2*arctan(d*x+c)*x*a*b^2*c*e^2+1/4*I/d*e^2*a*b^2*ln(I+d
*x+c)^2+1/2*I/d*e^2*a*b^2*dilog(1/2*I*(d*x+c-I))-1/4*I/d*e^2*a*b^2*ln(d*x+c
-I)^2-1/2*I/d*e^2*a*b^2*dilog(-1/2*I*(I+d*x+c))+1/4*I/d*e^2*b^3*csgn(I/((1+
I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)*arctan(d*x+c)^2*csgn(I*(1+I*(d*x+c))^2/(1+
(d*x+c)^2)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)*csgn(I*(1+I*(d*x+c))^2/(1+(
d*x+c)^2))*Pi-1/2/d*a^2*b*c^2*e^2+1/4*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)
+1))*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^2*arctan(d*x+c)^2*Pi*x*b^3
*e^2+1/8/d*csgn(I*(1+I*(d*x+c))^4/(1+(d*x+c)^2)^2+2*I*(1+I*(d*x+c))^2/(1+(d
*x+c)^2)+I)*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+I)^2*arctan(d*x+c)^2*Pi*b^
3*c*e^2-1/8/d*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*csgn(I*((1+I*(d*x
+c))^2/(1+(d*x+c)^2)+1)^2)*arctan(d*x+c)^2*Pi*b^3*c*e^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 7/8*b^3*c^4*e^2*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 3*a*b^2*c^4*e
^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - (3*arctan(d*x + c)*arctan(
(d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2*c^4*e^2 - 7/32*(6
*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)*arctan((
d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^3*c^4*e^2 + 1/3*a^3*d^
2*e^2*x^3 + 7/8*b^3*c^2*e^2*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 2
8*b^3*d^4*e^2*integrate(1/32*x^4*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2
 + 1), x) + 3*b^3*d^4*e^2*integrate(1/32*x^4*arctan(d*x + c)*log(d^2*x^2 +
2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 96*a*b^2*d^4*e^2*i
ntegrate(1/32*x^4*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 112
*b^3*c*d^3*e^2*integrate(1/32*x^3*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^
2 + 1), x) + 4*b^3*d^4*e^2*integrate(1/32*x^4*arctan(d*x + c)*log(d^2*x^2 +
 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*c*d^3*e^2*in
tegrate(1/32*x^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^
2 + 2*c*d*x + c^2 + 1), x) + 384*a*b^2*c*d^3*e^2*integrate(1/32*x^3*arctan(
d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 168*b^3*c^2*d^2*e^2*integrat
e(1/32*x^2*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 16*b^3*c*d
^3*e^2*integrate(1/32*x^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/
(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 18*b^3*c^2*d^2*e^2*integrate(1/32*x^2*a
rctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2
+ 1), x) + 576*a*b^2*c^2*d^2*e^2*integrate(1/32*x^2*arctan(d*x + c)^2/(d^2*
x^2 + 2*c*d*x + c^2 + 1), x) + 112*b^3*c^3*d*e^2*integrate(1/32*x*arctan(d*
x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 24*b^3*c^2*d^2*e^2*integrate(1
/32*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x
 + c^2 + 1), x) + 12*b^3*c^3*d*e^2*integrate(1/32*x*arctan(d*x + c)*log(d^2
```

```
*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 384*a*b^2*c
^3*d*e^2*integrate(1/32*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1),
x) + 12*b^3*c^3*d*e^2*integrate(1/32*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*
x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*c^4*e^2*integrate(1/
32*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x +
c^2 + 1), x) + a^3*c*d*e^2*x^2 + 3*a*b^2*c^2*e^2*arctan(d*x + c)^2*arctan((
d^2*x + c*d)/d)/d - 4*b^3*d^3*e^2*integrate(1/32*x^3*arctan(d*x + c)^2/(d^2
*x^2 + 2*c*d*x + c^2 + 1), x) + b^3*d^3*e^2*integrate(1/32*x^3*log(d^2*x^2
+ 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 12*b^3*c*d^2*e^2
*integrate(1/32*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3
*b^3*c*d^2*e^2*integrate(1/32*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x
^2 + 2*c*d*x + c^2 + 1), x) - 12*b^3*c^2*d*e^2*integrate(1/32*x*arctan(d*x
+ c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*c^2*d*e^2*integrate(1/32*x
*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - (3*
arctan(d*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*
a*b^2*c^2*e^2 - 7/32*(6*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*a
rctan(d*x + c)*arctan((d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b
^3*c^2*e^2 + 3*(x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*x +
c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a^2*b*c*d*e^2 + 1/2*
(2*x^3*arctan(d*x + c) - d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2
*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*a^2*b
*d^2*e^2 + a^3*c^2*e^2*x + 28*b^3*d^2*e^2*integrate(1/32*x^2*arctan(d*x + c
)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*d^2*e^2*integrate(1/32*x^2*ar
ctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 +
 1), x) + 96*a*b^2*d^2*e^2*integrate(1/32*x^2*arctan(d*x + c)^2/(d^2*x^2 +
2*c*d*x + c^2 + 1), x) + 56*b^3*c*d*e^2*integrate(1/32*x*arctan(d*x + c)^3/
(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*c*d*e^2*integrate(1/32*x*arctan(d
*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x
) + 192*a*b^2*c*d*e^2*integrate(1/32*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x
 + c^2 + 1), x) + 3*b^3*c^2*e^2*integrate(1/32*arctan(d*x + c)*log(d^2*x^2
+ 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3/2*(2*(d*x + c)
*arctan(d*x + c) - log((d*x + c)^2 + 1))*a^2*b*c^2*e^2/d + 1/24*(b^3*d^2*e^
2*x^3 + 3*b^3*c*d*e^2*x^2 + 3*b^3*c^2*e^2*x)*arctan(d*x + c)^3 - 1/32*(b^3*
d^2*e^2*x^3 + 3*b^3*c*d*e^2*x^2 + 3*b^3*c^2*e^2*x)*arctan(d*x + c)*log(d^2*
x^2 + 2*c*d*x + c^2 + 1)^2
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^2\,(a+b\,\mathrm{atan}\,(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*atan(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)^2*(a + b*atan(c + d*x))^3, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$e^2\left(\int a^3c^2\,dx+\int a^3d^2x^2\,dx+\int b^3c^2\,\mathrm{atan}^3\,(c+dx)\,dx+\int 3ab^2c^2\,\mathrm{atan}^2\,(c+dx)\,dx+\int 3a^2bc^2\,\mathrm{atan}\,(c+d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*atan(d*x+c))**3,x)
```

```
[Out] e**2*(Integral(a**3*c**2, x) + Integral(a**3*d**2*x**2, x) + Integral(b**3*
c**2*atan(c + d*x)**3, x) + Integral(3*a*b**2*c**2*atan(c + d*x)**2, x) + I
ntegral(3*a**2*b*c**2*atan(c + d*x), x) + Integral(2*a**3*c*d*x, x) + Integ
ral(b**3*d**2*x**2*atan(c + d*x)**3, x) + Integral(3*a*b**2*d**2*x**2*atan(
c + d*x)**2, x) + Integral(3*a**2*b*d**2*x**2*atan(c + d*x), x) + Integral(
2*b**3*c*d*x*atan(c + d*x)**3, x) + Integral(6*a*b**2*c*d*x*atan(c + d*x)**
2, x) + Integral(6*a**2*b*c*d*x*atan(c + d*x), x))
```

3.16 $\int (ce+dex)\left(a+b\tan^{-1}(c+dx)\right)^3\,dx$

Optimal. Leaf size=164

$$-\frac{3b^2e\log\left(\frac{2}{1+i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{d}-\frac{3ibe\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}-\frac{3be(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}+\frac{e(c-}{}$$

```
[Out] -3/2*I*b*e*(a+b*arctan(d*x+c))^2/d-3/2*b*e*(d*x+c)*(a+b*arctan(d*x+c))^2/d+
1/2*e*(a+b*arctan(d*x+c))^3/d+1/2*e*(d*x+c)^2*(a+b*arctan(d*x+c))^3/d-3*b^2
*e*(a+b*arctan(d*x+c))*ln(2/(1+I*(d*x+c)))/d-3/2*I*b^3*e*polylog(2,1-2/(1+I
*(d*x+c)))/d
```

Rubi [A] time = 0.24, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {5043, 12, 4852, 4916, 4846, 4920, 4854, 2402, 2315, 4884}

$$-\frac{3ib^3e\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{2d}-\frac{3b^2e\log\left(\frac{2}{1+i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{d}-\frac{3ibe\left(a+b\tan^{-1}(c+dx)\right)^2}{2d}-\frac{3be(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] (((-3*I)/2)*b*e*(a + b*ArcTan[c + d*x])^2)/d - (3*b*e*(c + d*x)*(a + b*ArcT
an[c + d*x])^2)/(2*d) + (e*(a + b*ArcTan[c + d*x])^3)/(2*d) + (e*(c + d*x)^
2*(a + b*ArcTan[c + d*x])^3)/(2*d) - (3*b^2*e*(a + b*ArcTan[c + d*x])*Log[2
/(1 + I*(c + d*x))])/d - (((3*I)/2)*b^3*e*PolyLog[2, 1 - 2/(1 + I*(c + d*x)
)])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
```

```
erQ[m]) && NeQ[m, -1]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (ce+dex)\left(a+b\tan^{-1}(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int ex\left(a+b\tan^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$
$$= \frac{e\,\text{Subst}\left(\int x\left(a+b\tan^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$
$$= \frac{e(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)^3}{2d} - \frac{(3be)\,\text{Subst}\left(\int \frac{x^2\left(a+b\tan^{-1}(x)\right)^2}{1+x^2}\,dx,x,\right.}{2d}$$
$$= \frac{e(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)^3}{2d} - \frac{(3be)\,\text{Subst}\left(\int \left(a+b\tan^{-1}(x)\right)^2\,dx\right.}{2d}$$
$$= -\frac{3be(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{2d} + \frac{e\left(a+b\tan^{-1}(c+dx)\right)^3}{2d} + \frac{e(c+}{}$$
$$= -\frac{3ibe\left(a+b\tan^{-1}(c+dx)\right)^2}{2d} - \frac{3be(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{2d} + \frac{e(}{}$$
$$= -\frac{3ibe\left(a+b\tan^{-1}(c+dx)\right)^2}{2d} - \frac{3be(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{2d} + \frac{e(}{}$$
$$= -\frac{3ibe\left(a+b\tan^{-1}(c+dx)\right)^2}{2d} - \frac{3be(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{2d} + \frac{e(}{}$$
$$= -\frac{3ibe\left(a+b\tan^{-1}(c+dx)\right)^2}{2d} - \frac{3be(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{2d} + \frac{e(}{}$$

Mathematica [A] time = 0.52, size = 196, normalized size = 1.20

$$\frac{e\left(3b\tan^{-1}(c+dx)\left(a\left(a\left(c^2+2cdx+d^2x^2+1\right)-2b(c+dx)\right)-2b^2\log\left(1+e^{2i\tan^{-1}(c+dx)}\right)\right)+a\left(a(c+dx)(ac+a\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] (e*(3*b^2*(-I + c + d*x)*(-b + a*(I + c + d*x))*ArcTan[c + d*x]^2 + b^3*(1
+ c^2 + 2*c*d*x + d^2*x^2)*ArcTan[c + d*x]^3 + 3*b*ArcTan[c + d*x]*(a*(-2*b
*(c + d*x) + a*(1 + c^2 + 2*c*d*x + d^2*x^2)) - 2*b^2*Log[1 + E^((2*I)*ArcT
an[c + d*x])]) + a*(a*(c + d*x)*(-3*b + a*c + a*d*x) - 6*b^2*Log[1/Sqrt[1 +
 (c + d*x)^2]]) + (3*I)*b^3*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])]))/(2*d)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(a^3dex+a^3ce+\left(b^3dex+b^3ce\right)\arctan\left(dx+c\right)^3+3\left(ab^2dex+ab^2ce\right)\arctan\left(dx+c\right)^2+3\left(a^2bdex+a^2b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*d*e*x + a^3*c*e + (b^3*d*e*x + b^3*c*e)*arctan(d*x + c)^3 + 3*
(a*b^2*d*e*x + a*b^2*c*e)*arctan(d*x + c)^2 + 3*(a^2*b*d*e*x + a^2*b*c*e)*a
rctan(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.14, size = 567, normalized size = 3.46

$$\frac{3ie\,b^3 \operatorname{dilog}\left(\frac{i(dx+c-i)}{2}\right)}{4d}-\frac{3ie\,b^3 \ln\left(dx+c-i\right)^2}{8d}+\frac{3ie\,b^3 \ln\left(dx+c+i\right)^2}{8d}-\frac{3ie\,b^3 \operatorname{dilog}\left(-\frac{i(dx+c+i)}{2}\right)}{4d}+\frac{3ea\,b^2 \ln\left(1+(\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arctan(d*x+c))^3,x)
```

```
[Out] 3/2/d*e*a*b^2*ln(1+(d*x+c)^2)-3/4*I/d*e*b^3*dilog(-1/2*I*(I+d*x+c))-3/8*I/d
*e*b^3*ln(d*x+c-I)^2+3/4*I/d*e*b^3*dilog(1/2*I*(d*x+c-I))+3/8*I/d*e*b^3*ln(
I+d*x+c)^2-3*arctan(d*x+c)*x*a*b^2*e+arctan(d*x+c)^3*x*b^3*c*e+1/2/d*arctan
(d*x+c)^3*b^3*c^2*e-3/2/d*arctan(d*x+c)^2*b^3*c*e+3/2/d*e*a^2*b*arctan(d*x+
c)+3/2/d*e*b^3*arctan(d*x+c)*ln(1+(d*x+c)^2)+3/2/d*e*a*b^2*arctan(d*x+c)^2-
3/2/d*a^2*b*c*e+1/2*d*arctan(d*x+c)^3*x^2*b^3*e+3*arctan(d*x+c)^2*x*a*b^2*c
*e-3/4*I/d*e*b^3*ln(I+d*x+c)*ln(1+(d*x+c)^2)-3/4*I/d*e*b^3*ln(d*x+c-I)*ln(-
1/2*I*(I+d*x+c))+3/4*I/d*e*b^3*ln(d*x+c-I)*ln(1+(d*x+c)^2)+3/2/d*arctan(d*x
+c)^2*a*b^2*c^2*e+3/4*I/d*e*b^3*ln(I+d*x+c)*ln(1/2*I*(d*x+c-I))+1/2/d*a^3*c
^2*e-3/2*e*x*a^2*b+x*a^3*c*e+1/2*d*x^2*a^3*e-3/2*arctan(d*x+c)^2*x*b^3*e+1/
2/d*e*b^3*arctan(d*x+c)^3+3*arctan(d*x+c)*x*a^2*b*c*e+3/2/d*arctan(d*x+c)*a
^2*b*c^2*e-3/d*arctan(d*x+c)*a*b^2*c*e+3/2*d*arctan(d*x+c)^2*x^2*a*b^2*e+3/
2*d*arctan(d*x+c)*x^2*a^2*b*e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^3*d*e*x^2 + 3/2*(x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d
^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a^2*b*d*e + a
^3*c*e*x + 3/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*a^2*b*c
*e/d + 1/32*(8*(b^3*d^2*e*x^2 + 2*b^3*c*d*e*x + (b^3*c^2 + b^3)*e)*arctan(d
*x + c)^3 + 12*(a*b^2*d^2*e*x^2 + (2*a*b^2*c - b^3)*d*e*x)*arctan(d*x + c)^
2 - 3*(a*b^2*d^2*e*x^2 + (2*a*b^2*c - b^3)*d*e*x)*log(d^2*x^2 + 2*c*d*x + c
^2 + 1)^2 + 4*(4*b^3*c^3*e*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 18
*a*b^2*c^3*e*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 6*(3*arctan(d*x
+ c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2*c^3*e
 - (6*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)*arc
tan((d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^3*c^3*e - 3*b^3*c
^2*e*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d + 4*b^3*c*e*arctan(d*x + c
)^3*arctan((d^2*x + c*d)/d)/d + 128*b^3*d^3*e*integrate(1/32*x^3*arctan(d*x
 + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 576*a*b^2*d^3*e*integrate(1/32*
x^3*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 384*b^3*c*d^2*e*i
ntegrate(1/32*x^2*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 48*
a*b^2*d^3*e*integrate(1/32*x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2
+ 2*c*d*x + c^2 + 1), x) + 1728*a*b^2*c*d^2*e*integrate(1/32*x^2*arctan(d*x
 + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 384*b^3*c^2*d*e*integrate(1/32*
x*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 96*a*b^2*d^3*e*inte
grate(1/32*x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 +
1), x) + 144*a*b^2*c*d^2*e*integrate(1/32*x^2*log(d^2*x^2 + 2*c*d*x + c^2 +
 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 1728*a*b^2*c^2*d*e*integrate(1/32
*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 288*a*b^2*c*d^2*e*
```

```
integrate(1/32*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^
2 + 1), x) + 144*a*b^2*c^2*d*e*integrate(1/32*x*log(d^2*x^2 + 2*c*d*x + c^2
 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*a*b^2*c^2*d*e*integrate(1/3
2*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 48
*a*b^2*c^3*e*integrate(1/32*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2
*c*d*x + c^2 + 1), x) + (3*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^2/d - ar
ctan((d^2*x + c*d)/d)^3/d)*b^3*c^2*e + 18*a*b^2*c*e*arctan(d*x + c)^2*arcta
n((d^2*x + c*d)/d)/d - 96*b^3*d^2*e*integrate(1/32*x^2*arctan(d*x + c)^2/(d
^2*x^2 + 2*c*d*x + c^2 + 1), x) - 24*b^3*d^2*e*integrate(1/32*x^2*log(d^2*x
^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 192*a*b^2*d^2
*e*integrate(1/32*x^2*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 1
92*b^3*c*d*e*integrate(1/32*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 +
1), x) - 96*b^3*d^2*e*integrate(1/32*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(
d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 48*b^3*c*d*e*integrate(1/32*x*log(d^2*x^
2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 384*a*b^2*c*d*
e*integrate(1/32*x*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 96*b
^3*c*d*e*integrate(1/32*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d
*x + c^2 + 1), x) - 24*b^3*c^2*e*integrate(1/32*log(d^2*x^2 + 2*c*d*x + c^2
 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 6*(3*arctan(d*x + c)*arctan((d^
2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2*c*e - (6*arctan(d*x
+ c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)*arctan((d^2*x + c*d)
/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^3*c*e - 3*b^3*e*arctan(d*x + c)^2*
arctan((d^2*x + c*d)/d)/d + 128*b^3*d*e*integrate(1/32*x*arctan(d*x + c)^3/
(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 576*a*b^2*d*e*integrate(1/32*x*arctan(d
*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 48*a*b^2*d*e*integrate(1/32*x
*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192
*b^3*d*e*integrate(1/32*x*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)
 + 48*a*b^2*c*e*integrate(1/32*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2
+ 2*c*d*x + c^2 + 1), x) + (3*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^2/d -
 arctan((d^2*x + c*d)/d)^3/d)*b^3*e - 24*b^3*e*integrate(1/32*log(d^2*x^2 +
 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x))*d)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)\,\left(a+b\,\mathrm{atan}\left(c+d\,x\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*atan(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)*(a + b*atan(c + d*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int a^{3}c\,dx + \int a^{3}dx\,dx + \int b^{3}c\operatorname{atan}^{3}{\left(c + dx \right)}\,dx + \int 3ab^{2}c\operatorname{atan}^{2}{\left(c + dx \right)}\,dx + \int 3a^{2}bc\operatorname{atan}{\left(c + dx \right)}\,dx + \int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*atan(d*x+c))**3,x)
```

```
[Out] e*(Integral(a**3*c, x) + Integral(a**3*d*x, x) + Integral(b**3*c*atan(c + d
*x)**3, x) + Integral(3*a*b**2*c*atan(c + d*x)**2, x) + Integral(3*a**2*b*c
*atan(c + d*x), x) + Integral(b**3*d*x*atan(c + d*x)**3, x) + Integral(3*a*
b**2*d*x*atan(c + d*x)**2, x) + Integral(3*a**2*b*d*x*atan(c + d*x), x))
```

3.17 $\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{ce+dex}\,dx$

Optimal. Leaf size=279

$$-\frac{3b^2\text{Li}_3\left(1-\frac{2}{i(c+dx)+1}\right)\left(a+b\tan^{-1}(c+dx)\right)}{2de}+\frac{3b^2\text{Li}_3\left(\frac{2}{i(c+dx)+1}-1\right)\left(a+b\tan^{-1}(c+dx)\right)}{2de}-\frac{3ib\text{Li}_2\left(1-\frac{2}{i(c+dx)+\cdots}\right.}{\cdots}$$

```
[Out] -2*(a+b*arctan(d*x+c))^3*arctanh(-1+2/(1+I*(d*x+c)))/d/e-3/2*I*b*(a+b*arcta
n(d*x+c))^2*polylog(2,1-2/(1+I*(d*x+c)))/d/e+3/2*I*b*(a+b*arctan(d*x+c))^2*
polylog(2,-1+2/(1+I*(d*x+c)))/d/e-3/2*b^2*(a+b*arctan(d*x+c))*polylog(3,1-2
/(1+I*(d*x+c)))/d/e+3/2*b^2*(a+b*arctan(d*x+c))*polylog(3,-1+2/(1+I*(d*x+c)
))/d/e+3/4*I*b^3*polylog(4,1-2/(1+I*(d*x+c)))/d/e-3/4*I*b^3*polylog(4,-1+2/
(1+I*(d*x+c)))/d/e
```

Rubi [A] time = 0.46, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5043, 12, 4850, 4988, 4884, 4994, 4998, 6610}

$$-\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{2de}+\frac{3b^2\text{PolyLog}\left(3,-1+\frac{2}{1+i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{2de}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x),x]
```

```
[Out] (2*(a + b*ArcTan[c + d*x])^3*ArcTanh[1 - 2/(1 + I*(c + d*x))])/(d*e) - (((3
*I)/2)*b*(a + b*ArcTan[c + d*x])^2*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/(d*
e) + (((3*I)/2)*b*(a + b*ArcTan[c + d*x])^2*PolyLog[2, -1 + 2/(1 + I*(c + d
*x))])/(d*e) - (3*b^2*(a + b*ArcTan[c + d*x])*PolyLog[3, 1 - 2/(1 + I*(c +
d*x))])/(2*d*e) + (3*b^2*(a + b*ArcTan[c + d*x])*PolyLog[3, -1 + 2/(1 + I*(
c + d*x))])/(2*d*e) + (((3*I)/4)*b^3*PolyLog[4, 1 - 2/(1 + I*(c + d*x))])/(
d*e) - (((3*I)/4)*b^3*PolyLog[4, -1 + 2/(1 + I*(c + d*x))])/(d*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4998

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{ce+dex}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^3}{ex}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^3}{x}\,dx, x, c+dx\right)}{de}$$

$$= \frac{2\left(a+b\tan^{-1}(c+dx)\right)^3\tanh^{-1}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} - \frac{(6b)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2\tanh^{-1}(}{1+x^2}\right.}{de}$$

$$= \frac{2\left(a+b\tan^{-1}(c+dx)\right)^3\tanh^{-1}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} - \frac{(3b)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2\log\left(2-\right.}{1+x^2}\right.}{de}$$

$$= \frac{2\left(a+b\tan^{-1}(c+dx)\right)^3\tanh^{-1}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} - \frac{3ib\left(a+b\tan^{-1}(c+dx)\right)^2\text{Li}_2\left(1\right.}{2de}$$

$$= \frac{2\left(a+b\tan^{-1}(c+dx)\right)^3\tanh^{-1}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} - \frac{3ib\left(a+b\tan^{-1}(c+dx)\right)^2\text{Li}_2\left(1\right.}{2de}$$

$$= \frac{2\left(a+b\tan^{-1}(c+dx)\right)^3\tanh^{-1}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} - \frac{3ib\left(a+b\tan^{-1}(c+dx)\right)^2\text{Li}_2\left(1\right.}{2de}$$

Mathematica [A] time = 0.15, size = 252, normalized size = 0.90

$$6b^2\mathrm{Li}_3\left(-\frac{c+dx+i}{c+dx-i}\right)\left(a+b\tan^{-1}(c+dx)\right)-6b^2\mathrm{Li}_3\left(\frac{c+dx+i}{c+dx-i}\right)\left(a+b\tan^{-1}(c+dx)\right)+6ib\mathrm{Li}_2\left(-\frac{c+dx+i}{c+dx-i}\right)\left(a+b\tan^{-1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x),x]
```

```
[Out] (8*(a + b*ArcTan[c + d*x])^3*ArcTanh[(I + c + d*x)/(-I + c + d*x)] + (6*I)*
b*(a + b*ArcTan[c + d*x])^2*PolyLog[2, -((I + c + d*x)/(-I + c + d*x))] - (
6*I)*b*(a + b*ArcTan[c + d*x])^2*PolyLog[2, (I + c + d*x)/(-I + c + d*x)] +
 6*b^2*(a + b*ArcTan[c + d*x])*PolyLog[3, -((I + c + d*x)/(-I + c + d*x))]
- 6*b^2*(a + b*ArcTan[c + d*x])*PolyLog[3, (I + c + d*x)/(-I + c + d*x)] -
(3*I)*b^3*PolyLog[4, -((I + c + d*x)/(-I + c + d*x))] + (3*I)*b^3*PolyLog[4
, (I + c + d*x)/(-I + c + d*x)])/(4*d*e)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\arctan\left(dx+c\right)^3+3\,ab^2\arctan\left(dx+c\right)^2+3\,a^2b\arctan\left(dx+c\right)+a^3}{dex+ce},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arcta
n(d*x + c) + a^3)/(d*e*x + c*e), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.28, size = 2894, normalized size = 10.37

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^3/(d*e*x+c*e),x)
```

```
[Out] 1/d*a^3/e*ln(d*x+c)+1/2*I/d*b^3/e*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-
1))*csgn(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csgn(I*((1+I*(d*x+c))^2/(1+(d
*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*arctan(d*x+c)^3-3/2*I/d*a*b^
2/e*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)
^2)+1))*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^
2)+1))^2*arctan(d*x+c)^2+3/2*I/d*a*b^2/e*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x
+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csgn(((1+I*(d*x+c))^2/(1+(d*x+
c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*arctan(d*x+c)^2-3/2*I/d*a*b^2/e
*Pi*csgn(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csgn(I*((1+I*(d*x+c))^2/(1+(d
*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*arctan(d*x+c)^2-3/2*I/d*a*
b^2/e*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1))*csgn(I*((1+I*(d*x+c))^2/
(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*arctan(d*x+c)^2-3/2*I
/d*a^2*b/e*dilog(1-I*(d*x+c))+3/2*I/d*a^2*b/e*dilog(1+I*(d*x+c))+1/2*I/d*b^
3/e*Pi*arctan(d*x+c)^3+3/d*a*b^2/e*ln(d*x+c)*arctan(d*x+c)^2-3/d*a*b^2/e*ar
ctan(d*x+c)^2*ln((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)+3/d*a*b^2/e*arctan(d*x+c)
```

```
^2*ln(1+(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+3/d*a*b^2/e*arctan(d*x+c)^2*ln(1
-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))-3*I/d*b^3/e*arctan(d*x+c)^2*polylog(2,(
1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+3/2*I/d*b^3/e*arctan(d*x+c)^2*polylog(2,-
(1+I*(d*x+c))^2/(1+(d*x+c)^2))-3*I/d*b^3/e*arctan(d*x+c)^2*polylog(2,-(1+I*
(d*x+c))/(1+(d*x+c)^2)^(1/2))+3/d*a^2*b/e*ln(d*x+c)*arctan(d*x+c)-6*I/d*a*b
^2/e*arctan(d*x+c)*polylog(2,(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+3*I/d*a*b^2
/e*arctan(d*x+c)*polylog(2,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))-1/2*I/d*b^3/e*Pi
*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^
2*arctan(d*x+c)^3+1/2*I/d*b^3/e*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)
/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^3*arctan(d*x+c)^3+1/2*I/d*b^3/e*Pi*csgn
(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^3*arc
tan(d*x+c)^3-3/2*I/d*a^2*b/e*ln(d*x+c)*ln(1-I*(d*x+c))-6*I/d*a*b^2/e*arctan
(d*x+c)*polylog(2,-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+3/2*I/d*a^2*b/e*ln(d*
x+c)*ln(1+I*(d*x+c))+3/2*I/d*a*b^2/e*Pi*arctan(d*x+c)^2-1/2*I/d*b^3/e*Pi*cs
gn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*c
sgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*
arctan(d*x+c)^3-3/2*I/d*a*b^2/e*Pi*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(
(1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*arctan(d*x+c)^2-1/2*I/d*b^3/e*Pi*csgn(I
/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1
)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*arctan(d*x+c)^3-1/2*I/d*b^3/e*Pi*csg
n(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1))*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2
)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*arctan(d*x+c)^3+3/2*I/d*a*b^2/e*P
i*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1
))^3*arctan(d*x+c)^2+1/2*I/d*b^3/e*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)
-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-
1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*arctan(d*x+c)^3+6*I/d*b^3/e*polylog(4
,(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+6*I/d*b^3/e*polylog(4,-(1+I*(d*x+c))/(1
+(d*x+c)^2)^(1/2))-3/4*I/d*b^3/e*polylog(4,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))-
3/2/d*b^3/e*arctan(d*x+c)*polylog(3,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))+1/d*b^3
/e*ln(d*x+c)*arctan(d*x+c)^3-1/d*b^3/e*arctan(d*x+c)^3*ln((1+I*(d*x+c))^2/(
1+(d*x+c)^2)-1)+1/d*b^3/e*arctan(d*x+c)^3*ln(1+(1+I*(d*x+c))/(1+(d*x+c)^2)^
(1/2))+3/2*I/d*a*b^2/e*Pi*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x
+c))^2/(1+(d*x+c)^2)+1))^3*arctan(d*x+c)^2+6/d*b^3/e*arctan(d*x+c)*polylog(
3,-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+1/d*b^3/e*arctan(d*x+c)^3*ln(1-(1+I*(
d*x+c))/(1+(d*x+c)^2)^(1/2))+6/d*b^3/e*arctan(d*x+c)*polylog(3,(1+I*(d*x+c)
)/(1+(d*x+c)^2)^(1/2))+6/d*a*b^2/e*polylog(3,-(1+I*(d*x+c))/(1+(d*x+c)^2)^(
1/2))+6/d*a*b^2/e*polylog(3,(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))-3/2/d*a*b^2/
e*polylog(3,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))+3/2*I/d*a*b^2/e*Pi*csgn(I*((1+I
*(d*x+c))^2/(1+(d*x+c)^2)-1))*csgn(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csg
n(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*ar
ctan(d*x+c)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^3\log(dex+ce)}{de}+\int\frac{28\,b^3\arctan(dx+c)^3+3\,b^3\arctan(dx+c)\log\left(d^2x^2+2\,cdx+c^2+1\right)^2+96\,ab^2\arctan(d}{32\,(dex+ce)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] a^3*log(d*e*x + c*e)/(d*e) + integrate(1/32*(28*b^3*arctan(d*x + c)^3 + 3*b
^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 96*a*b^2*arctan(d*x
 + c)^2 + 96*a^2*b*arctan(d*x + c))/(d*e*x + c*e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{(a+b\,\mathrm{atan}(c+d\,x))^3}{c\,e+d\,e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))^3/(c*e + d*e*x),x)
```

```
[Out] int((a + b*atan(c + d*x))^3/(c*e + d*e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^3}{c+dx}\,dx + \int \frac{b^3 \operatorname{atan}^3(c+dx)}{c+dx}\,dx + \int \frac{3ab^2 \operatorname{atan}^2(c+dx)}{c+dx}\,dx + \int \frac{3a^2 b \operatorname{atan}(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**3/(d*e*x+c*e),x)
```

```
[Out] (Integral(a**3/(c + d*x), x) + Integral(b**3*atan(c + d*x)**3/(c + d*x), x)
 + Integral(3*a*b**2*atan(c + d*x)**2/(c + d*x), x) + Integral(3*a**2*b*ata
n(c + d*x)/(c + d*x), x))/e
```

3.18 $\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{(ce+dex)^2}\,dx$

Optimal. Leaf size=163

$$-\frac{3ib^2\text{Li}_2\left(\frac{2}{1-i(c+dx)}-1\right)\left(a+b\tan^{-1}(c+dx)\right)}{de^2}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{de^2(c+dx)}-\frac{i\left(a+b\tan^{-1}(c+dx)\right)^3}{de^2}+\frac{3b\log\left(2-\frac{2}{1-i(c+}\right.}{}$$

```
[Out] -I*(a+b*arctan(d*x+c))^3/d/e^2-(a+b*arctan(d*x+c))^3/d/e^2/(d*x+c)+3*b*(a+b
*arctan(d*x+c))^2*ln(2-2/(1-I*(d*x+c)))/d/e^2-3*I*b^2*(a+b*arctan(d*x+c))*p
olylog(2,-1+2/(1-I*(d*x+c)))/d/e^2+3/2*b^3*polylog(3,-1+2/(1-I*(d*x+c)))/d/
e^2
```

Rubi [A] time = 0.30, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5043, 12, 4852, 4924, 4868, 4884, 4992, 6610}

$$-\frac{3ib^2\text{PolyLog}\left(2,-1+\frac{2}{1-i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{de^2}+\frac{3b^3\text{PolyLog}\left(3,-1+\frac{2}{1-i(c+dx)}\right)}{2de^2}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{de^2(c+dx)}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x)^2,x]
```

```
[Out] ((-I)*(a + b*ArcTan[c + d*x])^3)/(d*e^2) - (a + b*ArcTan[c + d*x])^3/(d*e^2
*(c + d*x)) + (3*b*(a + b*ArcTan[c + d*x])^2*Log[2 - 2/(1 - I*(c + d*x))])/
(d*e^2) - ((3*I)*b^2*(a + b*ArcTan[c + d*x])*PolyLog[2, -1 + 2/(1 - I*(c +
d*x))])/(d*e^2) + (3*b^3*PolyLog[3, -1 + 2/(1 - I*(c + d*x))])/(2*d*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{(ce+dex)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^3}{e^2x^2}\,dx,x,c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^3}{x^2}\,dx,x,c+dx\right)}{de^2}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{(3b)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{x\left(1+x^2\right)}\,dx,x,c+dx\right)}{de^2}$$

$$= -\frac{i\left(a+b\tan^{-1}(c+dx)\right)^3}{de^2} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{(3ib)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x\right.}{x(i+x)}\right.}{de^2}$$

$$= -\frac{i\left(a+b\tan^{-1}(c+dx)\right)^3}{de^2} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{3b\left(a+b\tan^{-1}(c+dx)\right)^2}{de^2}$$

$$= -\frac{i\left(a+b\tan^{-1}(c+dx)\right)^3}{de^2} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{3b\left(a+b\tan^{-1}(c+dx)\right)^2}{de^2}$$

$$= -\frac{i\left(a+b\tan^{-1}(c+dx)\right)^3}{de^2} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{3b\left(a+b\tan^{-1}(c+dx)\right)^2}{de^2}$$

Mathematica [A] time = 0.69, size = 263, normalized size = 1.61

$$-\frac{2a^3}{c+dx} - 3a^2b\log\left(c^2+2cdx+d^2x^2+1\right) + 6a^2b\log(c+dx) - \frac{6a^2b\tan^{-1}(c+dx)}{c+dx} + 6ab^2\left(\tan^{-1}(c+dx)\left(\left(-\frac{1}{c+dx} - i\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x)^2,x]
```

```
[Out] ((-2*a^3)/(c + d*x) - (6*a^2*b*ArcTan[c + d*x])/(c + d*x) + 6*a^2*b*Log[c +
 d*x] - 3*a^2*b*Log[1 + c^2 + 2*c*d*x + d^2*x^2] + 6*a*b^2*(ArcTan[c + d*x]
*((-I - (c + d*x)^(-1))*ArcTan[c + d*x] + 2*Log[1 - E^((2*I)*ArcTan[c + d*x
])]) - I*PolyLog[2, E^((2*I)*ArcTan[c + d*x])]) + 2*b^3*((-1/8*I)*Pi^3 + I*
ArcTan[c + d*x]^3 - ArcTan[c + d*x]^3/(c + d*x) + 3*ArcTan[c + d*x]^2*Log[1
 - E^((-2*I)*ArcTan[c + d*x])] + (3*I)*ArcTan[c + d*x]*PolyLog[2, E^((-2*I)
*ArcTan[c + d*x])] + (3*PolyLog[3, E^((-2*I)*ArcTan[c + d*x])])/2))/(2*d*e^
2)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3 \arctan(dx+c)^3 + 3\,ab^2 \arctan(dx+c)^2 + 3\,a^2 b \arctan(dx+c) + a^3}{d^2e^2x^2 + 2\,cde^2x + c^2e^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arcta
n(d*x + c) + a^3)/(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.44, size = 2696, normalized size = 16.54

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^2,x)
```

```
[Out] -3/2*I/d*a*b^2/e^2*ln(I+d*x+c)*ln(1/2*I*(d*x+c-I))+3*I/d*a*b^2/e^2*ln(d*x+c
)*ln(1+I*(d*x+c))-3/2*I/d*a*b^2/e^2*ln(d*x+c-I)*ln(1+(d*x+c)^2)-3*I/d*a*b^2
/e^2*ln(d*x+c)*ln(1-I*(d*x+c))-1/d*b^3/e^2/(d*x+c)*arctan(d*x+c)^3-3/d*b^3/
e^2*arctan(d*x+c)^2*ln((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)+3/d*b^3/e^2*arctan(
d*x+c)^2*ln(1+(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+3/d*b^3/e^2*arctan(d*x+c)^
2*ln((1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+3/d*b^3/e^2*arctan(d*x+c)^2*ln(1-(1
+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+3/d*b^3/e^2*ln(d*x+c)*arctan(d*x+c)^2-3/2/
d*b^3/e^2*arctan(d*x+c)^2*ln(1+(d*x+c)^2)+3/d*a^2*b/e^2*ln(d*x+c)-3/2/d*a^2
*b/e^2*ln(1+(d*x+c)^2)+3/d*b^3/e^2*arctan(d*x+c)^2*ln(2)-I/d*b^3/e^2*arctan
(d*x+c)^3+3/2*I/d*a*b^2/e^2*ln(I+d*x+c)*ln(1+(d*x+c)^2)-3/2*I/d*b^3/e^2*Pi*
csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2
*arctan(d*x+c)^2+3/2*I/d*b^3/e^2*Pi*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/
((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^3*arctan(d*x+c)^2+3/4*I/d*b^3/e^2*arctan
(d*x+c)^2*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^3-3/4*I/d*b^3/e^2*
arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))^3-3/4*I/d*b^3/e^2*
arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/((1+I*(d*x+c))^2/(1
+(d*x+c)^2)+1)^2)^3+3/2*I/d*b^3/e^2*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2
)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^3*arctan(d*x+c)^2+3/2*I/d*a*b^2/e^2
*ln(d*x+c-I)*ln(-1/2*I*(I+d*x+c))-1/d*a^3/e^2/(d*x+c)+6/d*b^3/e^2*polylog(3
```

```
,(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+6/d*b^3/e^2*polylog(3,-(1+I*(d*x+c))/(1
+(d*x+c)^2)^(1/2))-3/d*a*b^2/e^2/(d*x+c)*arctan(d*x+c)^2+6/d*a*b^2/e^2*arct
an(d*x+c)*ln(d*x+c)-3/d*a*b^2/e^2*arctan(d*x+c)*ln(1+(d*x+c)^2)-3/d*a^2*b/e
^2/(d*x+c)*arctan(d*x+c)-6*I/d*b^3/e^2*arctan(d*x+c)*polylog(2,-(1+I*(d*x+c
))/(1+(d*x+c)^2)^(1/2))-3/4*I/d*b^3/e^2*arctan(d*x+c)^2*Pi*csgn(I/((1+I*(d*
x+c))^2/(1+(d*x+c)^2)+1)^2)*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))*csgn(I*(1
+I*(d*x+c))^2/(1+(d*x+c)^2)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)+3/2*I/d*b^
3/e^2*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1))*csgn(I/((1+I*(d*x+c))^2/
(1+(d*x+c)^2)+1))*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2
/(1+(d*x+c)^2)+1))*arctan(d*x+c)^2-3/4*I/d*b^3/e^2*arctan(d*x+c)^2*Pi*csgn(
I*(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))^2*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)
)+3/4*I/d*b^3/e^2*arctan(d*x+c)^2*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+
1))^2*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)+3/2*I/d*b^3/e^2*arctan(d*
x+c)^2*Pi*csgn(I*(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))*csgn(I*(1+I*(d*x+c))^2/
(1+(d*x+c)^2))^2+3/4*I/d*b^3/e^2*arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))^2/
(1+(d*x+c)^2))*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/((1+I*(d*x+c))^2/(1+(d*
x+c)^2)+1)^2)^2-3/2*I/d*b^3/e^2*arctan(d*x+c)^2*Pi*csgn(I*((1+I*(d*x+c))^2/
(1+(d*x+c)^2)+1))*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^2+3/2*I/d*b^3
/e^2*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c
)^2)+1))*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)
^2)+1))*arctan(d*x+c)^2+3*I/d*a*b^2/e^2*dilog(1+I*(d*x+c))+3/4*I/d*a*b^2/e^
2*ln(d*x+c-I)^2+3/2*I/d*a*b^2/e^2*dilog(-1/2*I*(I+d*x+c))-6*I/d*b^3/e^2*arc
tan(d*x+c)*polylog(2,(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))-3/4*I/d*a*b^2/e^2*l
n(I+d*x+c)^2-3/2*I/d*a*b^2/e^2*dilog(1/2*I*(d*x+c-I))+3/2*I/d*b^3/e^2*Pi*ar
ctan(d*x+c)^2-3*I/d*a*b^2/e^2*dilog(1-I*(d*x+c))-3/2*I/d*b^3/e^2*Pi*csgn(I*
((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1))*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)
/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*arctan(d*x+c)^2+3/4*I/d*b^3/e^2*arcta
n(d*x+c)^2*Pi*csgn(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)*csgn(I*(1+I*(d*x+
c))^2/(1+(d*x+c)^2)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^2-3/2*I/d*b^3/e^2*
Pi*csgn(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csgn(I*((1+I*(d*x+c))^2/(1+(d*
x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*arctan(d*x+c)^2-3/2*I/d*b^3
/e^2*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c
)^2)+1))*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)
^2)+1))^2*arctan(d*x+c)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{3}{2}\left(d\left(\frac{\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^2e^2}-\frac{2\,\log\left(dx+c\right)}{d^2e^2}\right)+\frac{2\,\arctan\left(dx+c\right)}{d^2e^2x+cde^2}\right)a^2b-\frac{a^3}{d^2e^2x+cde^2}-\frac{\frac{15}{2}\,b^3\arctan\left(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] -3/2*(d*(log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*e^2) - 2*log(d*x + c)/(d^2*e
^2)) + 2*arctan(d*x + c)/(d^2*e^2*x + c*d*e^2))*a^2*b - a^3/(d^2*e^2*x + c*
d*e^2) - 1/32*(4*b^3*arctan(d*x + c)^3 - 3*b^3*arctan(d*x + c)*log(d^2*x^2
+ 2*c*d*x + c^2 + 1)^2 - 32*(d^2*e^2*x + c*d*e^2)*integrate(1/32*(28*(b^3*d
^2*x^2 + 2*b^3*c*d*x + b^3*c^2 + b^3)*arctan(d*x + c)^3 + 12*(8*a*b^2*d^2*x
^2 + 8*a*b^2*c^2 + b^3*c + 8*a*b^2 + (16*a*b^2*c + b^3)*d*x)*arctan(d*x + c
)^2 - 12*(b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2)*arctan(d*x + c)*log(d^2*x^2
+ 2*c*d*x + c^2 + 1) - 3*(b^3*d*x + b^3*c - (b^3*d^2*x^2 + 2*b^3*c*d*x + b^
3*c^2 + b^3)*arctan(d*x + c))*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2)/(d^4*e^2*
x^4 + 4*c*d^3*e^2*x^3 + (6*c^2 + 1)*d^2*e^2*x^2 + 2*(2*c^3 + c)*d*e^2*x + (
c^4 + c^2)*e^2), x))/(d^2*e^2*x + c*d*e^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{atan}\left(c+d\,x\right)\right)}^3}{{\left(c\,e+d\,e\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))^3/(c*e + d*e*x)^2,x)
```

```
[Out] int((a + b*atan(c + d*x))^3/(c*e + d*e*x)^2, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^3}{c^2+2cdx+d^2x^2}\,dx + \int \frac{b^3\operatorname{atan}^3(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{3ab^2\operatorname{atan}^2(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{3a^2b\operatorname{atan}(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**3/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**3*atan(c + d*
x)**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(3*a*b**2*atan(c + d*x)**2
/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(3*a**2*b*atan(c + d*x)/(c**2 +
 2*c*d*x + d**2*x**2), x))/e**2
```

3.19 $\int \frac{(a+b\tan^{-1}(c+dx))^3}{(ce+dex)^3}\,dx$

Optimal. Leaf size=180

$$\frac{3b^2\log\left(2-\frac{2}{1-i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{de^3}-\frac{3b\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3(c+dx)}-\frac{3ib\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{2de^3(c+dx)^2}$$

```
[Out] -3/2*I*b*(a+b*arctan(d*x+c))^2/d/e^3-3/2*b*(a+b*arctan(d*x+c))^2/d/e^3/(d*x
+c)-1/2*(a+b*arctan(d*x+c))^3/d/e^3-1/2*(a+b*arctan(d*x+c))^3/d/e^3/(d*x+c)
^2+3*b^2*(a+b*arctan(d*x+c))*ln(2-2/(1-I*(d*x+c)))/d/e^3-3/2*I*b^3*polylog(
2,-1+2/(1-I*(d*x+c)))/d/e^3
```

Rubi [A] time = 0.32, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5043, 12, 4852, 4918, 4924, 4868, 2447, 4884}

$$-\frac{3ib^3\text{PolyLog}\left(2,-1+\frac{2}{1-i(c+dx)}\right)}{2de^3}+\frac{3b^2\log\left(2-\frac{2}{1-i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{de^3}-\frac{3b\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3(c+dx)}-\frac{3ib\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x)^3,x]
```

```
[Out] (((-3*I)/2)*b*(a + b*ArcTan[c + d*x])^2)/(d*e^3) - (3*b*(a + b*ArcTan[c + d
*x])^2)/(2*d*e^3*(c + d*x)) - (a + b*ArcTan[c + d*x])^3/(2*d*e^3) - (a + b*
ArcTan[c + d*x])^3/(2*d*e^3*(c + d*x)^2) + (3*b^2*(a + b*ArcTan[c + d*x])*L
og[2 - 2/(1 - I*(c + d*x))])/(d*e^3) - (((3*I)/2)*b^3*PolyLog[2, -1 + 2/(1
- I*(c + d*x))])/(d*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{(ce+dex)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^3}{e^3x^3}\,dx,x,c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^3}{x^3}\,dx,x,c+dx\right)}{de^3}\\
&= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{2de^3(c+dx)^2}+\frac{(3b)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{x^2\left(1+x^2\right)}\,dx,x,c+dx\right)}{2de^3}\\
&= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{2de^3(c+dx)^2}+\frac{(3b)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{x^2}\,dx,x,c+dx\right)}{2de^3}-\frac{(3b)\,\text{Subst}}{\cdots}\\
&= -\frac{3b\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3(c+dx)}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{2de^3}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{2de^3(c+dx)^2}+\cdots\\
&= -\frac{3ib\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3}-\frac{3b\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3(c+dx)}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{2de^3}\cdots\\
&= -\frac{3ib\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3}-\frac{3b\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3(c+dx)}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{2de^3}\cdots\\
&= -\frac{3ib\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3}-\frac{3b\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^3(c+dx)}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{2de^3}\cdots
\end{aligned}
$$

Mathematica [A] time = 0.31, size = 225, normalized size = 1.25

$$-\frac{a^3+3a^2b\left(\left((c+dx)^2+1\right)\tan^{-1}(c+dx)+c+dx\right)+3ab^2\left(-2(c+dx)^2\log\left(\frac{c+dx}{\sqrt{(c+dx)^2+1}}\right)+2(c+dx)\tan^{-1}(c+dx)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x)^3,x]
```

```
[Out] -1/2*(a^3 + b^3*(1 + c^2 + 2*c*d*x + d^2*x^2)*ArcTan[c + d*x]^3 + 3*a^2*b*(
c + d*x + (1 + (c + d*x)^2)*ArcTan[c + d*x]) + 3*a*b^2*(2*(c + d*x)*ArcTan[
c + d*x] + (1 + (c + d*x)^2)*ArcTan[c + d*x]^2 - 2*(c + d*x)^2*Log[(c + d*x
)/Sqrt[1 + (c + d*x)^2]]) + 3*b^3*(c + d*x)*(ArcTan[c + d*x]^2 - 2*(c + d*x
)*ArcTan[c + d*x]*Log[1 - E^((2*I)*ArcTan[c + d*x])] + I*(c + d*x)*(ArcTan[
c + d*x]^2 + PolyLog[2, E^((2*I)*ArcTan[c + d*x])]))))/(d*e^3*(c + d*x)^2)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\arctan(dx+c)^3+3ab^2\arctan(dx+c)^2+3a^2b\arctan(dx+c)+a^3}{d^3e^3x^3+3cd^2e^3x^2+3c^2de^3x+c^3e^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arcta
n(d*x + c) + a^3)/(d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3)
, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.16, size = 631, normalized size = 3.51

$$\frac{3ib^3\ln(dx+c+i)\ln\left(1+(dx+c)^2\right)}{4d\,e^3}+\frac{3ib^3\ln(dx+c-i)\ln\left(-\frac{i(dx+c+i)}{2}\right)}{4d\,e^3}+\frac{3ib^3\ln(dx+c)\ln(1+i(dx+c))}{2d\,e^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^3,x)
```

```
[Out] 3/4*I/d*b^3/e^3*ln(I+d*x+c)*ln(1+(d*x+c)^2)+3/4*I/d*b^3/e^3*ln(d*x+c-I)*ln(
-1/2*I*(I+d*x+c))-3/2/d*a*b^2/e^3/(d*x+c)^2*arctan(d*x+c)^2-3/2/d*a^2*b/e^3
/(d*x+c)^2*arctan(d*x+c)+3/2*I/d*b^3/e^3*ln(d*x+c)*ln(1+I*(d*x+c))-3/2*I/d*
b^3/e^3*ln(d*x+c)*ln(1-I*(d*x+c))-3/4*I/d*b^3/e^3*ln(d*x+c-I)*ln(1+(d*x+c)^
2)-3/4*I/d*b^3/e^3*ln(I+d*x+c)*ln(1/2*I*(d*x+c-I))-3/d*a*b^2/e^3*arctan(d*x
+c)/(d*x+c)-3/2/d*b^3/e^3*arctan(d*x+c)*ln(1+(d*x+c)^2)+3/8*I/d*b^3/e^3*ln(
d*x+c-I)^2-3/8*I/d*b^3/e^3*ln(I+d*x+c)^2+3/2*I/d*b^3/e^3*dilog(1+I*(d*x+c))
-3/4*I/d*b^3/e^3*dilog(1/2*I*(d*x+c-I))+3/4*I/d*b^3/e^3*dilog(-1/2*I*(I+d*x
+c))-3/2*I/d*b^3/e^3*dilog(1-I*(d*x+c))-3/2/d*a^2*b/e^3/(d*x+c)-3/2/d*a*b^2
/e^3*ln(1+(d*x+c)^2)-3/2/d*a^2*b/e^3*arctan(d*x+c)-3/2/d*a*b^2/e^3*arctan(d
*x+c)^2+3/d*a*b^2/e^3*ln(d*x+c)-3/2/d*b^3/e^3*arctan(d*x+c)^2/(d*x+c)+3/d*b
^3/e^3*ln(d*x+c)*arctan(d*x+c)-1/2/d*b^3/e^3/(d*x+c)^2*arctan(d*x+c)^3-1/2/
d*a^3/e^3/(d*x+c)^2-1/2/d*b^3/e^3*arctan(d*x+c)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{3}{2}\left(d\left(\frac{1}{d^3e^3x+cd^2e^3}+\frac{\arctan\left(\frac{d^2x+cd}{d}\right)}{d^2e^3}\right)+\frac{\arctan(dx+c)}{d^3e^3x^2+2\,cd^2e^3x+c^2de^3}\right)a^2b-\frac{3}{2}\left(2\,d\left(\frac{1}{d^3e^3x+cd^2e^3}+\frac{\arctan\left(\frac{d^2x+c}{d}\right.}{d^2e^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] -3/2*(d*(1/(d^3*e^3*x + c*d^2*e^3) + arctan((d^2*x + c*d)/d)/(d^2*e^3)) + a
rctan(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3))*a^2*b - 3/2*(2*d*
(1/(d^3*e^3*x + c*d^2*e^3) + arctan((d^2*x + c*d)/d)/(d^2*e^3))*arctan(d*x
+ c) - (arctan(d*x + c)^2 - log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*log(d*x +
c))/(d*e^3))*a*b^2 - 3/2*a*b^2*arctan(d*x + c)^2/(d^3*e^3*x^2 + 2*c*d^2*e^3
*x + c^2*d*e^3) - 1/32*(8*(d^2*x^2 + 2*c*d*x + c^2 + 1)*arctan(d*x + c)^3 +
 12*(d*x + c)*arctan(d*x + c)^2 - 3*(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 +
 1)^2 - 32*(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)*integrate(1/32*(16*(d^
2*x^2 + 2*c*d*x + c^2 + 1)*arctan(d*x + c)^3 + 12*(d^3*x^3 + 3*c*d^2*x^2 +
c^3 + (3*c^2 + 1)*d*x + c)*arctan(d*x + c)^2 + 3*(d^3*x^3 + 3*c*d^2*x^2 + c
^3 + (3*c^2 + 1)*d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 24*(d^2*x^2
+ 2*c*d*x + c^2)*arctan(d*x + c) - 12*(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x +
c^3)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^5*e^3*x^5 + 5*c*d^4*e^3*x^4 + (10
*c^2 + 1)*d^3*e^3*x^3 + (10*c^3 + 3*c)*d^2*e^3*x^2 + (5*c^4 + 3*c^2)*d*e^3*
x + (c^5 + c^3)*e^3), x))*b^3/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3) - 1
/2*a^3/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atan}\,(c+d\,x))^3}{(c\,e+d\,e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))^3/(c*e + d*e*x)^3,x)
```

```
[Out] int((a + b*atan(c + d*x))^3/(c*e + d*e*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^3}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{b^3\,\mathrm{atan}^3\,(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{3ab^2\,\mathrm{atan}^2\,(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{3a^2b\,\mathrm{atan}\,(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**3/(d*e*x+c*e)**3,x)
```

```
[Out] (Integral(a**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integr
al(b**3*atan(c + d*x)**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x
) + Integral(3*a*b**2*atan(c + d*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 +
 d**3*x**3), x) + Integral(3*a**2*b*atan(c + d*x)/(c**3 + 3*c**2*d*x + 3*c*
d**2*x**2 + d**3*x**3), x))/e**3
```

3.20 $\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{(ce+dex)^4} dx$

Optimal. Leaf size=287

$$\frac{ib^2\text{Li}_2\left(\frac{2}{1-i(c+dx)}-1\right)\left(a+b\tan^{-1}(c+dx)\right)}{de^4}-\frac{b^2\left(a+b\tan^{-1}(c+dx)\right)}{de^4(c+dx)}-\frac{b\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2}-\frac{b\left(a+b\tan^{-1}(\right.}{2de^4}$$

```
[Out] -b^2*(a+b*arctan(d*x+c))/d/e^4/(d*x+c)-1/2*b*(a+b*arctan(d*x+c))^2/d/e^4-1/
2*b*(a+b*arctan(d*x+c))^2/d/e^4/(d*x+c)^2+1/3*I*(a+b*arctan(d*x+c))^3/d/e^4
-1/3*(a+b*arctan(d*x+c))^3/d/e^4/(d*x+c)^3+b^3*ln(d*x+c)/d/e^4-1/2*b^3*ln(1
+(d*x+c)^2)/d/e^4-b*(a+b*arctan(d*x+c))^2*ln(2-2/(1-I*(d*x+c)))/d/e^4+I*b^2
*(a+b*arctan(d*x+c))*polylog(2,-1+2/(1-I*(d*x+c)))/d/e^4-1/2*b^3*polylog(3,
-1+2/(1-I*(d*x+c)))/d/e^4
```

Rubi [A] time = 0.50, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {5043, 12, 4852, 4918, 266, 36, 29, 31, 4884, 4924, 4868, 4992, 6610}

$$\frac{ib^2\text{PolyLog}\left(2,-1+\frac{2}{1-i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{de^4}-\frac{b^3\text{PolyLog}\left(3,-1+\frac{2}{1-i(c+dx)}\right)}{2de^4}-\frac{b^2\left(a+b\tan^{-1}(c+dx)\right)}{de^4(c+dx)}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x)^4,x]
```

```
[Out] -((b^2*(a + b*ArcTan[c + d*x]))/(d*e^4*(c + d*x))) - (b*(a + b*ArcTan[c + d
*x])^2)/(2*d*e^4) - (b*(a + b*ArcTan[c + d*x])^2)/(2*d*e^4*(c + d*x)^2) + (
(I/3)*(a + b*ArcTan[c + d*x])^3)/(d*e^4) - (a + b*ArcTan[c + d*x])^3/(3*d*e
^4*(c + d*x)^3) + (b^3*Log[c + d*x])/(d*e^4) - (b^3*Log[1 + (c + d*x)^2])/(
2*d*e^4) - (b*(a + b*ArcTan[c + d*x])^2*Log[2 - 2/(1 - I*(c + d*x))])/(d*e^
4) + (I*b^2*(a + b*ArcTan[c + d*x])*PolyLog[2, -1 + 2/(1 - I*(c + d*x))])/(
d*e^4) - (b^3*PolyLog[3, -1 + 2/(1 - I*(c + d*x))])/(2*d*e^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{(ce+dex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^3}{e^4x^4}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^3}{x^4}\,dx, x, c+dx\right)}{de^4}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{3de^4(c+dx)^3} + \frac{b\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{x^3(1+x^2)}\,dx, x, c+dx\right)}{de^4}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{3de^4(c+dx)^3} + \frac{b\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{x^3}\,dx, x, c+dx\right)}{de^4} - \frac{b\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{b\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} + \frac{i\left(a+b\tan^{-1}(c+dx)\right)^3}{3de^4} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{3de^4(c+dx)^3}$$

$$= -\frac{b\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} + \frac{i\left(a+b\tan^{-1}(c+dx)\right)^3}{3de^4} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{3de^4(c+dx)^3}$$

$$= -\frac{b^2\left(a+b\tan^{-1}(c+dx)\right)}{de^4(c+dx)} - \frac{b\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^4} - \frac{b\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2}$$

$$= -\frac{b^2\left(a+b\tan^{-1}(c+dx)\right)}{de^4(c+dx)} - \frac{b\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^4} - \frac{b\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2}$$

$$= -\frac{b^2\left(a+b\tan^{-1}(c+dx)\right)}{de^4(c+dx)} - \frac{b\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^4} - \frac{b\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2}$$

$$= -\frac{b^2\left(a+b\tan^{-1}(c+dx)\right)}{de^4(c+dx)} - \frac{b\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^4} - \frac{b\left(a+b\tan^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2}$$

Mathematica [A] time = 1.22, size = 360, normalized size = 1.25

$$-\frac{8a^3}{(c+dx)^3} + 12a^2b\log\left(c^2+2cdx+d^2x^2+1\right) - \frac{12a^2b}{(c+dx)^2} - 24a^2b\log(c+dx) - \frac{24a^2b\tan^{-1}(c+dx)}{(c+dx)^3} + 24ab^2\left(i\text{Li}_2\left(e^{2i\tan}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x)^4,x]
```

```
[Out] ((-8*a^3)/(c + d*x)^3 - (12*a^2*b)/(c + d*x)^2 - (24*a^2*b*ArcTan[c + d*x])
/(c + d*x)^3 - 24*a^2*b*Log[c + d*x] + 12*a^2*b*Log[1 + c^2 + 2*c*d*x + d^2
*x^2] + 24*a*b^2*(-(((c + d*x)^2 + ArcTan[c + d*x]^2)/(c + d*x)^3) + ArcTan
[c + d*x]*(-1 - (c + d*x)^(-2) + I*ArcTan[c + d*x] - 2*Log[1 - E^((2*I)*Arc
Tan[c + d*x])]) + I*PolyLog[2, E^((2*I)*ArcTan[c + d*x])]) + b^3*(I*Pi^3 -
(24*ArcTan[c + d*x])/(c + d*x) - 12*ArcTan[c + d*x]^2 - (12*ArcTan[c + d*x]
^2)/(c + d*x)^2 - (8*I)*ArcTan[c + d*x]^3 - (8*ArcTan[c + d*x]^3)/(c + d*x)
^3 - 24*ArcTan[c + d*x]^2*Log[1 - E^((-2*I)*ArcTan[c + d*x])] + 24*Log[(c +
 d*x)/Sqrt[1 + (c + d*x)^2]] - (24*I)*ArcTan[c + d*x]*PolyLog[2, E^((-2*I)*
ArcTan[c + d*x])] - 12*PolyLog[3, E^((-2*I)*ArcTan[c + d*x])]))/(24*d*e^4)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3 \arctan\left(dx+c\right)^3 + 3\,ab^2 \arctan\left(dx+c\right)^2 + 3\,a^2 b \arctan\left(dx+c\right) + a^3}{d^4 e^4 x^4 + 4\,cd^3 e^4 x^3 + 6\,c^2 d^2 e^4 x^2 + 4\,c^3 de^4 x + c^4 e^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arcta
n(d*x + c) + a^3)/(d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^
3*d*e^4*x + c^4*e^4), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 1.63, size = 7083, normalized size = 24.68

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^4,x)
```

```
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atan}\left(c+d\,x\right)\right)}^3}{{\left(c\,e+d\,e\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))^3/(c*e + d*e*x)^4,x)
```

```
[Out] int((a + b*atan(c + d*x))^3/(c*e + d*e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^3}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{b^3\operatorname{atan}^3{(c+dx)}}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{3ab^2\operatorname{atan}^2{(c+dx)}}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{3a^2l}{c^4+4c^3dx+6c}}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**3/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a**3/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4
*x**4), x) + Integral(b**3*atan(c + d*x)**3/(c**4 + 4*c**3*d*x + 6*c**2*d**
2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(3*a*b**2*atan(c + d*x)**
2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) +
Integral(3*a**2*b*atan(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c
*d**3*x**3 + d**4*x**4), x))/e**4
```

3.21 $\int \frac{\tan^{-1}(1+x)}{2+2x}\,dx$

Optimal. Leaf size=31

$$\frac{1}{4}i\text{Li}_2(-i(x+1))-\frac{1}{4}i\text{Li}_2(i(x+1))$$

```
[Out] 1/4*I*polylog(2,-I*(1+x))-1/4*I*polylog(2,I*(1+x))
```

Rubi [A] time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5043, 12, 4848, 2391}

$$\frac{1}{4}i\text{PolyLog}(2,-i(x+1))-\frac{1}{4}i\text{PolyLog}(2,i(x+1))$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[1 + x]/(2 + 2*x),x]
```

```
[Out] (I/4)*PolyLog[2, (-I)*(1 + x)] - (I/4)*PolyLog[2, I*(1 + x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(1+x)}{2+2x}\,dx &= \text{Subst}\left(\int \frac{\tan^{-1}(x)}{2x}\,dx,x,1+x\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{\tan^{-1}(x)}{x}\,dx,x,1+x\right)\\
&= \frac{1}{4}i\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,1+x\right)-\frac{1}{4}i\,\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx,x,1+x\right)\\
&= \frac{1}{4}i\text{Li}_2(-i(1+x))-\frac{1}{4}i\text{Li}_2(i(1+x))
\end{aligned}$$

Mathematica [A] time = 0.00, size = 31, normalized size = 1.00

$$\frac{1}{4}i\text{Li}_2(-i(x+1))-\frac{1}{4}i\text{Li}_2(i(x+1))$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[1 + x]/(2 + 2*x),x]
```

```
[Out] (I/4)*PolyLog[2, (-I)*(1 + x)] - (I/4)*PolyLog[2, I*(1 + x)]
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\arctan\left(x+1\right)}{2\left(x+1\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(1+x)/(2+2*x),x, algorithm="fricas")
```

```
[Out] integral(1/2*arctan(x + 1)/(x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(1+x)/(2+2*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.06, size = 68, normalized size = 2.19

$$\frac{\ln\left(x+1\right)\arctan\left(x+1\right)}{2}+\frac{i\ln\left(x+1\right)\ln\left(1+i\left(x+1\right)\right)}{4}-\frac{i\ln\left(x+1\right)\ln\left(1-i\left(x+1\right)\right)}{4}+\frac{i\,\mathrm{dilog}\left(1+i\left(x+1\right)\right)}{4}-\frac{i\,\mathrm{di}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x+1)/(2+2*x),x)
```

```
[Out] 1/2*ln(x+1)*arctan(x+1)+1/4*I*ln(x+1)*ln(1+I*(x+1))-1/4*I*ln(x+1)*ln(1-I*(x
+1))+1/4*I*dilog(1+I*(x+1))-1/4*I*dilog(1-I*(x+1))
```

maxima [B] time = 0.46, size = 44, normalized size = 1.42

$$-\frac{1}{4}\arctan\left(x+1,0\right)\log\left(x^2+2\,x+2\right)+\frac{1}{2}\arctan\left(x+1\right)\log\left(|x+1|\right)-\frac{1}{4}i\,\mathrm{Li}_2\left(i\,x+i+1\right)+\frac{1}{4}i\,\mathrm{Li}_2\left(-i\,x-i+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(1+x)/(2+2*x),x, algorithm="maxima")
```

```
[Out] -1/4*arctan2(x + 1, 0)*log(x^2 + 2*x + 2) + 1/2*arctan(x + 1)*log(abs(x + 1
)) - 1/4*I*dilog(I*x + I + 1) + 1/4*I*dilog(-I*x - I + 1)
```

mupad [B] time = 0.08, size = 25, normalized size = 0.81

$$-\frac{\mathrm{Li}_2\left(1-x\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{4}+\frac{\mathrm{Li}_2\left(x\,1\mathrm{i}+1+1\mathrm{i}\right)\,1\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x + 1)/(2*x + 2),x)
```

```
[Out] (dilog(x*1i + (1 + 1i))*1i)/4 - (dilog((1 - 1i) - x*1i)*1i)/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}{\left(x+1\right)}}{x+1}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(1+x)/(2+2*x),x)
```

```
[Out] Integral(atan(x + 1)/(x + 1), x)/2
```

3.22 $\int \frac{\tan^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx$

Optimal. Leaf size=41

$$\frac{i\text{Li}_2(-i(a+bx))}{2d}-\frac{i\text{Li}_2(i(a+bx))}{2d}$$

```
[Out] 1/2*I*polylog(2,-I*(b*x+a))/d-1/2*I*polylog(2,I*(b*x+a))/d
```

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {5043, 12, 4848, 2391}

$$\frac{i\text{PolyLog}(2,-i(a+bx))}{2d}-\frac{i\text{PolyLog}(2,i(a+bx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] ((I/2)*PolyLog[2, (-I)*(a + b*x)])/d - ((I/2)*PolyLog[2, I*(a + b*x)])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 5043

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx &= \frac{\text{Subst}\left(\int \frac{b\tan^{-1}(x)}{dx}\,dx,x,a+bx\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{\tan^{-1}(x)}{x}\,dx,x,a+bx\right)}{d}\\
&= \frac{i\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,a+bx\right)}{2d}-\frac{i\,\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx,x,a+bx\right)}{2d}\\
&= \frac{i\text{Li}_2(-i(a+bx))}{2d}-\frac{i\text{Li}_2(i(a+bx))}{2d}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 0.83

$$\frac{i(\text{Li}_2(-i(a+bx))-\text{Li}_2(i(a+bx)))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] ((I/2)*(PolyLog[2, (-I)*(a + b*x)] - PolyLog[2, I*(a + b*x)]))/d
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\arctan(bx+a)}{bdx+ad},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")
```

```
[Out] integral(b*arctan(b*x + a)/(b*d*x + a*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(a*d/b+d*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.06, size = 98, normalized size = 2.39

$$\frac{\ln(bx+a)\arctan(bx+a)}{d}+\frac{i\ln(bx+a)\ln(1+i(bx+a))}{2d}-\frac{i\ln(bx+a)\ln(1-i(bx+a))}{2d}+\frac{i\,\text{dilog}(1+i(bx+a)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(a*d/b+d*x),x)
```

```
[Out] 1/d*ln(b*x+a)*arctan(b*x+a)+1/2*I/d*ln(b*x+a)*ln(1+I*(b*x+a))-1/2*I/d*ln(b*
x+a)*ln(1-I*(b*x+a))+1/2*I/d*dilog(1+I*(b*x+a))-1/2*I/d*dilog(1-I*(b*x+a))
```

maxima [B] time = 0.49, size = 123, normalized size = 3.00

$$\frac{\arctan(bx+a)\log\left(dx+\frac{ad}{b}\right)}{d}-\frac{\arctan\left(\frac{b^2x+ab}{b}\right)\log\left(dx+\frac{ad}{b}\right)}{d}-\frac{\arctan(bx+a,0)\log\left(b^2x^2+2\,abx+a^2+1\right)-2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")
```

```
[Out] arctan(b*x + a)*log(d*x + a*d/b)/d - arctan((b^2*x + a*b)/b)*log(d*x + a*d/
b)/d - 1/2*(arctan2(b*x + a, 0)*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*arctan
(b*x + a)*log(abs(b*x + a)) + I*dilog(I*b*x + I*a + 1) - I*dilog(-I*b*x - I
*a + 1))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\text{atan}(a+b\,x)}{d\,x+\frac{a\,d}{b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)/(d*x + (a*d)/b),x)
```

```
[Out] int(atan(a + b*x)/(d*x + (a*d)/b), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b\int\frac{\operatorname{atan}(a+bx)}{a+bx}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(a*d/b+d*x),x)
```

```
[Out] b*Integral(atan(a + b*x)/(a + b*x), x)/d
```

3.23 $\int (a+bx)^2\sqrt{\tan^{-1}(a+bx)}\,dx$

Optimal. Leaf size=21

$$\mathrm{Int}\left((a+bx)^2\sqrt{\tan^{-1}(a+bx)},x\right)$$

```
[Out] Unintegrable((b*x+a)^2*arctan(b*x+a)^(1/2),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+bx)^2\sqrt{\tan^{-1}(a+bx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*x)^2*Sqrt[ArcTan[a + b*x]],x]
[Out] Defer[Int][(a + b*x)^2*Sqrt[ArcTan[a + b*x]], x]
```

Rubi steps

$$\int (a+bx)^2\sqrt{\tan^{-1}(a+bx)}\,dx = \int (a+bx)^2\sqrt{\tan^{-1}(a+bx)}\,dx$$

Mathematica [A] time = 6.11, size = 0, normalized size = 0.00

$$\int (a+bx)^2\sqrt{\tan^{-1}(a+bx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*x)^2*Sqrt[ArcTan[a + b*x]],x]
[Out] Integrate[(a + b*x)^2*Sqrt[ArcTan[a + b*x]], x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)^(1/2),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 3.23, size = 0, normalized size = 0.00

$$\int (bx+a)^2\sqrt{\arctan(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*arctan(b*x+a)^(1/2),x)
```

```
[Out] int((b*x+a)^2*arctan(b*x+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \sqrt{\operatorname{atan}\left(a+b\,x\right)}\,{\left(a+b\,x\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)^(1/2)*(a + b*x)^2,x)
```

```
[Out] int(atan(a + b*x)^(1/2)*(a + b*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + bx\right)^{2} \sqrt{\operatorname{atan}{\left(a + bx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*atan(b*x+a)**(1/2),x)
```

```
[Out] Integral((a + b*x)**2*sqrt(atan(a + b*x)), x)
```

3.24 $\int (e+fx)^3 \left(a+b\tan^{-1}(c+dx)\right) dx$

Optimal. Leaf size=233

$$\frac{(e+fx)^4\left(a+b\tan^{-1}(c+dx)\right)}{4f}-\frac{bfx\left(-\left(1-6c^2\right)f^2-12cdef+6d^2e^2\right)}{4d^3}-\frac{b\left(-6\left(1-c^2\right)d^2e^2f^2+4c\left(3-c^2\right)def^3\right.}{}$$

```
[Out] -1/4*b*f*(6*d^2*e^2-12*c*d*e*f-(-6*c^2+1)*f^2)*x/d^3-1/2*b*f^2*(-c*f+d*e)*(
d*x+c)^2/d^4-1/12*b*f^3*(d*x+c)^3/d^4-1/4*b*(d^4*e^4-4*c*d^3*e^3*f-6*(-c^2+
1)*d^2*e^2*f^2+4*c*(-c^2+3)*d*e*f^3+(c^4-6*c^2+1)*f^4)*arctan(d*x+c)/d^4/f+
1/4*(f*x+e)^4*(a+b*arctan(d*x+c))/f-1/2*b*(-c*f+d*e)*(-c*f+d*e+f)*(d*e-(1+c
)*f)*ln(1+(d*x+c)^2)/d^4
```

Rubi [A] time = 0.38, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5047, 4862, 702, 635, 203, 260}

$$\frac{(e+fx)^4\left(a+b\tan^{-1}(c+dx)\right)}{4f}-\frac{bfx\left(-\left(1-6c^2\right)f^2-12cdef+6d^2e^2\right)}{4d^3}-\frac{b\left(-6\left(1-c^2\right)d^2e^2f^2+4c\left(3-c^2\right)def^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] -(b*f*(6*d^2*e^2 - 12*c*d*e*f - (1 - 6*c^2)*f^2)*x)/(4*d^3) - (b*f^2*(d*e -
 c*f)*(c + d*x)^2)/(2*d^4) - (b*f^3*(c + d*x)^3)/(12*d^4) - (b*(d^4*e^4 - 4
*c*d^3*e^3*f - 6*(1 - c^2)*d^2*e^2*f^2 + 4*c*(3 - c^2)*d*e*f^3 + (1 - 6*c^2
 + c^4)*f^4)*ArcTan[c + d*x])/(4*d^4*f) + ((e + f*x)^4*(a + b*ArcTan[c + d*
x]))/(4*f) - (b*(d*e - c*f)*(d*e + f - c*f)*(d*e - (1 + c)*f)*Log[1 + (c +
d*x)^2])/(2*d^4)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
```

```
c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\int (e+fx)^3\left(a+b\tan^{-1}(c+dx)\right)dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^3\left(a+b\tan^{-1}(x)\right)dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^4\left(a+b\tan^{-1}(c+dx)\right)}{4f} - \frac{b\,\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^4}{1+x^2}dx, x, c+dx\right)}{4f}$$

$$= \frac{(e+fx)^4\left(a+b\tan^{-1}(c+dx)\right)}{4f} - \frac{b\,\text{Subst}\left(\int \left(\frac{f^2\left(6d^2e^2-12cdef-\left(1-6c^2\right)f^2\right)}{d^4}\right.\right.}{}$$

$$= -\frac{bf\left(6d^2e^2-12cdef-\left(1-6c^2\right)f^2\right)x}{4d^3} - \frac{bf^2(de-cf)(c+dx)^2}{2d^4} - \frac{bf^3(}{1}$$

$$= -\frac{bf\left(6d^2e^2-12cdef-\left(1-6c^2\right)f^2\right)x}{4d^3} - \frac{bf^2(de-cf)(c+dx)^2}{2d^4} - \frac{bf^3(}{1}$$

$$= -\frac{bf\left(6d^2e^2-12cdef-\left(1-6c^2\right)f^2\right)x}{4d^3} - \frac{bf^2(de-cf)(c+dx)^2}{2d^4} - \frac{bf^3(}{1}$$

Mathematica [C] time = 0.30, size = 157, normalized size = 0.67

$$\frac{(e+fx)^4\left(a+b\tan^{-1}(c+dx)\right) - \frac{b\left(6df^2x\left(\left(6c^2-1\right)f^2-12cdef+6d^2e^2\right)+12f^3(c+dx)^2(de-cf)-3i(de-(c-i)f)^4\log(-c-dx+i)+3i(de-(c+i)f\right.}{6d^4}}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^3*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] ((e + f*x)^4*(a + b*ArcTan[c + d*x]) - (b*(6*d*f^2*(6*d^2*e^2 - 12*c*d*e*f
+ (-1 + 6*c^2)*f^2)*x + 12*f^3*(d*e - c*f)*(c + d*x)^2 + 2*f^4*(c + d*x)^3
- (3*I)*(d*e - (-I + c)*f)^4*Log[I - c - d*x] + (3*I)*(d*e - (I + c)*f)^4*L
og[I + c + d*x]))/(6*d^4))/(4*f)
```

fricas [A] time = 0.48, size = 317, normalized size = 1.36

$$\frac{3\,ad^4f^3x^4 + \left(12\,ad^4ef^2 - bd^3f^3\right)x^3 + 3\left(6\,ad^4e^2f - 2\,bd^3ef^2 + bcd^2f^3\right)x^2 + 3\left(4\,ad^4e^3 - 6\,bd^3e^2f + 8\,bcd^2ef^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(a+b*arctan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*d^4*f^3*x^4 + (12*a*d^4*e*f^2 - b*d^3*f^3)*x^3 + 3*(6*a*d^4*e^2*f
 - 2*b*d^3*e*f^2 + b*c*d^2*f^3)*x^2 + 3*(4*a*d^4*e^3 - 6*b*d^3*e^2*f + 8*b*
```

```
c*d^2*e*f^2 - (3*b*c^2 - b)*d*f^3)*x + 3*(b*d^4*f^3*x^4 + 4*b*d^4*e*f^2*x^3
 + 6*b*d^4*e^2*f*x^2 + 4*b*d^4*e^3*x + 4*b*c*d^3*e^3 - 6*(b*c^2 - b)*d^2*e^
2*f + 4*(b*c^3 - 3*b*c)*d*e*f^2 - (b*c^4 - 6*b*c^2 + b)*f^3)*arctan(d*x + c
) - 6*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + (3*b*c^2 - b)*d*e*f^2 - (b*c^3 - b*c)*
f^3)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/d^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.05, size = 494, normalized size = 2.12

$$\frac{b f^3 \ln\left(1+(dx+c)^2\right) c^3}{2d^4}+\frac{5b f^2 c^2 e}{2d^3}-\frac{3b f e^2 c}{2d^2}+\frac{3a f x^2 e^2}{2}+\frac{b f^3 \arctan(dx+c) x^4}{4}+\arctan(dx+c) x b e^3-\frac{b \ln\left(1+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*(a+b*arctan(d*x+c)),x)
```

```
[Out] 1/2/d^4*b*f^3*ln(1+(d*x+c)^2)*c^3+5/2/d^3*b*f^2*c^2*e-3/2/d^2*b*f*e^2*c+3/2
*a*f*x^2*e^2+1/4*b*f^3*arctan(d*x+c)*x^4+arctan(d*x+c)*x*b*e^3-1/2/d*b*ln(1
+(d*x+c)^2)*e^3-1/4/d^4*b*f^3*arctan(d*x+c)-1/12/d*b*f^3*x^3+1/4*b/d^3*f^3*
x+a*f^2*x^3*e+1/4/d^4*b*f^3*c-13/12/d^4*b*f^3*c^3+1/4*a/f*e^4-3/d^3*b*f^2*a
rctan(d*x+c)*c*e-3/2/d^2*b*f*arctan(d*x+c)*e^2*c^2+1/d^3*b*f^2*arctan(d*x+c
)*c^3*e+3/2/d^2*b*f*ln(1+(d*x+c)^2)*c*e^2-3/2/d^3*b*f^2*ln(1+(d*x+c)^2)*c^2
*e+2*b/d^2*f^2*c*e*x+1/d*arctan(d*x+c)*b*c*e^3+1/2/d^3*b*f^2*ln(1+(d*x+c)^2
)*e+3/2/d^4*b*f^3*arctan(d*x+c)*c^2-1/2/d^4*b*f^3*ln(1+(d*x+c)^2)*c-1/2/d*b
*f^2*e*x^2+1/4/d^2*b*f^3*x^2*c-3/4*b/d^3*f^3*c^2*x-3/2*b/d*f*e^2*x-1/4/d^4*
b*f^3*arctan(d*x+c)*c^4+3/2/d^2*b*f*arctan(d*x+c)*e^2+3/2*b*f*arctan(d*x+c)
*e^2*x^2+b*f^2*arctan(d*x+c)*e*x^3+a*x*e^3+1/4*a*f^3*x^4
```

maxima [A] time = 0.43, size = 346, normalized size = 1.48

$$\frac{1}{4} a f^3 x^4 + a e f^2 x^3 + \frac{3}{2} a e^2 f x^2 + \frac{3}{2}\left(x^2 \arctan(dx+c) - d\left(\frac{x}{d^2} + \frac{\left(c^2-1\right)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3} - \frac{c \log\left(d^2x^2+2cdx+c^2+\right.}{d^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(a+b*arctan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*a*f^3*x^4 + a*e*f^2*x^3 + 3/2*a*e^2*f*x^2 + 3/2*(x^2*arctan(d*x + c) -
d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x
+ c^2 + 1)/d^3))*b*e^2*f + 1/2*(2*x^3*arctan(d*x + c) - d*((d*x^2 - 4*c*x)/
d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 +
 2*c*d*x + c^2 + 1)/d^4))*b*e*f^2 + 1/12*(3*x^4*arctan(d*x + c) - d*((d^2*x
^3 - 3*c*d*x^2 + 3*(3*c^2 - 1)*x)/d^4 + 3*(c^4 - 6*c^2 + 1)*arctan((d^2*x +
 c*d)/d)/d^5 - 6*(c^3 - c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^5))*b*f^3 + a
*e^3*x + 1/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*b*e^3/d
```

mupad [B] time = 1.02, size = 787, normalized size = 3.38

$$\mathrm{atan}(c+d\,x)\left(b\,e^3\,x+\frac{3\,b\,e^2\,f\,x^2}{2}+b\,e\,f^2\,x^3+\frac{b\,f^3\,x^4}{4}\right)+x\left(\frac{e\left(6\,a\,c^2\,f^2+12\,a\,c\,d\,e\,f+2\,a\,d^2\,e^2-3\,b\,d\,e\,f+6\,a\,f\right.}{2\,d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3*(a + b*atan(c + d*x)),x)
```

```
[Out] atan(c + d*x)*((b*f^3*x^4)/4 + b*e^3*x + (3*b*e^2*f*x^2)/2 + b*e*f^2*x^3) +
 x*((e*(6*a*f^2 + 6*a*c^2*f^2 + 2*a*d^2*e^2 - 3*b*d*e*f + 12*a*c*d*e*f))/(2
*d^2) - ((4*c^2 + 4)*((f^2*(8*a*c*f - b*f + 12*a*d*e))/(4*d) - (2*a*c*f^3)/
d))/(4*d^2) + (2*c*((2*c*((f^2*(8*a*c*f - b*f + 12*a*d*e))/(4*d) - (2*a*c*f
^3)/d))/d - (4*a*f^3 + 4*a*c^2*f^3 - 4*b*d*e*f^2 + 12*a*d^2*e^2*f + 24*a*c*
d*e*f^2)/(4*d^2) + (a*f^3*(4*c^2 + 4))/(4*d^2)))/d) - x^2*((c*((f^2*(8*a*c*
f - b*f + 12*a*d*e))/(4*d) - (2*a*c*f^3)/d))/d - (4*a*f^3 + 4*a*c^2*f^3 - 4
*b*d*e*f^2 + 12*a*d^2*e^2*f + 24*a*c*d*e*f^2)/(8*d^2) + (a*f^3*(4*c^2 + 4))
/(8*d^2)) + x^3*((f^2*(8*a*c*f - b*f + 12*a*d*e))/(12*d) - (2*a*c*f^3)/(3*d
)) + (a*f^3*x^4)/4 - (log(c^2 + d^2*x^2 + 2*c*d*x + 1)*(64*b*d^7*e^3 - 64*b
*c^3*d^4*f^3 + 64*b*c*d^4*f^3 - 64*b*d^5*e*f^2 - 192*b*c*d^6*e^2*f + 192*b*
c^2*d^5*e*f^2))/(128*d^8) - (b*atan((4*d^3*((c*(f^3 - 6*c^2*f^3 + c^4*f^3 -
 4*c*d^3*e^3 - 6*d^2*e^2*f + 6*c^2*d^2*e^2*f + 12*c*d*e*f^2 - 4*c^3*d*e*f^2
))/(4*d^3) + (x*(f^3 - 6*c^2*f^3 + c^4*f^3 - 4*c*d^3*e^3 - 6*d^2*e^2*f + 6*
c^2*d^2*e^2*f + 12*c*d*e*f^2 - 4*c^3*d*e*f^2))/(4*d^2)))/(f^3 - 6*c^2*f^3 +
 c^4*f^3 - 4*c*d^3*e^3 - 6*d^2*e^2*f + 6*c^2*d^2*e^2*f + 12*c*d*e*f^2 - 4*c
^3*d*e*f^2))*(f^3 - 6*c^2*f^3 + c^4*f^3 - 4*c*d^3*e^3 - 6*d^2*e^2*f + 6*c^2
*d^2*e^2*f + 12*c*d*e*f^2 - 4*c^3*d*e*f^2))/(4*d^4)
```

sympy [A] time = 25.36, size = 654, normalized size = 2.81

$$\begin{cases} ae^3x + \frac{3ae^2fx^2}{2} + aef^2x^3 + \frac{af^3x^4}{4} - \frac{bc^4f^3\operatorname{atan}(c+dx)}{4d^4} + \frac{bc^3ef^2\operatorname{atan}(c+dx)}{d^3} + \frac{bc^3f^3\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{d^4} - \frac{ibc^3f^3\operatorname{atan}(c+dx)}{d^4} - \frac{3bc^2e^2f \\ (a+b\operatorname{atan}(c))\left(e^3x + \frac{3e^2fx^2}{2} + ef^2x^3 + \frac{f^3x^4}{4}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*(a+b*atan(d*x+c)),x)
```

```
[Out] Piecewise((a*e**3*x + 3*a*e**2*f*x**2/2 + a*e*f**2*x**3 + a*f**3*x**4/4 - b
*c**4*f**3*atan(c + d*x)/(4*d**4) + b*c**3*e*f**2*atan(c + d*x)/d**3 + b*c*
*3*f**3*log(c/d + x - I/d)/d**4 - I*b*c**3*f**3*atan(c + d*x)/d**4 - 3*b*c*
*2*e**2*f*atan(c + d*x)/(2*d**2) - 3*b*c**2*e*f**2*log(c/d + x - I/d)/d**3
+ 3*I*b*c**2*e*f**2*atan(c + d*x)/d**3 - 3*b*c**2*f**3*x/(4*d**3) + 3*b*c**
2*f**3*atan(c + d*x)/(2*d**4) + b*c*e**3*atan(c + d*x)/d + 3*b*c*e**2*f*log
(c/d + x - I/d)/d**2 - 3*I*b*c*e**2*f*atan(c + d*x)/d**2 + 2*b*c*e*f**2*x/d
**2 + b*c*f**3*x**2/(4*d**2) - 3*b*c*e*f**2*atan(c + d*x)/d**3 - b*c*f**3*l
og(c/d + x - I/d)/d**4 + I*b*c*f**3*atan(c + d*x)/d**4 + b*e**3*x*atan(c +
d*x) + 3*b*e**2*f*x**2*atan(c + d*x)/2 + b*e*f**2*x**3*atan(c + d*x) + b*f*
*3*x**4*atan(c + d*x)/4 - b*e**3*log(c/d + x - I/d)/d + I*b*e**3*atan(c + d
*x)/d - 3*b*e**2*f*x/(2*d) - b*e*f**2*x**2/(2*d) - b*f**3*x**3/(12*d) + 3*b
*e**2*f*atan(c + d*x)/(2*d**2) + b*e*f**2*log(c/d + x - I/d)/d**3 - I*b*e*f
**2*atan(c + d*x)/d**3 + b*f**3*x/(4*d**3) - b*f**3*atan(c + d*x)/(4*d**4),
 Ne(d, 0)), ((a + b*atan(c))*(e**3*x + 3*e**2*f*x**2/2 + e*f**2*x**3 + f**3
*x**4/4), True))
```

3.25 $\int (e+fx)^2 \left(a+b\tan^{-1}(c+dx)\right) dx$

Optimal. Leaf size=155

$$\frac{(e+fx)^3\left(a+b\tan^{-1}(c+dx)\right)}{3f}-\frac{b\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)\log\left((c+dx)^2+1\right)}{6d^3}-\frac{b(de-cf)\left(-\left(3-c^2\right)f^2\right.}{}$$

```
[Out] -b*f*(-c*f+d*e)*x/d^2-1/6*b*f^2*(d*x+c)^2/d^3-1/3*b*(-c*f+d*e)*(d^2*e^2-2*c
*d*e*f-(-c^2+3)*f^2)*arctan(d*x+c)/d^3/f+1/3*(f*x+e)^3*(a+b*arctan(d*x+c))/
f-1/6*b*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*ln(1+(d*x+c)^2)/d^3
```

Rubi [A] time = 0.19, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5047, 4862, 702, 635, 203, 260}

$$\frac{(e+fx)^3\left(a+b\tan^{-1}(c+dx)\right)}{3f}-\frac{b\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)\log\left((c+dx)^2+1\right)}{6d^3}-\frac{b(de-cf)\left(-\left(3-c^2\right)f^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] -((b*f*(d*e - c*f)*x)/d^2) - (b*f^2*(c + d*x)^2)/(6*d^3) - (b*(d*e - c*f)*(
d^2*e^2 - 2*c*d*e*f - (3 - c^2)*f^2)*ArcTan[c + d*x])/(3*d^3*f) + ((e + f*x
)^3*(a + b*ArcTan[c + d*x]))/(3*f) - (b*(3*d^2*e^2 - 6*c*d*e*f - (1 - 3*c^2
)*f^2)*Log[1 + (c + d*x)^2])/(6*d^3)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\int (e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2\left(a+b\tan^{-1}(x)\right)dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^3\left(a+b\tan^{-1}(c+dx)\right)}{3f} - \frac{b\,\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^3}{1+x^2}dx, x, c+dx\right)}{3f}$$

$$= \frac{(e+fx)^3\left(a+b\tan^{-1}(c+dx)\right)}{3f} - \frac{b\,\text{Subst}\left(\int \left(\frac{3f^2(de-cf)}{d^3}+\frac{f^3x}{d^3}+\frac{(de-cf)}{}\right.\right.}{}$$

$$= -\frac{bf(de-cf)x}{d^2} - \frac{bf^2(c+dx)^2}{6d^3} + \frac{(e+fx)^3\left(a+b\tan^{-1}(c+dx)\right)}{3f} - \frac{b\,\text{S}}{}$$

$$= -\frac{bf(de-cf)x}{d^2} - \frac{bf^2(c+dx)^2}{6d^3} + \frac{(e+fx)^3\left(a+b\tan^{-1}(c+dx)\right)}{3f} - \frac{(b(}{}$$

$$= -\frac{bf(de-cf)x}{d^2} - \frac{bf^2(c+dx)^2}{6d^3} - \frac{b(de-cf)\left(d^2e^2-2cdef-\left(3-c^2\right)f^2\right.}{3d^3f}$$

Mathematica [C] time = 0.16, size = 118, normalized size = 0.76

$$\frac{(e+fx)^3\left(a+b\tan^{-1}(c+dx)\right)-\frac{b\left(6df^2x(de-cf)-i(de-(c-i)f)^3\log(-c-dx+i)+i(de-(c+i)f)^3\log(c+dx+i)+f^3(c+dx)^2\right)}{2d^3}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] ((e + f*x)^3*(a + b*ArcTan[c + d*x]) - (b*(6*d*f^2*(d*e - c*f)*x + f^3*(c +
 d*x)^2 - I*(d*e - (-I + c)*f)^3*Log[I - c - d*x] + I*(d*e - (I + c)*f)^3*L
og[I + c + d*x]))/(2*d^3))/(3*f)
```

fricas [A] time = 0.47, size = 199, normalized size = 1.28

$$\frac{2\,ad^3f^2x^3+\left(6\,ad^3ef-bd^2f^2\right)x^2+2\left(3\,ad^3e^2-3\,bd^2ef+2\,bcdf^2\right)x+2\left(bd^3f^2x^3+3\,bd^3efx^2+3\,bd^3e^2x+3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*d^3*f^2*x^3 + (6*a*d^3*e*f - b*d^2*f^2)*x^2 + 2*(3*a*d^3*e^2 - 3*b
*d^2*e*f + 2*b*c*d*f^2)*x + 2*(b*d^3*f^2*x^3 + 3*b*d^3*e*f*x^2 + 3*b*d^3*e^
2*x + 3*b*c*d^2*e^2 - 3*(b*c^2 - b)*d*e*f + (b*c^3 - 3*b*c)*f^2)*arctan(d*x
 + c) - (3*b*d^2*e^2 - 6*b*c*d*e*f + (3*b*c^2 - b)*f^2)*log(d^2*x^2 + 2*c*d
*x + c^2 + 1))/d^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.05, size = 283, normalized size = 1.83

$$\frac{\arctan(dx+c)\,bce^2}{d}+\frac{bf^2\ln\left(1+(dx+c)^2\right)}{6d^3}+\frac{bf^2\arctan(dx+c)\,x^3}{3}+\frac{bf^2\arctan(dx+c)\,c^3}{3d^3}+\frac{af^2x^3}{3}+\frac{ae^3}{3f}-\frac{bf^2}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*(a+b*arctan(d*x+c)),x)
```

```
[Out] 1/d*arctan(d*x+c)*b*c*e^2+1/6/d^3*b*f^2*ln(1+(d*x+c)^2)+1/3*b*f^2*arctan(d*
x+c)*x^3+1/3/d^3*b*f^2*arctan(d*x+c)*c^3+1/3*a*f^2*x^3+1/3*a/f*e^3-1/6/d*b*
f^2*x^2-1/d^3*b*f^2*arctan(d*x+c)*c+arctan(d*x+c)*x*b*e^2+1/d^2*b*f*ln(1+(d
*x+c)^2)*c*e+5/6/d^3*b*f^2*c^2+b*f*arctan(d*x+c)*e*x^2-1/d^2*b*f*arctan(d*x
+c)*e*c^2+1/d^2*b*f*arctan(d*x+c)*e-1/2/d^3*b*f^2*ln(1+(d*x+c)^2)*c^2-b/d*f
*e*x+2/3*b/d^2*f^2*c*x+a*x*e^2-1/d^2*b*f*c*e+a*f*x^2*e-1/2*b*e^2*ln(1+(d*x+
c)^2)/d
```

maxima [A] time = 0.43, size = 220, normalized size = 1.42

$$\frac{1}{3}af^2x^3+aefx^2+\left(x^2\arctan(dx+c)-d\left(\frac{x}{d^2}+\frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3}-\frac{c\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^3}\right)\right)bef+\frac{1}{6}\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*a*f^2*x^3 + a*e*f*x^2 + (x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arc
tan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*b*e*f +
 1/6*(2*x^3*arctan(d*x + c) - d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan
((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*
b*f^2 + a*e^2*x + 1/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*
b*e^2/d
```

mupad [B] time = 0.78, size = 411, normalized size = 2.65

$$x^2\left(\frac{f\,\left(6\,a\,c\,f-b\,f+6\,a\,d\,e\right)}{6\,d}-\frac{a\,c\,f^2}{d}\right)-x\left(\frac{2\,c\,\left(\frac{f\,\left(6\,a\,c\,f-b\,f+6\,a\,d\,e\right)}{3\,d}-\frac{2\,a\,c\,f^2}{d}\right)}{d}-\frac{3\,a\,c^2\,f^2+12\,a\,c\,d\,e\,f+3\,a\,d^2\,e^2-3}{3\,d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2*(a + b*atan(c + d*x)),x)
```

```
[Out] x^2*((f*(6*a*c*f - b*f + 6*a*d*e))/(6*d) - (a*c*f^2)/d) - x*((2*c*((f*(6*a*
c*f - b*f + 6*a*d*e))/(3*d) - (2*a*c*f^2)/d))/d - (3*a*f^2 + 3*a*c^2*f^2 +
3*a*d^2*e^2 - 3*b*d*e*f + 12*a*c*d*e*f)/(3*d^2) + (a*f^2*(3*c^2 + 3))/(3*d^
2)) + atan(c + d*x)*((b*f^2*x^3)/3 + b*e^2*x + b*e*f*x^2) + (a*f^2*x^3)/3 -
 (log(c^2 + d^2*x^2 + 2*c*d*x + 1)*(36*b*d^5*e^2 - 12*b*d^3*f^2 + 36*b*c^2*
d^3*f^2 - 72*b*c*d^4*e*f))/(72*d^6) + (b*atan((3*d^2*((c*(c^3*f^2 - 3*c*f^2
 + 3*c*d^2*e^2 + 3*d*e*f - 3*c^2*d*e*f))/(3*d^2) + (x*(c^3*f^2 - 3*c*f^2 +
3*c*d^2*e^2 + 3*d*e*f - 3*c^2*d*e*f))/(3*d)))/(c^3*f^2 - 3*c*f^2 + 3*c*d^2*
```

```
e^2 + 3*d*e*f - 3*c^2*d*e*f))*(c^3*f^2 - 3*c*f^2 + 3*c*d^2*e^2 + 3*d*e*f -
3*c^2*d*e*f))/(3*d^3)
```

sympy [A] time = 10.66, size = 376, normalized size = 2.43

$$\begin{cases} ae^2x + aefx^2 + \frac{af^2x^3}{3} + \frac{bc^3f^2\operatorname{atan}(c+dx)}{3d^3} - \frac{bc^2ef\operatorname{atan}(c+dx)}{d^2} - \frac{bc^2f^2\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{d^3} + \frac{ibc^2f^2\operatorname{atan}(c+dx)}{d^3} + \frac{bce^2\operatorname{atan}(c+dx)}{d} + \frac{2b \\ (a+b\operatorname{atan}(c))\left(e^2x+efx^2+\frac{f^2x^3}{3}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(a+b*atan(d*x+c)),x)
```

```
[Out] Piecewise((a*e**2*x + a*e*f*x**2 + a*f**2*x**3/3 + b*c**3*f**2*atan(c + d*x
)/(3*d**3) - b*c**2*e*f*atan(c + d*x)/d**2 - b*c**2*f**2*log(c/d + x - I/d)
/d**3 + I*b*c**2*f**2*atan(c + d*x)/d**3 + b*c*e**2*atan(c + d*x)/d + 2*b*c
*e*f*log(c/d + x - I/d)/d**2 - 2*I*b*c*e*f*atan(c + d*x)/d**2 + 2*b*c*f**2*
x/(3*d**2) - b*c*f**2*atan(c + d*x)/d**3 + b*e**2*x*atan(c + d*x) + b*e*f*x
**2*atan(c + d*x) + b*f**2*x**3*atan(c + d*x)/3 - b*e**2*log(c/d + x - I/d)
/d + I*b*e**2*atan(c + d*x)/d - b*e*f*x/d - b*f**2*x**2/(6*d) + b*e*f*atan(
c + d*x)/d**2 + b*f**2*log(c/d + x - I/d)/(3*d**3) - I*b*f**2*atan(c + d*x)
/(3*d**3), Ne(d, 0)), ((a + b*atan(c))*(e**2*x + e*f*x**2 + f**2*x**3/3), T
rue))
```

3.26 $\int (e+fx)\left(a+b\tan^{-1}(c+dx)\right)\,dx$

Optimal. Leaf size=97

$$\frac{(e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)}{2f}-\frac{b(de-cf)\log\left((c+dx)^2+1\right)}{2d^2}-\frac{b(-cf+de+f)(de-(c+1)f)\tan^{-1}(c+dx)}{2d^2f}-\frac{bf}{2}$$

```
[Out] -1/2*b*f*x/d-1/2*b*(-c*f+d*e+f)*(d*e-(1+c)*f)*arctan(d*x+c)/d^2/f+1/2*(f*x+
e)^2*(a+b*arctan(d*x+c))/f-1/2*b*(-c*f+d*e)*ln(1+(d*x+c)^2)/d^2
```

Rubi [A] time = 0.11, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5047, 4862, 702, 635, 203, 260}

$$\frac{(e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)}{2f}-\frac{b(de-cf)\log\left((c+dx)^2+1\right)}{2d^2}-\frac{b(-cf+de+f)(de-(c+1)f)\tan^{-1}(c+dx)}{2d^2f}-\frac{bf}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] -(b*f*x)/(2*d) - (b*(d*e + f - c*f)*(d*e - (1 + c)*f)*ArcTan[c + d*x])/(2*d
^2*f) + ((e + f*x)^2*(a + b*ArcTan[c + d*x]))/(2*f) - (b*(d*e - c*f)*Log[1
+ (c + d*x)^2])/(2*d^2)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
```

```
tQ[p, 0]
```

Rubi steps

$$\int (e+fx)\left(a+b\tan^{-1}(c+dx)\right)dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)\left(a+b\tan^{-1}(x)\right)dx, x, c+dx\right)}{d}$$
$$= \frac{(e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)}{2f} - \frac{b\,\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2}{1+x^2}dx, x, c+dx\right)}{2f}$$
$$= \frac{(e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)}{2f} - \frac{b\,\text{Subst}\left(\int \left(\frac{f^2}{d^2}+\frac{(de-f-cf)(de+f-cf)+2f(}{d^2(1+x^2)}\right.\right.}{2f}$$
$$= -\frac{bfx}{2d} + \frac{(e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)}{2f} - \frac{b\,\text{Subst}\left(\int \frac{(de-f-cf)(de+f-cf)+}{1+x^2}\right.}{2d^2f}$$
$$= -\frac{bfx}{2d} + \frac{(e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)}{2f} - \frac{(b(de-cf))\,\text{Subst}\left(\int \frac{x}{1+x^2}dx\right.}{d^2}$$
$$= -\frac{bfx}{2d} - \frac{b(de+f-cf)(de-(1+c)f)\tan^{-1}(c+dx)}{2d^2f} + \frac{(e+fx)^2\left(a+b\,\mathrm{t}\right.}{2f}$$

Mathematica [C] time = 0.07, size = 163, normalized size = 1.68

$$aex+\frac{1}{2}afx^2-\frac{be\left(\log\left(c^2+2cdx+d^2x^2+1\right)-2c\tan^{-1}(c+dx)\right)}{2d}+\frac{bf\left(\frac{1}{2}d\left(\frac{c+dx}{d}-\frac{c}{d}\right)^2\tan^{-1}(c+dx)-\frac{1}{2}d\left(-\frac{i(-c}{}\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] a*e*x + (a*f*x^2)/2 + b*e*x*ArcTan[c + d*x] + (b*f*((d*(-(c/d) + (c + d*x)/
d)^2*ArcTan[c + d*x])/2 - (d*(x/d - ((I/2)*(I - c)^2*Log[I - c - d*x])/d^2
+ ((I/2)*(I + c)^2*Log[I + c + d*x])/d^2))/2))/d - (b*e*(-2*c*ArcTan[c + d*
x] + Log[1 + c^2 + 2*c*d*x + d^2*x^2]))/(2*d)
```

fricas [A] time = 0.46, size = 103, normalized size = 1.06

$$\frac{ad^2fx^2+\left(2\,ad^2e-bdf\right)x+\left(bd^2fx^2+2\,bd^2ex+2\,bcde-\left(bc^2-b\right)f\right)\arctan\left(dx+c\right)-\left(bde-bcf\right)\log\left(d^2x^2\right.}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d^2*f*x^2 + (2*a*d^2*e - b*d*f)*x + (b*d^2*f*x^2 + 2*b*d^2*e*x + 2*b
*c*d*e - (b*c^2 - b)*f)*arctan(d*x + c) - (b*d*e - b*c*f)*log(d^2*x^2 + 2*c
*d*x + c^2 + 1))/d^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.06, size = 146, normalized size = 1.51

$$\frac{a\,x^2 f}{2}-\frac{a f\,c^2}{2d^2}+aex+\frac{ace}{d}+\frac{bf\arctan\left(dx+c\right)x^2}{2}-\frac{bf\arctan\left(dx+c\right)c^2}{2d^2}+\arctan\left(dx+c\right)xbe+\frac{\arctan\left(dx+c\right)bce}{d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(a+b*arctan(d*x+c)),x)
```

```
[Out] 1/2*a*x^2*f-1/2/d^2*a*f*c^2+a*e*x+1/d*a*c*e+1/2*b*f*arctan(d*x+c)*x^2-1/2/d
^2*b*f*arctan(d*x+c)*c^2+arctan(d*x+c)*x*b*e+1/d*arctan(d*x+c)*b*c*e-1/2*b*
f*x/d-1/2/d^2*b*c*f+1/2/d^2*b*ln(1+(d*x+c)^2)*c*f-1/2/d*b*ln(1+(d*x+c)^2)*e
+1/2/d^2*b*f*arctan(d*x+c)
```

maxima [A] time = 0.42, size = 116, normalized size = 1.20

$$\frac{1}{2}afx^2+\frac{1}{2}\left(x^2\arctan\left(dx+c\right)-d\left(\frac{x}{d^2}+\frac{\left(c^2-1\right)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3}-\frac{c\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^3}\right)\right)bf+aex+\frac{\left(2\left(d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*a*f*x^2 + 1/2*(x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*x
 + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*b*f + a*e*x + 1/2
*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*b*e/d
```

mupad [B] time = 1.80, size = 136, normalized size = 1.40

$$a\,e\,x+\frac{a\,f\,x^2}{2}-\frac{b\,e\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)}{2\,d}+\frac{b\,f\,\mathrm{atan}\left(c+d\,x\right)}{2\,d^2}+\frac{b\,f\,x^2\,\mathrm{atan}\left(c+d\,x\right)}{2}-\frac{b\,f\,x}{2\,d}+b\,e\,x\,\mathrm{atan}\left(c+d\,x\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)*(a + b*atan(c + d*x)),x)
```

```
[Out] a*e*x + (a*f*x^2)/2 - (b*e*log(c^2 + d^2*x^2 + 2*c*d*x + 1))/(2*d) + (b*f*a
tan(c + d*x))/(2*d^2) + (b*f*x^2*atan(c + d*x))/2 - (b*f*x)/(2*d) + b*e*x*a
tan(c + d*x) - (b*c^2*f*atan(c + d*x))/(2*d^2) + (b*c*f*log(c^2 + d^2*x^2 +
 2*c*d*x + 1))/(2*d^2) + (b*c*e*atan(c + d*x))/d
```

sympy [A] time = 4.54, size = 177, normalized size = 1.82

$$\begin{cases} aex+\frac{afx^2}{2}-\frac{bc^2f\,\mathrm{atan}\left(c+dx\right)}{2d^2}+\frac{bce\,\mathrm{atan}\left(c+dx\right)}{d}+\frac{bcf\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{d^2}-\frac{ibcf\,\mathrm{atan}\left(c+dx\right)}{d^2}+bex\,\mathrm{atan}\left(c+dx\right)+\frac{bfx^2\,\mathrm{atan}\left(c+dx\right)}{2}- \\ \left(a+b\,\mathrm{atan}\left(c\right)\right)\left(ex+\frac{fx^2}{2}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*atan(d*x+c)),x)
```

```
[Out] Piecewise((a*e*x + a*f*x**2/2 - b*c**2*f*atan(c + d*x)/(2*d**2) + b*c*e*ata
n(c + d*x)/d + b*c*f*log(c/d + x - I/d)/d**2 - I*b*c*f*atan(c + d*x)/d**2 +
 b*e*x*atan(c + d*x) + b*f*x**2*atan(c + d*x)/2 - b*e*log(c/d + x - I/d)/d
+ I*b*e*atan(c + d*x)/d - b*f*x/(2*d) + b*f*atan(c + d*x)/(2*d**2), Ne(d, 0
)), ((a + b*atan(c))*(e*x + f*x**2/2), True))
```

3.27 $\int \left(a+b\tan^{-1}(c+dx)\right)\,dx$

Optimal. Leaf size=38

$$ax-\frac{b\log\left((c+dx)^2+1\right)}{2d}+\frac{b(c+dx)\tan^{-1}(c+dx)}{d}$$

```
[Out] a*x+b*(d*x+c)*arctan(d*x+c)/d-1/2*b*ln(1+(d*x+c)^2)/d
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5039, 4846, 260}

$$ax-\frac{b\log\left((c+dx)^2+1\right)}{2d}+\frac{b(c+dx)\tan^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTan[c + d*x],x]
```

```
[Out] a*x + (b*(c + d*x)*ArcTan[c + d*x])/d - (b*Log[1 + (c + d*x)^2])/(2*d)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5039

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\tan^{-1}(c+dx)\right)\,dx &= ax+b\int \tan^{-1}(c+dx)\,dx\\ &= ax+\frac{b\,\text{Subst}\left(\int \tan^{-1}(x)\,dx,x,c+dx\right)}{d}\\ &= ax+\frac{b(c+dx)\tan^{-1}(c+dx)}{d}-\frac{b\,\text{Subst}\left(\int \frac{x}{1+x^2}\,dx,x,c+dx\right)}{d}\\ &= ax+\frac{b(c+dx)\tan^{-1}(c+dx)}{d}-\frac{b\log\left(1+(c+dx)^2\right)}{2d}\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 1.29

$$ax-\frac{b\left(\log\left(c^2+2cdx+d^2x^2+1\right)-2c\tan^{-1}(c+dx)\right)}{2d}+bx\tan^{-1}(c+dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTan[c + d*x],x]
```

```
[Out] a*x + b*x*ArcTan[c + d*x] - (b*(-2*c*ArcTan[c + d*x] + Log[1 + c^2 + 2*c*d*
x + d^2*x^2]))/(2*d)
```

fricas [A] time = 0.43, size = 48, normalized size = 1.26

$$\frac{2\,adx + 2\,(bdx + bc)\arctan\left(dx + c\right) - b\log\left(d^2x^2 + 2\,cdx + c^2 + 1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*d*x + 2*(b*d*x + b*c)*arctan(d*x + c) - b*log(d^2*x^2 + 2*c*d*x +
c^2 + 1))/d
```

giac [A] time = 0.11, size = 36, normalized size = 0.95

$$ax + \frac{\left(2\,(dx + c)\arctan\left(dx + c\right) - \log\left((dx + c)^2 + 1\right)\right)b}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(d*x+c),x, algorithm="giac")
```

```
[Out] a*x + 1/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*b/d
```

maple [A] time = 0.04, size = 42, normalized size = 1.11

$$ax + b\arctan\left(dx + c\right)x + \frac{b\arctan\left(dx + c\right)c}{d} - \frac{b\ln\left(1 + (dx + c)^2\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctan(d*x+c),x)
```

```
[Out] a*x+b*arctan(d*x+c)*x+b/d*arctan(d*x+c)*c-1/2*b*ln(1+(d*x+c)^2)/d
```

maxima [A] time = 0.32, size = 36, normalized size = 0.95

$$ax + \frac{\left(2\,(dx + c)\arctan\left(dx + c\right) - \log\left((dx + c)^2 + 1\right)\right)b}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + 1/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*b/d
```

mupad [B] time = 1.10, size = 49, normalized size = 1.29

$$a\,x + b\,x\,\mathrm{atan}\left(c + d\,x\right) - \frac{b\,\ln\left(c^2 + 2\,c\,d\,x + d^2\,x^2 + 1\right)}{2\,d} + \frac{b\,c\,\mathrm{atan}\left(c + d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*atan(c + d*x),x)
```

```
[Out] a*x + b*x*atan(c + d*x) - (b*log(c^2 + d^2*x^2 + 2*c*d*x + 1))/(2*d) + (b*c
*atan(c + d*x))/d
```

sympy [A] time = 0.37, size = 51, normalized size = 1.34

$$ax + b\left(\begin{cases} \frac{c\,\mathrm{atan}\,(c+dx)}{d} + x\,\mathrm{atan}\,(c+dx) - \frac{\log\left(c^2+2cdx+d^2x^2+1\right)}{2d} & \text{for } d \neq 0 \\ x\,\mathrm{atan}\,(c) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atan(d*x+c),x)
```

```
[Out] a*x + b*Piecewise((c*atan(c + d*x)/d + x*atan(c + d*x) - log(c**2 + 2*c*d*x
 + d**2*x**2 + 1)/(2*d), Ne(d, 0)), (x*atan(c), True))
```

3.28 $\int \frac{a+b\tan^{-1}(c+dx)}{e+fx}\,dx$

Optimal. Leaf size=162

$$\frac{\left(a+b\tan^{-1}(c+dx)\right)\log\left(\frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{f}-\frac{\log\left(\frac{2}{1-i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{f}-\frac{ib\text{Li}_2\left(1-\frac{2d(e+fx)}{(de-cf+if)(1-i(c+dx}\right.}{2f}$$

```
[Out] -(a+b*arctan(d*x+c))*ln(2/(1-I*(d*x+c)))/f+(a+b*arctan(d*x+c))*ln(2*d*(f*x+
e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f+1/2*I*b*polylog(2,1-2/(1-I*(d*x+c)))/f-1/
2*I*b*polylog(2,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f
```

Rubi [A] time = 0.15, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5047, 4856, 2402, 2315, 2447}

$$-\frac{ib\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{2f}+\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{2f}+\frac{\left(a+b\tan^{-1}(c+dx)\right)\log\left(\frac{2d(e+fx)}{(1-i(c+dx))(-cf+d\right.}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])/(e + f*x),x]
```

```
[Out] -(((a + b*ArcTan[c + d*x])*Log[2/(1 - I*(c + d*x))])/f) + ((a + b*ArcTan[c
+ d*x])*Log[(2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f + ((I
/2)*b*PolyLog[2, 1 - 2/(1 - I*(c + d*x))])/f - ((I/2)*b*PolyLog[2, 1 - (2*d
*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
```

```
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(c+dx)}{e+fx}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{\frac{de-cf}{d}+\frac{fx}{d}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)\log\left(\frac{2}{1-i(c+dx)}\right)}{f} + \frac{\left(a+b\tan^{-1}(c+dx)\right)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+\cdots}\right.}{f}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)\log\left(\frac{2}{1-i(c+dx)}\right)}{f} + \frac{\left(a+b\tan^{-1}(c+dx)\right)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+\cdots}\right.}{f}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)\log\left(\frac{2}{1-i(c+dx)}\right)}{f} + \frac{\left(a+b\tan^{-1}(c+dx)\right)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+\cdots}\right.}{f}$$

Mathematica [A] time = 0.12, size = 160, normalized size = 0.99

$$\frac{2a\log(d(e+fx)) - ib\text{Li}_2\left(\frac{f(c+dx-i)}{(c-i)f-de}\right) + ib\text{Li}_2\left(\frac{f(c+dx+i)}{(c+i)f-de}\right) + ib\log(1-i(c+dx))\log\left(\frac{d(e+fx)}{de-(c+i)f}\right) - ib\log(1+i(c+\cdots}{2f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c + d*x])/(e + f*x),x]
```

```
[Out] (2*a*Log[d*(e + f*x)] + I*b*Log[(d*(e + f*x))/(d*e - (I + c)*f)]*Log[1 - I*
(c + d*x)] - I*b*Log[(d*(e + f*x))/(d*e + I*f - c*f)]*Log[1 + I*(c + d*x)]
- I*b*PolyLog[2, (f*(-I + c + d*x))/(-(d*e) + (-I + c)*f)] + I*b*PolyLog[2,
 (f*(I + c + d*x))/(-(d*e) + (I + c)*f)])/(2*f)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\arctan(dx+c)+a}{fx+e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(d*x + c) + a)/(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.10, size = 224, normalized size = 1.38

$$\frac{a\ln\left(f\left(dx+c\right)-cf+de\right)}{f}+\frac{b\ln\left(f\left(dx+c\right)-cf+de\right)\arctan\left(dx+c\right)}{f}+\frac{ib\ln\left(f\left(dx+c\right)-cf+de\right)\ln\left(\frac{if-f(dx+c)}{-cf+de+if}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))/(f*x+e),x)
```

```
[Out] a*ln(f*(d*x+c)-c*f+d*e)/f+b*ln(f*(d*x+c)-c*f+d*e)/f*arctan(d*x+c)+1/2*I*b*l
n(f*(d*x+c)-c*f+d*e)/f*ln((I*f-f*(d*x+c))/(d*e+I*f-c*f))-1/2*I*b*ln(f*(d*x+
c)-c*f+d*e)/f*ln((I*f+f*(d*x+c))/(I*f+c*f-d*e))+1/2*I*b/f*dilog((I*f-f*(d*x
+c))/(d*e+I*f-c*f))-1/2*I*b/f*dilog((I*f+f*(d*x+c))/(I*f+c*f-d*e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$2\,b\int\frac{\arctan\left(dx+c\right)}{2\left(fx+e\right)}\,dx+\frac{a\log\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e),x, algorithm="maxima")
```

```
[Out] 2*b*integrate(1/2*arctan(d*x + c)/(f*x + e), x) + a*log(f*x + e)/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{a+b\,\mathrm{atan}\left(c+d\,x\right)}{e+f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))/(e + f*x),x)
```

```
[Out] int((a + b*atan(c + d*x))/(e + f*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\,\mathrm{atan}\left(c+dx\right)}{e+fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))/(f*x+e),x)
```

```
[Out] Integral((a + b*atan(c + d*x))/(e + f*x), x)
```

3.29 $\int \frac{a+b\tan^{-1}(c+dx)}{(e+fx)^2}\,dx$

Optimal. Leaf size=151

$$-\frac{a+b\tan^{-1}(c+dx)}{f(e+fx)}-\frac{bd\log\left(c^2+2cdx+d^2x^2+1\right)}{2\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)}+\frac{bd\log(e+fx)}{\left(c^2+1\right)f^2-2cdef+d^2e^2}+\frac{bd(de-cf)\tan^{-1}(c+dx)}{f\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)}$$

```
[Out] b*d*(-c*f+d*e)*arctan(d*x+c)/f/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+(-a-b*arctan
(d*x+c))/f/(f*x+e)+b*d*ln(f*x+e)/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-1/2*b*d*ln
(d^2*x^2+2*c*d*x+c^2+1)/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)
```

Rubi [A] time = 0.12, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5045, 1982, 705, 31, 634, 618, 204, 628}

$$-\frac{a+b\tan^{-1}(c+dx)}{f(e+fx)}-\frac{bd\log\left(c^2+2cdx+d^2x^2+1\right)}{2\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)}+\frac{bd\log(e+fx)}{\left(c^2+1\right)f^2-2cdef+d^2e^2}+\frac{bd(de-cf)\tan^{-1}(c+dx)}{f\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])/(e + f*x)^2,x]
```

```
[Out] (b*d*(d*e - c*f)*ArcTan[c + d*x])/(f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2))
 - (a + b*ArcTan[c + d*x])/(f*(e + f*x)) + (b*d*Log[e + f*x])/(d^2*e^2 - 2*
c*d*e*f + (1 + c^2)*f^2) - (b*d*Log[1 + c^2 + 2*c*d*x + d^2*x^2])/(2*(d^2*e
^2 - 2*c*d*e*f + (1 + c^2)*f^2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1982

```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Rule 5045

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[((e + f*x)^(m + 1)*(a + b*ArcTan[c + d*x])^p)/(f*(m +
 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcTan[c
 + d*x])^(p - 1))/(1 + (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tan^{-1}(c+dx)}{(e+fx)^2}\,dx &= -\frac{a+b\tan^{-1}(c+dx)}{f(e+fx)}+\frac{(bd)\int \frac{1}{(e+fx)\left(1+(c+dx)^2\right)}\,dx}{f}\\
&= -\frac{a+b\tan^{-1}(c+dx)}{f(e+fx)}+\frac{(bd)\int \frac{1}{(e+fx)\left(1+c^2+2cdx+d^2x^2\right)}\,dx}{f}\\
&= -\frac{a+b\tan^{-1}(c+dx)}{f(e+fx)}+\frac{(bd)\int \frac{d^2e-2cdf-d^2fx}{1+c^2+2cdx+d^2x^2}\,dx}{f\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)}+\frac{(bdf)\int \frac{1}{e+fx}\,dx}{d^2e^2-2cdef+\left(1+c^2\right)f^2}\\
&= -\frac{a+b\tan^{-1}(c+dx)}{f(e+fx)}+\frac{bd\log(e+fx)}{d^2e^2-2cdef+\left(1+c^2\right)f^2}-\frac{(bd)\int \frac{2cd+2d^2x}{1+c^2+2cdx+d^2x^2}\,dx}{2\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)}\\
&= -\frac{a+b\tan^{-1}(c+dx)}{f(e+fx)}+\frac{bd\log(e+fx)}{d^2e^2-2cdef+\left(1+c^2\right)f^2}-\frac{bd\log\left(1+c^2+2cdx+d^2x^2\right)}{2\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)}\\
&= \frac{bd(de-cf)\tan^{-1}(c+dx)}{f\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)}-\frac{a+b\tan^{-1}(c+dx)}{f(e+fx)}+\frac{bd\log(e+fx)}{d^2e^2-2cdef+\left(1+c^2\right)f^2}-
\end{aligned}$$

Mathematica [C] time = 0.21, size = 121, normalized size = 0.80

$$\frac{-\frac{a+b\tan^{-1}(c+dx)}{e+fx}+\frac{bd(i(-de+(c+i)f)\log(-c-dx+i)+i(-cf+de+if)\log(c+dx+i)+2f\log(d(e+fx)))}{2\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])/(e + f*x)^2,x]
```

```
[Out] (-((a + b*ArcTan[c + d*x])/(e + f*x)) + (b*d*(I*(-(d*e) + (I + c)*f)*Log[I
 - c - d*x] + I*(d*e + I*f - c*f)*Log[I + c + d*x] + 2*f*Log[d*(e + f*x)]))/
(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)))/f
```

fricas [A] time = 0.70, size = 190, normalized size = 1.26

$$-\frac{2\,ad^2e^2-4\,acdef+2\left(ac^2+a\right)f^2-2\left(bcdef-\left(bc^2+b\right)f^2+\left(bd^2ef-bcdf^2\right)x\right)\arctan\left(dx+c\right)+\left(bdf^2x+\right.}{2\left(d^2e^3f-2\,cde^2f^2+\left(c^2+1\right)ef^3+\left(d^2e^2f^2-2\,cdef^3+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d^2*e^2 - 4*a*c*d*e*f + 2*(a*c^2 + a)*f^2 - 2*(b*c*d*e*f - (b*c^2
 + b)*f^2 + (b*d^2*e*f - b*c*d*f^2)*x)*arctan(d*x + c) + (b*d*f^2*x + b*d*e
*f)*log(d^2*x^2 + 2*c*d*x + c^2 + 1) - 2*(b*d*f^2*x + b*d*e*f)*log(f*x + e)
)/(d^2*e^3*f - 2*c*d*e^2*f^2 + (c^2 + 1)*e*f^3 + (d^2*e^2*f^2 - 2*c*d*e*f^3
 + (c^2 + 1)*f^4)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.05, size = 205, normalized size = 1.36

$$-\frac{da}{\left(dfx+de\right)f}-\frac{db\arctan\left(dx+c\right)}{\left(dfx+de\right)f}+\frac{db\ln\left(f\left(dx+c\right)-cf+de\right)}{c^2f^2-2cdef+d^2e^2+f^2}-\frac{db\ln\left(1+\left(dx+c\right)^2\right)}{2\left(c^2f^2-2cdef+d^2e^2+f^2\right)}-\frac{db\arctan}{c^2f^2-2cdef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))/(f*x+e)^2,x)
```

```
[Out] -d*a/(d*f*x+d*e)/f-d*b/(d*f*x+d*e)/f*arctan(d*x+c)+d*b/(c^2*f^2-2*c*d*e*f+d
^2*e^2+f^2)*ln(f*(d*x+c)-c*f+d*e)-1/2*d*b/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*l
n(1+(d*x+c)^2)-d*b/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)*c+d^2*b/f/
(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)*e
```

maxima [A] time = 0.42, size = 177, normalized size = 1.17

$$\frac{1}{2}\left(d\left(\frac{2\left(d^2e-cdf\right)\arctan\left(\frac{d^2x+cd}{d}\right)}{\left(d^2e^2f-2\,cdef^2+\left(c^2+1\right)f^3\right)d}-\frac{\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^2e^2-2\,cdef+\left(c^2+1\right)f^2}+\frac{2\,\log\left(fx+e\right)}{d^2e^2-2\,cdef+\left(c^2+1\right)f^2}\right)-\frac{2\,\text{arctar}}{f^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(d*(2*(d^2*e - c*d*f)*arctan((d^2*x + c*d)/d)/((d^2*e^2*f - 2*c*d*e*f^2
 + (c^2 + 1)*f^3)*d) - log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*e^2 - 2*c*d*e*
f + (c^2 + 1)*f^2) + 2*log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 + 1)*f^2))
- 2*arctan(d*x + c)/(f^2*x + e*f))*b - a/(f^2*x + e*f)
```

mupad [B] time = 1.83, size = 127, normalized size = 0.84

$$\frac{b\,d\,\ln\left(e+f\,x\right)}{d^2\,e^2-2\,c\,d\,e\,f+\left(c^2+1\right)\,f^2}-\frac{b\,\mathrm{atan}\left(c+d\,x\right)}{f\,\left(e+f\,x\right)}-\frac{a}{x\,f^2+e\,f}-\frac{b\,d\,\ln\left(c+d\,x-\mathrm{i}\right)\,1\mathrm{i}}{2\,f\,\left(d\,e-c\,f+f\,1\mathrm{i}\right)}-\frac{b\,d\,\ln\left(c+d\,x+1\mathrm{i}\right)}{2\,f\,\left(f-c\,f\,1\mathrm{i}+d\,e\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))/(e + f*x)^2,x)
```

```
[Out] (b*d*log(e + f*x))/(f^2*(c^2 + 1) + d^2*e^2 - 2*c*d*e*f) - (b*atan(c + d*x)
)/(f*(e + f*x)) - a/(e*f + f^2*x) - (b*d*log(c + d*x - 1i)*1i)/(2*f*(f*1i -
 c*f + d*e)) - (b*d*log(c + d*x + 1i))/(2*f*(f - c*f*1i + d*e*1i))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))/(f*x+e)**2,x)
```

```
[Out] Timed out
```

3.30 $\int \frac{a+b\tan^{-1}(c+dx)}{(e+fx)^3}\,dx$

Optimal. Leaf size=227

$$-\frac{a+b\tan^{-1}(c+dx)}{2f(e+fx)^2}-\frac{bd^2(de-cf)\log\left(c^2+2cdx+d^2x^2+1\right)}{2\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)^2}-\frac{bd}{2(e+fx)\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)}+\frac{bd^2}{\left((c^2\right.}$$

```
[Out] -1/2*b*d/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)/(f*x+e)+1/2*b*d^2*(-c*f+d*e+f)*(d*
e-(1+c)*f)*arctan(d*x+c)/f/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^2+1/2*(-a-b*arct
an(d*x+c))/f/(f*x+e)^2+b*d^2*(-c*f+d*e)*ln(f*x+e)/(d^2*e^2-2*c*d*e*f+(c^2+1
)*f^2)^2-1/2*b*d^2*(-c*f+d*e)*ln(d^2*x^2+2*c*d*x+c^2+1)/(d^2*e^2-2*c*d*e*f+
(c^2+1)*f^2)^2
```

Rubi [A] time = 0.30, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5045, 1982, 709, 800, 634, 618, 204, 628}

$$-\frac{a+b\tan^{-1}(c+dx)}{2f(e+fx)^2}-\frac{bd^2(de-cf)\log\left(c^2+2cdx+d^2x^2+1\right)}{2\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)^2}-\frac{bd}{2(e+fx)\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)}+\frac{bd^2}{\left((c^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])/(e + f*x)^3,x]
```

```
[Out] -(b*d)/(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)*(e + f*x)) + (b*d^2*(d*e +
f - c*f)*(d*e - (1 + c)*f)*ArcTan[c + d*x])/(2*f*(d^2*e^2 - 2*c*d*e*f + (1
+ c^2)*f^2)^2) - (a + b*ArcTan[c + d*x])/(2*f*(e + f*x)^2) + (b*d^2*(d*e -
c*f)*Log[e + f*x])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)^2 - (b*d^2*(d*e -
c*f)*Log[1 + c^2 + 2*c*d*x + d^2*x^2])/(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*
f^2)^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1982

```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Rule 5045

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[((e + f*x)^(m + 1)*(a + b*ArcTan[c + d*x])^p)/(f*(m +
 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcTan[c
 + d*x])^(p - 1))/(1 + (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(c+dx)}{(e+fx)^3}\,dx = -\frac{a+b\tan^{-1}(c+dx)}{2f(e+fx)^2}+\frac{(bd)\int \frac{1}{(e+fx)^2\left(1+(c+dx)^2\right)}\,dx}{2f}$$

$$= -\frac{a+b\tan^{-1}(c+dx)}{2f(e+fx)^2}+\frac{(bd)\int \frac{1}{(e+fx)^2\left(1+c^2+2cdx+d^2x^2\right)}\,dx}{2f}$$

$$= -\frac{bd}{2\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)(e+fx)}-\frac{a+b\tan^{-1}(c+dx)}{2f(e+fx)^2}+\frac{(bd)\int \frac{d(de-2cf)}{(e+fx)\left(1+c^2+\right.}}{2f\left(d^2e^2-2cdef+\right.}$$

$$= -\frac{bd}{2\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)(e+fx)}-\frac{a+b\tan^{-1}(c+dx)}{2f(e+fx)^2}+\frac{(bd)\int \left(\frac{2df^2}{\left(d^2e^2-2cdef+\right.}\right.}{}$$

$$= -\frac{bd}{2\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)(e+fx)}-\frac{a+b\tan^{-1}(c+dx)}{2f(e+fx)^2}+\frac{bd^2(de-cf)\log}{\left(d^2e^2-2cdef+(1\right.}$$

$$= -\frac{bd}{2\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)(e+fx)}-\frac{a+b\tan^{-1}(c+dx)}{2f(e+fx)^2}+\frac{bd^2(de-cf)\log}{\left(d^2e^2-2cdef+(1\right.}$$

$$= -\frac{bd}{2\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)(e+fx)}-\frac{a+b\tan^{-1}(c+dx)}{2f(e+fx)^2}+\frac{bd^2(de-cf)\log}{\left(d^2e^2-2cdef+(1\right.}$$

$$= -\frac{bd}{2\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)(e+fx)}+\frac{bd^2(de-f-cf)(de+f-cf)\tan^{-1}(c+}{2f\left(d^2e^2-2cdef+f^2+c^2f^2\right)^2}$$

Mathematica [C] time = 0.80, size = 175, normalized size = 0.77

$$\frac{-\frac{a+b\tan^{-1}(c+dx)}{(e+fx)^2}+\frac{1}{2}bd^2\left(-\frac{2f}{d(e+fx)\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)}-\frac{4f(cf-de)\log(d(e+fx))}{\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)^2}-\frac{i\log(-c-dx+i)}{(de-(c-i)f)^2}+\frac{i\log(c+dx+i)}{(de-(c+i)f)^2}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])/(e + f*x)^3,x]
```

```
[Out] (-((a + b*ArcTan[c + d*x])/(e + f*x)^2) + (b*d^2*((-2*f)/(d*(d^2*e^2 - 2*c*
d*e*f + (1 + c^2)*f^2)*(e + f*x)) - (I*Log[I - c - d*x])/(d*e - (-I + c)*f)
^2 + (I*Log[I + c + d*x])/(d*e - (I + c)*f)^2 - (4*f*(-(d*e) + c*f)*Log[d*(
e + f*x)])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)^2))/2)/(2*f)
```

fricas [B] time = 2.13, size = 682, normalized size = 3.00

$$-\frac{ad^4e^4-(4\,ac-b)d^3e^3f+2\left(3\,ac^2-bc+a\right)d^2e^2f^2-\left(4\,ac^3-bc^2+4\,ac-b\right)def^3+\left(ac^4+2\,ac^2+a\right)f^4+\left(bd\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a*d^4*e^4 - (4*a*c - b)*d^3*e^3*f + 2*(3*a*c^2 - b*c + a)*d^2*e^2*f^2
 - (4*a*c^3 - b*c^2 + 4*a*c - b)*d*e*f^3 + (a*c^4 + 2*a*c^2 + a)*f^4 + (b*d
^3*e^2*f^2 - 2*b*c*d^2*e*f^3 + (b*c^2 + b)*d*f^4)*x - (2*b*c*d^3*e^3*f - (5
*b*c^2 + 3*b)*d^2*e^2*f^2 + 4*(b*c^3 + b*c)*d*e*f^3 - (b*c^4 + 2*b*c^2 + b)
*f^4 + (b*d^4*e^2*f^2 - 2*b*c*d^3*e*f^3 + (b*c^2 - b)*d^2*f^4)*x^2 + 2*(b*d
^4*e^3*f - 2*b*c*d^3*e^2*f^2 + (b*c^2 - b)*d^2*e*f^3)*x)*arctan(d*x + c) +
(b*d^3*e^3*f - b*c*d^2*e^2*f^2 + (b*d^3*e*f^3 - b*c*d^2*f^4)*x^2 + 2*(b*d^3
*e^2*f^2 - b*c*d^2*e*f^3)*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1) - 2*(b*d^3*e^
3*f - b*c*d^2*e^2*f^2 + (b*d^3*e*f^3 - b*c*d^2*f^4)*x^2 + 2*(b*d^3*e^2*f^2
- b*c*d^2*e*f^3)*x)*log(f*x + e))/(d^4*e^6*f - 4*c*d^3*e^5*f^2 + 2*(3*c^2 +
 1)*d^2*e^4*f^3 - 4*(c^3 + c)*d*e^3*f^4 + (c^4 + 2*c^2 + 1)*e^2*f^5 + (d^4*
e^4*f^3 - 4*c*d^3*e^3*f^4 + 2*(3*c^2 + 1)*d^2*e^2*f^5 - 4*(c^3 + c)*d*e*f^6
 + (c^4 + 2*c^2 + 1)*f^7)*x^2 + 2*(d^4*e^5*f^2 - 4*c*d^3*e^4*f^3 + 2*(3*c^2
 + 1)*d^2*e^3*f^4 - 4*(c^3 + c)*d*e^2*f^5 + (c^4 + 2*c^2 + 1)*e*f^6)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.05, size = 438, normalized size = 1.93

$$-\frac{d^2a}{2\left(dfx+de\right)^2f}-\frac{d^2b\arctan\left(dx+c\right)}{2\left(dfx+de\right)^2f}-\frac{d^2b}{2\left(c^2f^2-2cdef+d^2e^2+f^2\right)\left(dfx+de\right)}-\frac{d^2bf\ln\left(f\left(dx+c\right)-cf+d\right.}{\left(c^2f^2-2cdef+d^2e^2+f^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))/(f*x+e)^3,x)
```

```
[Out] -1/2*d^2*a/(d*f*x+d*e)^2/f-1/2*d^2*b/(d*f*x+d*e)^2/f*arctan(d*x+c)-1/2*d^2*
b/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)/(d*f*x+d*e)-d^2*b*f/(c^2*f^2-2*c*d*e*f+d^
2*e^2+f^2)^2*ln(f*(d*x+c)-c*f+d*e)*c+d^3*b/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)^
```

```
2*ln(f*(d*x+c)-c*f+d*e)*e+1/2*d^2*b*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)^2*arc
tan(d*x+c)*c^2-d^3*b/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)^2*arctan(d*x+c)*c*e+1/
2*d^4*b/f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)^2*arctan(d*x+c)*e^2+1/2*d^2*b*f/(
c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)^2*ln(1+(d*x+c)^2)*c-1/2*d^3*b/(c^2*f^2-2*c*d
*e*f+d^2*e^2+f^2)^2*ln(1+(d*x+c)^2)*e-1/2*d^2*b*f/(c^2*f^2-2*c*d*e*f+d^2*e^
2+f^2)^2*arctan(d*x+c)
```

maxima [A] time = 0.43, size = 409, normalized size = 1.80

$$-\frac{1}{2}\left(d\left(\frac{\left(d^2e-cdf\right)\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^4e^4-4\,cd^3e^3f+2\left(3\,c^2+1\right)d^2e^2f^2-4\left(c^3+c\right)def^3+\left(c^4+2\,c^2+1\right)f^4}-\frac{2\left(d^2e-\right.}{d^4e^4-4\,cd^3e^3f+2\left(3\,c^2+1\right)d\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(d*((d^2*e - c*d*f)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^4*e^4 - 4*c*d^
3*e^3*f + 2*(3*c^2 + 1)*d^2*e^2*f^2 - 4*(c^3 + c)*d*e*f^3 + (c^4 + 2*c^2 +
1)*f^4) - 2*(d^2*e - c*d*f)*log(f*x + e)/(d^4*e^4 - 4*c*d^3*e^3*f + 2*(3*c^
2 + 1)*d^2*e^2*f^2 - 4*(c^3 + c)*d*e*f^3 + (c^4 + 2*c^2 + 1)*f^4) - (d^4*e^
2 - 2*c*d^3*e*f + (c^2 - 1)*d^2*f^2)*arctan((d^2*x + c*d)/d)/((d^4*e^4*f -
4*c*d^3*e^3*f^2 + 2*(3*c^2 + 1)*d^2*e^2*f^3 - 4*(c^3 + c)*d*e*f^4 + (c^4 +
2*c^2 + 1)*f^5)*d) + 1/(d^2*e^3 - 2*c*d*e^2*f + (c^2 + 1)*e*f^2 + (d^2*e^2*
f - 2*c*d*e*f^2 + (c^2 + 1)*f^3)*x)) + arctan(d*x + c)/(f^3*x^2 + 2*e*f^2*x
 + e^2*f))*b - 1/2*a/(f^3*x^2 + 2*e*f^2*x + e^2*f)
```

mupad [B] time = 7.53, size = 399, normalized size = 1.76

$$\frac{b\,d^3\,e\,\ln\left(e+f\,x\right)}{{\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)}^2}-\frac{a\,f}{2\,{\left(e+f\,x\right)}^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)}-\frac{b\,d\,e}{2\,{\left(e+f\,x\right)}^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))/(e + f*x)^3,x)
```

```
[Out] (b*d^2*log(c + d*x + 1i)*1i)/(4*f*(f*1i + c*f - d*e)^2) - (a*f)/(2*(e + f*x
)^2*(f^2 + c^2*f^2 + d^2*e^2 - 2*c*d*e*f)) - (b*d*e)/(2*(e + f*x)^2*(f^2 +
c^2*f^2 + d^2*e^2 - 2*c*d*e*f)) - (b*d^2*log(c + d*x - 1i)*1i)/(4*f*(f*1i -
 c*f + d*e)^2) - (b*atan(c + d*x))/(2*f*(e + f*x)^2) - (a*c^2*f)/(2*(e + f*
x)^2*(f^2 + c^2*f^2 + d^2*e^2 - 2*c*d*e*f)) + (b*d^3*e*log(e + f*x))/(f^2 +
 c^2*f^2 + d^2*e^2 - 2*c*d*e*f)^2 - (b*c*d^2*f*log(e + f*x))/(f^2 + c^2*f^2
 + d^2*e^2 - 2*c*d*e*f)^2 + (a*c*d*e)/((e + f*x)^2*(f^2 + c^2*f^2 + d^2*e^2
 - 2*c*d*e*f)) - (b*d*f*x)/(2*(e + f*x)^2*(f^2 + c^2*f^2 + d^2*e^2 - 2*c*d*
e*f)) - (a*d^2*e^2)/(2*f*(e + f*x)^2*(f^2 + c^2*f^2 + d^2*e^2 - 2*c*d*e*f))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))/(f*x+e)**3,x)
```

```
[Out] Timed out
```

3.31 $\int (e+fx)^2 \left(a+b\tan^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=382

$$\frac{i\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)\left(a+b\tan^{-1}(c+dx)\right)^2}{3d^3}-\frac{(de-cf)\left(-\left(3-c^2\right)f^2-2cdef+d^2e^2\right)\left(a+b\tan^{-1}\right.}{3d^3f}$$

```
[Out] 1/3*b^2*f^2*x/d^2-2*a*b*f*(-c*f+d*e)*x/d^2-1/3*b^2*f^2*arctan(d*x+c)/d^3-2*
b^2*f*(-c*f+d*e)*(d*x+c)*arctan(d*x+c)/d^3-1/3*b*f^2*(d*x+c)^2*(a+b*arctan(
d*x+c))/d^3+1/3*I*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*(a+b*arctan(d*x+c))^
2/d^3-1/3*(-c*f+d*e)*(d^2*e^2-2*c*d*e*f-(-c^2+3)*f^2)*(a+b*arctan(d*x+c))^2
/d^3/f+1/3*(f*x+e)^3*(a+b*arctan(d*x+c))^2/f+2/3*b*(3*d^2*e^2-6*c*d*e*f-(-3
*c^2+1)*f^2)*(a+b*arctan(d*x+c))*ln(2/(1+I*(d*x+c)))/d^3+b^2*f*(-c*f+d*e)*l
n(1+(d*x+c)^2)/d^3+1/3*I*b^2*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*polylog(2
,1-2/(1+I*(d*x+c)))/d^3
```

Rubi [A] time = 0.57, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.650, Rules used = {5047, 4864, 4846, 260, 4852, 321, 203, 4984, 4884, 4920, 4854, 2402, 2315}

$$\frac{ib^2\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{3d^3}+\frac{i\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)\left(a+b\tan^{-1}\right.}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (b^2*f^2*x)/(3*d^2) - (2*a*b*f*(d*e - c*f)*x)/d^2 - (b^2*f^2*ArcTan[c + d*x
])/(3*d^3) - (2*b^2*f*(d*e - c*f)*(c + d*x)*ArcTan[c + d*x])/d^3 - (b*f^2*(
c + d*x)^2*(a + b*ArcTan[c + d*x]))/(3*d^3) + ((I/3)*(3*d^2*e^2 - 6*c*d*e*f
 - (1 - 3*c^2)*f^2)*(a + b*ArcTan[c + d*x])^2)/d^3 - ((d*e - c*f)*(d^2*e^2
- 2*c*d*e*f - (3 - c^2)*f^2)*(a + b*ArcTan[c + d*x])^2)/(3*d^3*f) + ((e + f
*x)^3*(a + b*ArcTan[c + d*x])^2)/(3*f) + (2*b*(3*d^2*e^2 - 6*c*d*e*f - (1 -
 3*c^2)*f^2)*(a + b*ArcTan[c + d*x])*Log[2/(1 + I*(c + d*x))])/(3*d^3) + (b
^2*f*(d*e - c*f)*Log[1 + (c + d*x)^2])/d^3 + ((I/3)*b^2*(3*d^2*e^2 - 6*c*d*
e*f - (1 - 3*c^2)*f^2)*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d^3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4984

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\int (e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2\left(a+b\tan^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$=\frac{(e+fx)^3\left(a+b\tan^{-1}(c+dx)\right)^2}{3f}-\frac{(2b)\,\text{Subst}\left(\int\left(\frac{3f^2(de-cf)\left(a+b\tan^{-1}(x)\right)}{d^3}\right.\right.}{}$$

$$=\frac{(e+fx)^3\left(a+b\tan^{-1}(c+dx)\right)^2}{3f}-\frac{(2b)\,\text{Subst}\left(\int\frac{\left((de-cf)\left(d^2e^2-2cdef-3f^2\right.\right.}{}\right.}{}$$

$$=-\frac{2abf(de-cf)x}{d^2}-\frac{bf^2(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)}{3d^3}+\frac{(e+fx)^3\left(a\right.}{}$$

$$=\frac{b^2f^2x}{3d^2}-\frac{2abf(de-cf)x}{d^2}-\frac{2b^2f(de-cf)(c+dx)\tan^{-1}(c+dx)}{d^3}-\frac{bf^2}{}$$

$$=\frac{b^2f^2x}{3d^2}-\frac{2abf(de-cf)x}{d^2}-\frac{b^2f^2\tan^{-1}(c+dx)}{3d^3}-\frac{2b^2f(de-cf)(c+d}{d^3}$$

$$=\frac{b^2f^2x}{3d^2}-\frac{2abf(de-cf)x}{d^2}-\frac{b^2f^2\tan^{-1}(c+dx)}{3d^3}-\frac{2b^2f(de-cf)(c+d}{d^3}$$

$$=\frac{b^2f^2x}{3d^2}-\frac{2abf(de-cf)x}{d^2}-\frac{b^2f^2\tan^{-1}(c+dx)}{3d^3}-\frac{2b^2f(de-cf)(c+d}{d^3}$$

$$=\frac{b^2f^2x}{3d^2}-\frac{2abf(de-cf)x}{d^2}-\frac{b^2f^2\tan^{-1}(c+dx)}{3d^3}-\frac{2b^2f(de-cf)(c+d}{d^3}$$

Mathematica [B] time = 4.23, size = 801, normalized size = 2.10

$$\frac{1}{3}a^2f^2x^3+a^2efx^2+a^2e^2x+\frac{ab\left(-dfx(6de-4cf+dfx)+2\left(f^2c^3-3defc^2+3\left(d^2e^2-f^2\right)c+3def+d^3x\left(3e^2\right.\right.\right.}{3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)^2*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] a^2*e^2*x + a^2*e*f*x^2 + (a^2*f^2*x^3)/3 + (a*b*(-(d*f*x*(6*d*e - 4*c*f +
d*f*x)) + 2*(3*d*e*f - 3*c^2*d*e*f + c^3*f^2 + 3*c*(d^2*e^2 - f^2) + d^3*x*
(3*e^2 + 3*e*f*x + f^2*x^2))*ArcTan[c + d*x] + (-3*d^2*e^2 + 6*c*d*e*f + (1
 - 3*c^2)*f^2)*Log[1 + (c + d*x)^2]))/(3*d^3) + (b^2*e^2*(ArcTan[c + d*x]*(
(-I + c + d*x)*ArcTan[c + d*x] + 2*Log[1 + E^((2*I)*ArcTan[c + d*x])]) - I*
PolyLog[2, -E^((2*I)*ArcTan[c + d*x])]))/d + (b^2*e*f*((1 + (2*I)*c - c^2 +
```

```
 d^2*x^2)*ArcTan[c + d*x]^2 - 2*ArcTan[c + d*x]*(c + d*x + 2*c*Log[1 + E^((
2*I)*ArcTan[c + d*x])]) + Log[1 + (c + d*x)^2] + (2*I)*c*PolyLog[2, -E^((2*
I)*ArcTan[c + d*x])]))/d^2 + (b^2*f^2*(1 + (c + d*x)^2)^(3/2)*((c + d*x)/Sq
rt[1 + (c + d*x)^2] + (6*c*(c + d*x)*ArcTan[c + d*x])/Sqrt[1 + (c + d*x)^2]
 + (3*(c + d*x)*ArcTan[c + d*x]^2)/Sqrt[1 + (c + d*x)^2] + (3*c^2*(c + d*x)
*ArcTan[c + d*x]^2)/Sqrt[1 + (c + d*x)^2] + I*ArcTan[c + d*x]^2*Cos[3*ArcTa
n[c + d*x]] - (3*I)*c^2*ArcTan[c + d*x]^2*Cos[3*ArcTan[c + d*x]] - 2*ArcTan
[c + d*x]*Cos[3*ArcTan[c + d*x]]*Log[1 + E^((2*I)*ArcTan[c + d*x])] + 6*c^2
*ArcTan[c + d*x]*Cos[3*ArcTan[c + d*x]]*Log[1 + E^((2*I)*ArcTan[c + d*x])]
+ 6*c*Cos[3*ArcTan[c + d*x]]*Log[1/Sqrt[1 + (c + d*x)^2]] + ((3*I - 12*c -
(9*I)*c^2)*ArcTan[c + d*x]^2 + 2*ArcTan[c + d*x]*(-2 + (-3 + 9*c^2)*Log[1 +
 E^((2*I)*ArcTan[c + d*x])]) + 18*c*Log[1/Sqrt[1 + (c + d*x)^2]])/Sqrt[1 +
(c + d*x)^2] - ((4*I)*(-1 + 3*c^2)*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])])/
(1 + (c + d*x)^2)^(3/2) + Sin[3*ArcTan[c + d*x]] + 6*c*ArcTan[c + d*x]*Sin[
3*ArcTan[c + d*x]] - ArcTan[c + d*x]^2*Sin[3*ArcTan[c + d*x]] + 3*c^2*ArcTa
n[c + d*x]^2*Sin[3*ArcTan[c + d*x]]))/(12*d^3)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(a^2f^2x^2 + 2\,a^2efx + a^2e^2 + \left(b^2f^2x^2 + 2\,b^2efx + b^2e^2\right)\arctan\left(dx+c\right)^2 + 2\left(abf^2x^2 + 2\,abefx + abe^2\right)\text{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*f^2*x^2 + 2*a^2*e*f*x + a^2*e^2 + (b^2*f^2*x^2 + 2*b^2*e*f*x +
 b^2*e^2)*arctan(d*x + c)^2 + 2*(a*b*f^2*x^2 + 2*a*b*e*f*x + a*b*e^2)*arcta
n(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.15, size = 1622, normalized size = 4.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*(a+b*arctan(d*x+c))^2,x)
```

```
[Out] a^2*x*e^2+1/3/d^3*a*b*f^2*ln(1+(d*x+c)^2)+5/3/d^3*a*b*f^2*c^2+1/3*a^2/f*e^3
+1/3/d^3*b^2*f^2*c+a^2*f*x^2*e+1/3*b^2*f^2*arctan(d*x+c)^2*x^3+arctan(d*x+c
)^2*x*b^2*e^2-1/2*I/d^3*b^2*dilog(1/2*I*(d*x+c-I))*c^2*f^2+1/6*I/d^3*b^2*ln
(1+(d*x+c)^2)*ln(d*x+c-I)*f^2-1/6*I/d^3*b^2*ln(1+(d*x+c)^2)*ln(I+d*x+c)*f^2
+2/d*arctan(d*x+c)*a*b*c*e^2-1/d^3*a*b*f^2*ln(1+(d*x+c)^2)*c^2-2/d^3*a*b*f^
2*arctan(d*x+c)*c-2/d*b^2*f*arctan(d*x+c)*e*x+2/3/d^3*a*b*f^2*arctan(d*x+c)
*c^3-1/d^2*b^2*f*arctan(d*x+c)^2*c^2*e+1/3*a^2*f^2*x^3+4/3*a*b/d^2*f^2*c*x-
2*a*b/d*f*e*x+1/6*I/d^3*b^2*ln(I+d*x+c)*ln(1/2*I*(d*x+c-I))*f^2-1/4*I/d^3*b
^2*ln(I+d*x+c)^2*c^2*f^2+1/4*I/d^3*b^2*ln(d*x+c-I)^2*c^2*f^2-2/d^2*b^2*f*ar
ctan(d*x+c)*e*c+4/3/d^2*b^2*f^2*arctan(d*x+c)*c*x+2*a*b*f*arctan(d*x+c)*e*x
^2-1/d^3*b^2*f^2*arctan(d*x+c)*ln(1+(d*x+c)^2)*c^2+2/d^2*a*b*f*arctan(d*x+c
)*e-1/6*I/d^3*b^2*ln(d*x+c-I)*ln(-1/2*I*(I+d*x+c))*f^2+1/2*I/d^3*b^2*dilog(
-1/2*I*(I+d*x+c))*c^2*f^2+1/2*I/d*b^2*ln(1+(d*x+c)^2)*ln(I+d*x+c)*e^2-1/2*I
/d*b^2*ln(I+d*x+c)*ln(1/2*I*(d*x+c-I))*e^2-1/2*I/d*b^2*ln(1+(d*x+c)^2)*ln(d
*x+c-I)*e^2+1/2*I/d*b^2*ln(d*x+c-I)*ln(-1/2*I*(I+d*x+c))*e^2+1/3*b^2*f^2*x/
```

```
d^2-1/3*b^2*f^2*arctan(d*x+c)/d^3-I/d^2*b^2*ln(d*x+c-I)*ln(-1/2*I*(I+d*x+c)
)*c*e*f-I/d^2*b^2*ln(1+(d*x+c)^2)*ln(I+d*x+c)*c*e*f+I/d^2*b^2*ln(I+d*x+c)*l
n(1/2*I*(d*x+c-I))*c*e*f+I/d^2*b^2*ln(1+(d*x+c)^2)*ln(d*x+c-I)*c*e*f-1/d^3*
b^2*f^2*arctan(d*x+c)^2*c+1/d*arctan(d*x+c)^2*b^2*c*e^2+1/3/d^3*b^2*f^2*arc
tan(d*x+c)*ln(1+(d*x+c)^2)+1/d^2*b^2*f*ln(1+(d*x+c)^2)*e+1/d^2*b^2*f*arctan
(d*x+c)^2*e-1/2*I/d^3*b^2*ln(1+(d*x+c)^2)*ln(d*x+c-I)*c^2*f^2+1/2*I/d^3*b^2
*ln(d*x+c-I)*ln(-1/2*I*(I+d*x+c))*c^2*f^2-1/2*I/d^2*b^2*ln(d*x+c-I)^2*c*e*f
+1/2*I/d^3*b^2*ln(1+(d*x+c)^2)*ln(I+d*x+c)*c^2*f^2-I/d^2*b^2*dilog(-1/2*I*(
I+d*x+c))*c*e*f+I/d^2*b^2*dilog(1/2*I*(d*x+c-I))*c*e*f+1/2*I/d^2*b^2*ln(I+d
*x+c)^2*c*e*f-1/2*I/d^3*b^2*ln(I+d*x+c)*ln(1/2*I*(d*x+c-I))*c^2*f^2+1/3/d^3
*b^2*f^2*arctan(d*x+c)^2*c^3+5/3/d^3*b^2*f^2*arctan(d*x+c)*c^2-1/3/d*b^2*f^
2*arctan(d*x+c)*x^2-1/d^3*b^2*f^2*ln(1+(d*x+c)^2)*c+b^2*f*arctan(d*x+c)^2*e
*x^2-1/3/d*a*b*f^2*x^2+2/3*a*b*f^2*arctan(d*x+c)*x^3+2*arctan(d*x+c)*x*a*b*
e^2+1/6*I/d^3*b^2*dilog(1/2*I*(d*x+c-I))*f^2-1/6*I/d^3*b^2*dilog(-1/2*I*(I+
d*x+c))*f^2+1/12*I/d^3*b^2*ln(I+d*x+c)^2*f^2+1/2*I/d*b^2*dilog(-1/2*I*(I+d*
x+c))*e^2-1/2*I/d*b^2*dilog(1/2*I*(d*x+c-I))*e^2+1/4*I/d*b^2*ln(d*x+c-I)^2*
e^2-1/4*I/d*b^2*ln(I+d*x+c)^2*e^2-1/12*I/d^3*b^2*ln(d*x+c-I)^2*f^2-1/d*e^2*
a*b*ln(1+(d*x+c)^2)-1/d*e^2*b^2*arctan(d*x+c)*ln(1+(d*x+c)^2)-2/d^2*a*b*f*c
*e-2/d^2*a*b*f*arctan(d*x+c)*e*c^2+2/d^2*a*b*f*ln(1+(d*x+c)^2)*c*e+2/d^2*b^
2*f*arctan(d*x+c)*ln(1+(d*x+c)^2)*c*e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 3/4*b^2*c^2*e^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 1/4*(3*arctan
(d*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*b^2*c^
2*e^2 + 1/3*a^2*f^2*x^3 + 36*b^2*d^2*f^2*integrate(1/48*x^4*arctan(d*x + c)
^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*d^2*f^2*integrate(1/48*x^4*log
(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 72*b^2*
d^2*e*f*integrate(1/48*x^3*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1),
 x) + 72*b^2*c*d*f^2*integrate(1/48*x^3*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*
x + c^2 + 1), x) + 4*b^2*d^2*f^2*integrate(1/48*x^4*log(d^2*x^2 + 2*c*d*x +
 c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^2*d^2*e*f*integrate(1/48*
x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) +
6*b^2*c*d*f^2*integrate(1/48*x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^
2 + 2*c*d*x + c^2 + 1), x) + 36*b^2*d^2*e^2*integrate(1/48*x^2*arctan(d*x +
 c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 144*b^2*c*d*e*f*integrate(1/48*x^
2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 36*b^2*c^2*f^2*inte
grate(1/48*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^2
*d^2*e*f*integrate(1/48*x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c
*d*x + c^2 + 1), x) + 4*b^2*c*d*f^2*integrate(1/48*x^3*log(d^2*x^2 + 2*c*d*
x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*d^2*e^2*integrate(1/
48*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)
 + 12*b^2*c*d*e*f*integrate(1/48*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^
2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*c^2*f^2*integrate(1/48*x^2*log(d^2*x
^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 72*b^2*c*d*e^
2*integrate(1/48*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 72
*b^2*c^2*e*f*integrate(1/48*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 +
1), x) + 12*b^2*d^2*e^2*integrate(1/48*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)
/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^2*c*d*e*f*integrate(1/48*x^2*log(
d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^2*c*d*
e^2*integrate(1/48*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x
+ c^2 + 1), x) + 6*b^2*c^2*e*f*integrate(1/48*x*log(d^2*x^2 + 2*c*d*x + c^2
 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^2*c*d*e^2*integrate(1/48*x
*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2
*c^2*e^2*integrate(1/48*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d
```

```
*x + c^2 + 1), x) + a^2*e*f*x^2 + 3/4*b^2*e^2*arctan(d*x + c)^2*arctan((d^2
*x + c*d)/d)/d - 8*b^2*d*f^2*integrate(1/48*x^3*arctan(d*x + c)/(d^2*x^2 +
2*c*d*x + c^2 + 1), x) - 24*b^2*d*e*f*integrate(1/48*x^2*arctan(d*x + c)/(d
^2*x^2 + 2*c*d*x + c^2 + 1), x) - 24*b^2*d*e^2*integrate(1/48*x*arctan(d*x
+ c)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 1/4*(3*arctan(d*x + c)*arctan((d^2
*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*b^2*e^2 + 2*(x^2*arctan(d*x
 + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 +
2*c*d*x + c^2 + 1)/d^3))*a*b*e*f + 1/3*(2*x^3*arctan(d*x + c) - d*((d*x^2 -
 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d
^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*a*b*f^2 + a^2*e^2*x + 36*b^2*f^2*integrat
e(1/48*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*f^2*
integrate(1/48*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x +
c^2 + 1), x) + 72*b^2*e*f*integrate(1/48*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c
*d*x + c^2 + 1), x) + 6*b^2*e*f*integrate(1/48*x*log(d^2*x^2 + 2*c*d*x + c^
2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*e^2*integrate(1/48*log(d
^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + (2*(d*x +
 c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*a*b*e^2/d + 1/12*(b^2*f^2*x^3 +
 3*b^2*e*f*x^2 + 3*b^2*e^2*x)*arctan(d*x + c)^2 - 1/48*(b^2*f^2*x^3 + 3*b^2
*e*f*x^2 + 3*b^2*e^2*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(e+f\,x\right)^2\,\left(a+b\,\mathrm{atan}\left(c+d\,x\right)\right)^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2*(a + b*atan(c + d*x))^2,x)
```

```
[Out] int((e + f*x)^2*(a + b*atan(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{atan}\left(c+dx\right)\right)^2 \left(e+fx\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(a+b*atan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*atan(c + d*x))**2*(e + f*x)**2, x)
```

3.32 $\int (e+fx)\left(a+b\tan^{-1}(c+dx)\right)^2\,dx$

Optimal. Leaf size=222

$$\frac{i(de-cf)\left(a+b\tan^{-1}(c+dx)\right)^2}{d^2}-\frac{(-cf+de+f)(de-(c+1)f)\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^2f}+\frac{2b(de-cf)\log\left(\frac{2}{1+i(c\cdot}\right.}{}$$

```
[Out] -a*b*f*x/d-b^2*f*(d*x+c)*arctan(d*x+c)/d^2+I*(-c*f+d*e)*(a+b*arctan(d*x+c))
^2/d^2-1/2*(-c*f+d*e+f)*(d*e-(1+c)*f)*(a+b*arctan(d*x+c))^2/d^2/f+1/2*(f*x+
e)^2*(a+b*arctan(d*x+c))^2/f+2*b*(-c*f+d*e)*(a+b*arctan(d*x+c))*ln(2/(1+I*(
d*x+c)))/d^2+1/2*b^2*f*ln(1+(d*x+c)^2)/d^2+I*b^2*(-c*f+d*e)*polylog(2,1-2/(
1+I*(d*x+c)))/d^2
```

Rubi [A] time = 0.37, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {5047, 4864, 4846, 260, 4984, 4884, 4920, 4854, 2402, 2315}

$$\frac{ib^2(de-cf)\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2}+\frac{i(de-cf)\left(a+b\tan^{-1}(c+dx)\right)^2}{d^2}-\frac{(-cf+de+f)(de-(c+1)f)\left(a+b\right.}{2d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] -((a*b*f*x)/d) - (b^2*f*(c + d*x)*ArcTan[c + d*x])/d^2 + (I*(d*e - c*f)*(a
+ b*ArcTan[c + d*x])^2)/d^2 - ((d*e + f - c*f)*(d*e - (1 + c)*f)*(a + b*Arc
Tan[c + d*x])^2)/(2*d^2*f) + ((e + f*x)^2*(a + b*ArcTan[c + d*x])^2)/(2*f)
+ (2*b*(d*e - c*f)*(a + b*ArcTan[c + d*x])*Log[2/(1 + I*(c + d*x))])/d^2 +
(b^2*f*Log[1 + (c + d*x)^2])/(2*d^2) + (I*b^2*(d*e - c*f)*PolyLog[2, 1 - 2/
(1 + I*(c + d*x))])/d^2
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
```

```
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4984

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\int (e+fx)\left(a+b\tan^{-1}(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int\left(\frac{de-cf}{d}+\frac{fx}{d}\right)\left(a+b\tan^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$=\frac{(e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2f}-\frac{b\,\text{Subst}\left(\int\left(\frac{f^2\left(a+b\tan^{-1}(x)\right)}{d^2}+\frac{((de-f-c\ldots}{\ldots}\right.\right.}{\ldots}$$

$$=\frac{(e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2f}-\frac{b\,\text{Subst}\left(\int\frac{((de-f-cf)(de+f-cf)+2f(de-c\ldots}{1+x^2}\right.}{d^2f}$$

$$=-\frac{abfx}{d}+\frac{(e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2f}-\frac{b\,\text{Subst}\left(\int\left(\frac{(de+f-cf)(de-(1\ldots}{1\ldots}\right.\right.}{\ldots}$$

$$=-\frac{abfx}{d}-\frac{b^2f(c+dx)\tan^{-1}(c+dx)}{d^2}+\frac{(e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2f}\ldots$$

$$=-\frac{abfx}{d}-\frac{b^2f(c+dx)\tan^{-1}(c+dx)}{d^2}+\frac{i(de-cf)\left(a+b\tan^{-1}(c+dx)\right)^2}{d^2}\ldots$$

$$=-\frac{abfx}{d}-\frac{b^2f(c+dx)\tan^{-1}(c+dx)}{d^2}+\frac{i(de-cf)\left(a+b\tan^{-1}(c+dx)\right)^2}{d^2}\ldots$$

$$=-\frac{abfx}{d}-\frac{b^2f(c+dx)\tan^{-1}(c+dx)}{d^2}+\frac{i(de-cf)\left(a+b\tan^{-1}(c+dx)\right)^2}{d^2}\ldots$$

$$=-\frac{abfx}{d}-\frac{b^2f(c+dx)\tan^{-1}(c+dx)}{d^2}+\frac{i(de-cf)\left(a+b\tan^{-1}(c+dx)\right)^2}{d^2}\ldots$$

Mathematica [A] time = 0.45, size = 264, normalized size = 1.19

$$\frac{-a^2c^2f+2a^2cde+2a^2d^2ex+a^2d^2fx^2-2b\tan^{-1}(c+dx)\left(a\left(c^2f-2cde-2d^2ex-f\left(d^2x^2+1\right)\right)-2b(de-cf\ldots\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (2*a^2*c*d*e - 2*a*b*c*f - a^2*c^2*f + 2*a^2*d^2*e*x - 2*a*b*d*f*x + a^2*d^
2*f*x^2 + b^2*(-I + c + d*x)*(2*d*e + I*f - c*f + d*f*x)*ArcTan[c + d*x]^2
- 2*b*ArcTan[c + d*x]*(b*f*(c + d*x) + a*(-2*c*d*e + c^2*f - 2*d^2*e*x - f*
(1 + d^2*x^2)) - 2*b*(d*e - c*f)*Log[1 + E^((2*I)*ArcTan[c + d*x])]) + 4*a*
b*d*e*Log[1/Sqrt[1 + (c + d*x)^2]] - 2*b^2*f*Log[1/Sqrt[1 + (c + d*x)^2]] -
 4*a*b*c*f*Log[1/Sqrt[1 + (c + d*x)^2]] - (2*I)*b^2*(d*e - c*f)*PolyLog[2,
-E^((2*I)*ArcTan[c + d*x])])/(2*d^2)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(a^2fx+a^2e+\left(b^2fx+b^2e\right)\arctan\left(dx+c\right)^2+2\left(abfx+abe\right)\arctan\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*f*x + a^2*e + (b^2*f*x + b^2*e)*arctan(d*x + c)^2 + 2*(a*b*f*x
 + a*b*e)*arctan(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.14, size = 748, normalized size = 3.37

$$-\frac{ib^2\ln(dx+c-i)^2 cf}{4d^2}+\frac{ib^2\ln(dx+c+i)^2 cf}{4d^2}-\frac{ib^2\ln\left(1+(dx+c)^2\right)\ln(dx+c-i)e}{2d}-\frac{ib^2\ln(dx+c+i)\ln\left(\frac{i(dx+c}{2}\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(a+b*arctan(d*x+c))^2,x)
```

```
[Out] -1/2*I/d^2*b^2*ln(1+(d*x+c)^2)*ln(I+d*x+c)*c*f+1/2*I/d^2*b^2*ln(I+d*x+c)*ln
(1/2*I*(d*x+c-I))*c*f+1/2*I/d^2*b^2*ln(1+(d*x+c)^2)*ln(d*x+c-I)*c*f-1/2*I/d
^2*b^2*ln(-1/2*I*(I+d*x+c))*ln(d*x+c-I)*c*f-a*b*f*x/d+1/2*b^2*f*ln(1+(d*x+c
)^2)/d^2+1/2/d^2*b^2*arctan(d*x+c)^2*f+1/2*b^2*arctan(d*x+c)^2*f*x^2+arctan
(d*x+c)^2*x*b^2*e+1/d*a^2*c*e-1/2/d^2*a^2*f*c^2-1/2*I/d*b^2*ln(1+(d*x+c)^2)
*ln(d*x+c-I)*e-1/2*I/d*b^2*ln(I+d*x+c)*ln(1/2*I*(d*x+c-I))*e-1/2*I/d^2*b^2*
dilog(-1/2*I*(I+d*x+c))*c*f+1/2*I/d*b^2*ln(1+(d*x+c)^2)*ln(I+d*x+c)*e+1/2*I
/d*b^2*ln(-1/2*I*(I+d*x+c))*ln(d*x+c-I)*e-1/4*I/d^2*b^2*ln(d*x+c-I)^2*c*f+1
/2*I/d^2*b^2*dilog(1/2*I*(d*x+c-I))*c*f+1/d^2*a*b*ln(1+(d*x+c)^2)*c*f+2/d*a
rctan(d*x+c)*a*b*c*e+1/d^2*b^2*ln(1+(d*x+c)^2)*arctan(d*x+c)*c*f-1/d^2*a*b*
f*arctan(d*x+c)*c^2+1/4*I/d^2*b^2*ln(I+d*x+c)^2*c*f+1/d^2*a*b*f*arctan(d*x+
c)+1/d*arctan(d*x+c)^2*b^2*c*e-1/4*I/d*b^2*ln(I+d*x+c)^2*e-1/2*I/d*b^2*dilo
g(1/2*I*(d*x+c-I))*e+1/2*I/d*b^2*dilog(-1/2*I*(I+d*x+c))*e+1/4*I/d*b^2*ln(d
*x+c-I)^2*e-1/2/d^2*b^2*arctan(d*x+c)^2*f*c^2-1/d*b^2*arctan(d*x+c)*f*x-1/d
^2*b^2*arctan(d*x+c)*f*c+2*arctan(d*x+c)*x*a*b*e+a*b*f*arctan(d*x+c)*x^2-1/
d*b^2*ln(1+(d*x+c)^2)*arctan(d*x+c)*e-1/d*a*b*ln(1+(d*x+c)^2)*e-1/d^2*a*b*c
*f+1/2*a^2*x^2*f+a^2*x*e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 3/4*b^2*c^2*e*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 1/4*(3*arctan(d
*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*b^2*c^2*
e + 12*b^2*d^2*f*integrate(1/16*x^3*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x +
c^2 + 1), x) + b^2*d^2*f*integrate(1/16*x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1
)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^2*d^2*e*integrate(1/16*x^2*arc
tan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 24*b^2*c*d*f*integrate(1
/16*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 2*b^2*d^2*f*i
ntegrate(1/16*x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2
 + 1), x) + b^2*d^2*e*integrate(1/16*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2
/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 2*b^2*c*d*f*integrate(1/16*x^2*log(d^2
*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 24*b^2*c*d*
e*integrate(1/16*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12
*b^2*c^2*f*integrate(1/16*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1)
, x) + 4*b^2*d^2*e*integrate(1/16*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2
*x^2 + 2*c*d*x + c^2 + 1), x) + 2*b^2*c*d*f*integrate(1/16*x^2*log(d^2*x^2
+ 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 2*b^2*c*d*e*integr
```

```
ate(1/16*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1)
, x) + b^2*c^2*f*integrate(1/16*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x
^2 + 2*c*d*x + c^2 + 1), x) + 4*b^2*c*d*e*integrate(1/16*x*log(d^2*x^2 + 2*
c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + b^2*c^2*e*integrate(1/
16*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 1
/2*a^2*f*x^2 + 3/4*b^2*e*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 4*b^
2*d*f*integrate(1/16*x^2*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)
- 8*b^2*d*e*integrate(1/16*x*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 + 1),
 x) - 1/4*(3*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x +
c*d)/d)^3/d)*b^2*e + (x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^
2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a*b*f + a^2*e*
x + 12*b^2*f*integrate(1/16*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 +
1), x) + b^2*f*integrate(1/16*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2
 + 2*c*d*x + c^2 + 1), x) + b^2*e*integrate(1/16*log(d^2*x^2 + 2*c*d*x + c^
2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + (2*(d*x + c)*arctan(d*x + c) -
 log((d*x + c)^2 + 1))*a*b*e/d + 1/8*(b^2*f*x^2 + 2*b^2*e*x)*arctan(d*x + c
)^2 - 1/32*(b^2*f*x^2 + 2*b^2*e*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(e + f\,x\right)\,\left(a + b\,\mathrm{atan}\left(c + d\,x\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)*(a + b*atan(c + d*x))^2,x)
[Out] int((e + f*x)*(a + b*atan(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,\mathrm{atan}\,(c + dx)\right)^2 \left(e + fx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*atan(d*x+c))**2,x)
[Out] Integral((a + b*atan(c + d*x))**2*(e + f*x), x)
```

3.33 $\int \left(a+b\tan^{-1}(c+dx)\right)^2 dx$

Optimal. Leaf size=102

$$\frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{d}+\frac{i\left(a+b\tan^{-1}(c+dx)\right)^2}{d}+\frac{2b\log\left(\frac{2}{1+i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{d}+\frac{ib^2\text{Li}_2\left(1-\frac{}{i(c\cdot}\right.}{d}$$

```
[Out] I*(a+b*arctan(d*x+c))^2/d+(d*x+c)*(a+b*arctan(d*x+c))^2/d+2*b*(a+b*arctan(d
*x+c))*ln(2/(1+I*(d*x+c)))/d+I*b^2*polylog(2,1-2/(1+I*(d*x+c)))/d
```

Rubi [A] time = 0.11, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5039, 4846, 4920, 4854, 2402, 2315}

$$\frac{ib^2\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d}+\frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{d}+\frac{i\left(a+b\tan^{-1}(c+dx)\right)^2}{d}+\frac{2b\log\left(\frac{2}{1+i(c+dx)}\right)\left(a+b\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (I*(a + b*ArcTan[c + d*x])^2)/d + ((c + d*x)*(a + b*ArcTan[c + d*x])^2)/d +
 (2*b*(a + b*ArcTan[c + d*x])*Log[2/(1 + I*(c + d*x))])/d + (I*b^2*PolyLog[
2, 1 - 2/(1 + I*(c + d*x))])/d
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5039

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(a+b\tan^{-1}(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int \left(a+b\tan^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{d} - \frac{(2b)\,\text{Subst}\left(\int \frac{x\left(a+b\tan^{-1}(x)\right)}{1+x^2} dx, x, c+dx\right)}{d}$$

$$= \frac{i\left(a+b\tan^{-1}(c+dx)\right)^2}{d} + \frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{d} + \frac{(2b)\,\text{Subst}\left(\int \frac{a+b}{}\right.}{}$$

$$= \frac{i\left(a+b\tan^{-1}(c+dx)\right)^2}{d} + \frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{d} + \frac{2b\left(a+b\tan^{-1}(c\right.}{}$$

$$= \frac{i\left(a+b\tan^{-1}(c+dx)\right)^2}{d} + \frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{d} + \frac{2b\left(a+b\tan^{-1}(c\right.}{}$$

$$= \frac{i\left(a+b\tan^{-1}(c+dx)\right)^2}{d} + \frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{d} + \frac{2b\left(a+b\tan^{-1}(c\right.}{}$$

Mathematica [A] time = 0.09, size = 109, normalized size = 1.07

$$\frac{a\left(ac+adx+2b\log\left(\frac{1}{\sqrt{(c+dx)^2+1}}\right)\right)+2b\tan^{-1}(c+dx)\left(ac+adx+b\log\left(1+e^{2i\tan^{-1}(c+dx)}\right)\right)-ib^2\text{Li}_2\left(-e^{2i\tan^{-1}(}\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (b^2*(-I + c + d*x)*ArcTan[c + d*x]^2 + 2*b*ArcTan[c + d*x]*(a*c + a*d*x +
b*Log[1 + E^((2*I)*ArcTan[c + d*x])]) + a*(a*c + a*d*x + 2*b*Log[1/Sqrt[1 +
 (c + d*x)^2]]) - I*b^2*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])])/d
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2\arctan(dx+c)^2+2\,ab\arctan(dx+c)+a^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.33, size = 180, normalized size = 1.76

$$\arctan(dx+c)^2 x b^2 - \frac{i \arctan(dx+c)^2 b^2}{d} + \frac{\arctan(dx+c)^2 b^2 c}{d} + 2\arctan(dx+c) xab + \frac{2\ln\left(\frac{(1+i(dx+c))^2}{1+(dx+c)^2}+1\right) \mathrm{arc}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^2,x)
```

```
[Out] arctan(d*x+c)^2*x*b^2-I/d*arctan(d*x+c)^2*b^2+1/d*arctan(d*x+c)^2*b^2*c+2*a
rctan(d*x+c)*x*a*b+2/d*ln((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)*arctan(d*x+c)*b^
2-I/d*polylog(2,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))*b^2+2/d*arctan(d*x+c)*a*b*c
+a^2*x-1/d*a*b*ln(1+(d*x+c)^2)+a^2*c/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{16}\left(\frac{12\, c^2 \arctan(dx+c)^2 \arctan\left(\frac{d^2x+cd}{d}\right)}{d} - 4\left(\frac{3 \arctan(dx+c) \arctan\left(\frac{d^2x+cd}{d}\right)^2}{d} - \frac{\arctan\left(\frac{d^2x+cd}{d}\right)^3}{d}\right)c^2 + 4\, x \,\mathrm{ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/16*(12*c^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 4*(3*arctan(d*x
+ c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*c^2 + 4*x*a
rctan(d*x + c)^2 + 192*d^2*integrate(1/16*x^2*arctan(d*x + c)^2/(d^2*x^2 +
2*c*d*x + c^2 + 1), x) + 16*d^2*integrate(1/16*x^2*log(d^2*x^2 + 2*c*d*x +
c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 384*c*d*integrate(1/16*x*arc
tan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 64*d^2*integrate(1/16*x^
2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 32*c
*d*integrate(1/16*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x +
 c^2 + 1), x) + 64*c*d*integrate(1/16*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d
^2*x^2 + 2*c*d*x + c^2 + 1), x) + 16*c^2*integrate(1/16*log(d^2*x^2 + 2*c*d
*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - x*log(d^2*x^2 + 2*c*d*x
 + c^2 + 1)^2 + 12*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 12*arctan(
d*x + c)*arctan((d^2*x + c*d)/d)^2/d + 4*arctan((d^2*x + c*d)/d)^3/d - 128*
d*integrate(1/16*x*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 16*i
ntegrate(1/16*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 +
 1), x))*b^2 + a^2*x + (2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))
*a*b/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{atan}\,(c + d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))^2,x)
```

```
[Out] int((a + b*atan(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{atan}(c + dx))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*atan(c + d*x))**2, x)
```

3.34 $\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{e+fx}\,dx$

Optimal. Leaf size=261

$$-\frac{ib\left(a+b\tan^{-1}(c+dx)\right)\text{Li}_2\left(1-\frac{2d(e+fx)}{(de-cf+if)(1-i(c+dx))}\right)}{f}+\frac{\left(a+b\tan^{-1}(c+dx)\right)^2\log\left(\frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{f}+\frac{ib\text{Li}_2}{}$$

```
[Out] -(a+b*arctan(d*x+c))^2*ln(2/(1-I*(d*x+c)))/f+(a+b*arctan(d*x+c))^2*ln(2*d*(
f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f+I*b*(a+b*arctan(d*x+c))*polylog(2,1-2
/(1-I*(d*x+c)))/f-I*b*(a+b*arctan(d*x+c))*polylog(2,1-2*d*(f*x+e)/(d*e+I*f-
c*f)/(1-I*(d*x+c)))/f-1/2*b^2*polylog(3,1-2/(1-I*(d*x+c)))/f+1/2*b^2*polylo
g(3,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f
```

Rubi [A] time = 0.16, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5047, 4858}

$$-\frac{ib\left(a+b\tan^{-1}(c+dx)\right)\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{f}+\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)\left(a+b\tan^{-1}(c+\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^2/(e + f*x),x]
```

```
[Out] -(((a + b*ArcTan[c + d*x])^2*Log[2/(1 - I*(c + d*x))])/f) + ((a + b*ArcTan[
c + d*x])^2*Log[(2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f +
 (I*b*(a + b*ArcTan[c + d*x])*PolyLog[2, 1 - 2/(1 - I*(c + d*x))])/f - (I*b
*(a + b*ArcTan[c + d*x])*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*
(1 - I*(c + d*x)))])/f - (b^2*PolyLog[3, 1 - 2/(1 - I*(c + d*x))])/(2*f) +
(b^2*PolyLog[3, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])
/(2*f)
```

Rule 4858

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
-Simp[((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e, x] + (Simp[((a + b*ArcT
an[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x] + Simp[(I*
b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/e, x] - Simp[(I*b*(a +
 b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/
e, x] - Simp[(b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e), x] + Simp[(b^2*Poly
Log[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e), x]) /; FreeQ[
{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{e+fx}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{\frac{de-cf}{d}+\frac{fx}{d}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^2 \log\left(\frac{2}{1-i(c+dx)}\right)}{f} + \frac{\left(a+b\tan^{-1}(c+dx)\right)^2 \log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(}\right.}{f}$$

Mathematica [F] time = 6.51, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^2/(e + f*x),x]
```

```
[Out] Integrate[(a + b*ArcTan[c + d*x])^2/(e + f*x), x]
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\arctan\left(dx+c\right)^2+2\,ab\arctan\left(dx+c\right)+a^2}{fx+e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2)/(f*x + e), x
)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(f*x+e),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 2.31, size = 2149, normalized size = 8.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^2/(f*x+e),x)
```

```
[Out] b^2*ln(f*(d*x+c)-c*f+d*e)/f*arctan(d*x+c)^2-b^2/f*arctan(d*x+c)^2*ln(I*f*(1
+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x
+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)+b^2/(I*f+c*f-d*e)*arctan(d*x+c)*polylog(2
,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+1/2*b^2*c/(I*f+
c*f-d*e)*polylog(3,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f
))+1/2*I*b^2/(I*f+c*f-d*e)*polylog(3,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+
c)^2)/(d*e+I*f-c*f))-1/2*b^2/f*polylog(3,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))+a^
2*ln(f*(d*x+c)-c*f+d*e)/f+I*a*b*ln(f*(d*x+c)-c*f+d*e)/f*ln((I*f-f*(d*x+c))/
(d*e+I*f-c*f))+I*b^2/f*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(
d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/((1+
```

```
I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*Pi*arctan(d*x+c)^2+2*I*d*b^2/f*e*arctan(d*
x+c)*polylog(2,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))/(
2*I*f+2*c*f-2*d*e)+1/2*I*b^2/f*csgn(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*cs
gn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d
*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e))*csgn(I*(I*f*(1+I*(d*x+c))^2/
(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x
+c)^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*Pi*arctan(d*x+c)^2+b
^2*c/(I*f+c*f-d*e)*arctan(d*x+c)^2*ln(1-(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d
*x+c)^2)/(d*e+I*f-c*f))+2*a*b*ln(f*(d*x+c)-c*f+d*e)/f*arctan(d*x+c)-1/2*I*b
^2/f*csgn(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csgn(I*(I*f*(1+I*(d*x+c))^2/
(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x
+c)^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*Pi*arctan(d*x+c)^2
-1/2*I*b^2/f*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/
(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e))*csgn(I*(I*f*(
1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*
x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*Pi*
arctan(d*x+c)^2+I*a*b/f*dilog((I*f-f*(d*x+c))/(d*e+I*f-c*f))+I*b^2/(I*f+c*f
-d*e)*arctan(d*x+c)^2*ln(1-(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e
+I*f-c*f))+I*b^2/f*arctan(d*x+c)*polylog(2,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))-
I*a*b/f*dilog((I*f+f*(d*x+c))/(I*f+c*f-d*e))-I*b^2/f*Pi*arctan(d*x+c)^2-d*b
^2/f*e/(I*f+c*f-d*e)*arctan(d*x+c)^2*ln(1-(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+
(d*x+c)^2)/(d*e+I*f-c*f))-1/2*d*b^2/f*e/(I*f+c*f-d*e)*polylog(3,(I*f+c*f-d*
e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-I*a*b*ln(f*(d*x+c)-c*f+d*e)
/f*ln((I*f+f*(d*x+c))/(I*f+c*f-d*e))-I*b^2*c/(I*f+c*f-d*e)*arctan(d*x+c)*po
lylog(2,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-1/2*I*b^
2/f*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c
)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*
x+c)^2)+1))^3*Pi*arctan(d*x+c)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2\log\left(fx+e\right)}{f}+\int\frac{12\,b^2\arctan\left(dx+c\right)^2+b^2\log\left(d^2x^2+2\,cdx+c^2+1\right)^2+32\,ab\arctan\left(dx+c\right)}{16\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(f*x+e),x, algorithm="maxima")
```

```
[Out] a^2*log(f*x + e)/f + integrate(1/16*(12*b^2*arctan(d*x + c)^2 + b^2*log(d^2
*x^2 + 2*c*d*x + c^2 + 1)^2 + 32*a*b*arctan(d*x + c))/(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(a+b\,\mathrm{atan}\left(c+d\,x\right)\right)^2}{e+f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))^2/(e + f*x),x)
```

```
[Out] int((a + b*atan(c + d*x))^2/(e + f*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\,\mathrm{atan}\left(c+dx\right)\right)^2}{e+fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**2/(f*x+e),x)
```

```
[Out] Integral((a + b*atan(c + d*x))**2/(e + f*x), x)
```

3.35 $\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{(e+fx)^2}\,dx$

Optimal. Leaf size=568

$$\frac{2abd\log(e+fx)}{(de-cf)^2+f^2}-\frac{abd\log\left((c+dx)^2+1\right)}{(de-cf)^2+f^2}+\frac{2abd(de-cf)\tan^{-1}(c+dx)}{f\left((de-cf)^2+f^2\right)}-\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{f(e+fx)}+\frac{ib^2d\mathrm{Li}_2\left(1-\right.}{\left(c^2+1\right)f^2-2}$$

```
[Out] 2*a*b*d*(-c*f+d*e)*arctan(d*x+c)/f/(f^2+(-c*f+d*e)^2)+I*b^2*d*arctan(d*x+c)
^2/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+b^2*d*(-c*f+d*e)*arctan(d*x+c)^2/f/(d^2*
e^2-2*c*d*e*f+(c^2+1)*f^2)-(a+b*arctan(d*x+c))^2/f/(f*x+e)+2*a*b*d*ln(f*x+e
)/(f^2+(-c*f+d*e)^2)-2*b^2*d*arctan(d*x+c)*ln(2/(1-I*(d*x+c)))/(d^2*e^2-2*c
*d*e*f+(c^2+1)*f^2)+2*b^2*d*arctan(d*x+c)*ln(2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I
*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+2*b^2*d*arctan(d*x+c)*ln(2/(1+I*
(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-a*b*d*ln(1+(d*x+c)^2)/(f^2+(-c*f+
d*e)^2)+I*b^2*d*polylog(2,1-2/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2
)-I*b^2*d*polylog(2,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/(d^2*e^2-2*c
*d*e*f+(c^2+1)*f^2)+I*b^2*d*polylog(2,1-2/(1+I*(d*x+c)))/(d^2*e^2-2*c*d*e*f
+(c^2+1)*f^2)
```

Rubi [A] time = 1.35, antiderivative size = 568, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 25, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {5045, 1982, 705, 31, 634, 618, 204, 628, 6741, 5057, 706, 635, 203, 260, 6688, 12, 6725, 4856, 2402, 2315, 2447, 4984, 4884, 4920, 4854}

$$\frac{ib^2d\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{\left(c^2+1\right)f^2-2cdef+d^2e^2}-\frac{ib^2d\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(1-i(c+dx))(de+(-c+i)f)}\right)}{\left(c^2+1\right)f^2-2cdef+d^2e^2}+\frac{ib^2d\mathrm{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{\left(c^2+1\right)f^2-2cdef+d^2e^2}+\frac{2abd\,\mathrm{l}}{(de-}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^2/(e + f*x)^2,x]
```

```
[Out] (2*a*b*d*(d*e - c*f)*ArcTan[c + d*x])/(f*(f^2 + (d*e - c*f)^2)) + (I*b^2*d*
ArcTan[c + d*x]^2)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (b^2*d*(d*e - c*
f)*ArcTan[c + d*x]^2)/(f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)) - (a + b*Ar
cTan[c + d*x])^2/(f*(e + f*x)) + (2*a*b*d*Log[e + f*x])/(f^2 + (d*e - c*f)^
2) - (2*b^2*d*ArcTan[c + d*x]*Log[2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*
f + (1 + c^2)*f^2) + (2*b^2*d*ArcTan[c + d*x]*Log[(2*d*(e + f*x))/((d*e + (
I - c)*f)*(1 - I*(c + d*x)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (2*b
^2*d*ArcTan[c + d*x]*Log[2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 +
c^2)*f^2) - (a*b*d*Log[1 + (c + d*x)^2])/(f^2 + (d*e - c*f)^2) + (I*b^2*d*P
olyLog[2, 1 - 2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) -
 (I*b^2*d*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + (I - c)*f)*(1 - I*(c + d*x
)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (I*b^2*d*PolyLog[2, 1 - 2/(1
+ I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 1982

```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
```

```
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4984

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 5045

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[((e + f*x)^(m + 1)*(a + b*ArcTan[c + d*x])^p)/(f*(m +
 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcTan[c
 + d*x])^(p - 1))/(1 + (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 5057

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst
[Int[((d*e - c*f)/d + (f*x)/d)^m*(C/d^2 + (C*x^2)/d^2)^q*(a + b*ArcTan[x])^
p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &&
 EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{(e+fx)^2}\,dx = -\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{(2bd)\int \frac{a+b\tan^{-1}(c+dx)}{(e+fx)\left(1+(c+dx)^2\right)}\,dx}{f}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{(2bd)\int \frac{a+b\tan^{-1}(c+dx)}{(e+fx)\left(1+c^2+2cdx+d^2x^2\right)}\,dx}{f}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{(2b)\,\mathrm{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)(1+x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{(2b)\,\mathrm{Subst}\left(\int \frac{d\left(a+b\tan^{-1}(x)\right)}{(de-cf+fx)(1+x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{(2bd)\,\mathrm{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{(de-cf+fx)(1+x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{(2bd)\,\mathrm{Subst}\left(\int \left(\frac{a}{(de-cf+fx)(1+x^2)} + \frac{b\tan^{-1}(x)}{(de-cf+fx)(1+x^2)}\right)dx,\right.}{f}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{(2abd)\,\mathrm{Subst}\left(\int \frac{1}{(de-cf+fx)(1+x^2)}\,dx, x, c+dx\right)}{f} + \frac{\left(2b^2d\right)}{}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{\left(2b^2d\right)\mathrm{Subst}\left(\int \left(\frac{f^2\tan^{-1}(x)}{\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)(de-cf+fx)} + \frac{(d}{(d^2e^2-}\right.\right.}{f}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{2abd\log(e+fx)}{f^2+(de-cf)^2} + \frac{\left(2b^2d\right)\mathrm{Subst}\left(\int \frac{(de-cf-fx)\tan^{-1}(x)}{1+x^2}\,d\right.}{f\left(d^2e^2-2cdef+\left(1+c^2\right)\right.}$$

$$= \frac{2abd(de-cf)\tan^{-1}(c+dx)}{f\left(f^2+(de-cf)^2\right)} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{2abd\log(e+fx)}{f^2+(de-cf)^2} - \frac{2b^2d}{d}$$

$$= \frac{2abd(de-cf)\tan^{-1}(c+dx)}{f\left(f^2+(de-cf)^2\right)} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{2abd\log(e+fx)}{f^2+(de-cf)^2} - \frac{2b^2d}{d}$$

$$= \frac{2abd(de-cf)\tan^{-1}(c+dx)}{f\left(f^2+(de-cf)^2\right)} + \frac{ib^2d\tan^{-1}(c+dx)^2}{d^2e^2-2cdef+\left(1+c^2\right)f^2} + \frac{b^2d(de-cf)\tan^{-1}(}{f\left(d^2e^2-2cdef+(1\right.}$$

$$= \frac{2abd(de-cf)\tan^{-1}(c+dx)}{f\left(f^2+(de-cf)^2\right)} + \frac{ib^2d\tan^{-1}(c+dx)^2}{d^2e^2-2cdef+\left(1+c^2\right)f^2} + \frac{b^2d(de-cf)\tan^{-1}(}{f\left(d^2e^2-2cdef+(1\right.}$$

$$= \frac{2abd(de-cf)\tan^{-1}(c+dx)}{f\left(f^2+(de-cf)^2\right)} + \frac{ib^2d\tan^{-1}(c+dx)^2}{d^2e^2-2cdef+\left(1+c^2\right)f^2} + \frac{b^2d(de-cf)\tan^{-1}(}{f\left(d^2e^2-2cdef+(1\right.}$$

$$= \frac{2abd(de-cf)\tan^{-1}(c+dx)}{f\left(f^2+(de-cf)^2\right)} + \frac{ib^2d\tan^{-1}(c+dx)^2}{d^2e^2-2cdef+\left(1+c^2\right)f^2} + \frac{b^2d(de-cf)\tan^{-1}(}{f\left(d^2e^2-2cdef+(1\right.}$$

Mathematica [A] time = 7.49, size = 419, normalized size = 0.74

$$-\frac{a^2}{f}+\frac{2ab\left(d(e+fx)\log\left(\frac{d(e+fx)}{\sqrt{(c+dx)^2+1}}\right)-\tan^{-1}(c+dx)\left(c^2f-cde+cdfx-d^2ex+f\right)\right)}{\left(c^2+1\right)f^2-2cdef+d^2e^2}+\frac{b^2d(e+fx)\left(-\frac{(de-cf)\left(i\text{Li}_2\left(\exp\left(2i\left(\tan^{-1}\left(\frac{de-cf}{f}\right)+\tan^{-1}(c+dx)\right)\right)\right)-2(\text{ta}\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^2/(e + f*x)^2,x]
```

```
[Out] (-(a^2/f) + (2*a*b*(-((-(c*d*e) + f + c^2*f - d^2*e*x + c*d*f*x)*ArcTan[c +
 d*x]) + d*(e + f*x)*Log[(d*(e + f*x))/Sqrt[1 + (c + d*x)^2]]))/(d^2*e^2 -
2*c*d*e*f + (1 + c^2)*f^2) + (b^2*d*(e + f*x)*(-((E^(I*ArcTan[(d*e - c*f)/f
])*ArcTan[c + d*x]^2)/(f*Sqrt[1 + (d*e - c*f)^2/f^2])) + ((c + d*x)*ArcTan[
c + d*x]^2)/(d*(e + f*x)) - ((d*e - c*f)*((-I)*(Pi - 2*ArcTan[(d*e - c*f)/f
])*ArcTan[c + d*x] - Pi*Log[1 + E^((-2*I)*ArcTan[c + d*x])] - 2*(ArcTan[(d*
e - c*f)/f] + ArcTan[c + d*x])*Log[1 - E^((2*I)*(ArcTan[(d*e - c*f)/f] + Ar
cTan[c + d*x]))] + Pi*Log[1/Sqrt[1 + (c + d*x)^2]] + 2*ArcTan[(d*e - c*f)/f
]*Log[Sin[ArcTan[(d*e - c*f)/f] + ArcTan[c + d*x]]] + I*PolyLog[2, E^((2*I)
*(ArcTan[(d*e - c*f)/f] + ArcTan[c + d*x]))]))/(f^2*(1 + (d*e - c*f)^2/f^2)
)))/(d*e - c*f))/(e + f*x)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\arctan\left(dx+c\right)^2+2\,ab\arctan\left(dx+c\right)+a^2}{f^2x^2+2\,efx+e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2)/(f^2*x^2 + 2
*e*f*x + e^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.15, size = 1087, normalized size = 1.91

$$-\frac{d\,a^2}{\left(dfx+de\right)f}-\frac{d\,b^2\arctan\left(dx+c\right)^2}{\left(dfx+de\right)f}+\frac{2d\,b^2\arctan\left(dx+c\right)\ln\left(f\left(dx+c\right)-cf+de\right)}{c^2f^2-2cdef+d^2e^2+f^2}-\frac{d\,b^2\arctan\left(dx+c\right)\ln\left(\right.}{c^2f^2-2cdef+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^2/(f*x+e)^2,x)
```

```
[Out] -d*a^2/(d*f*x+d*e)/f-d*b^2/(d*f*x+d*e)/f*arctan(d*x+c)^2+2*d*b^2*arctan(d*x
+c)/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(f*(d*x+c)-c*f+d*e)-d*b^2*arctan(d*x+
c)/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(1+(d*x+c)^2)-d*b^2/(c^2*f^2-2*c*d*e*f
+d^2*e^2+f^2)*arctan(d*x+c)^2*c+d^2*b^2/f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*a
rctan(d*x+c)^2*e+1/4*I*d*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(d*x+c-I)^2-
```

```
1/2*I*d*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*dilog(1/2*I*(d*x+c-I))-I*d*b^2/
(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*dilog((I*f+f*(d*x+c))/(I*f+c*f-d*e))+1/2*I*
d*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*dilog(-1/2*I*(I+d*x+c))+I*d*b^2/(c^2*
f^2-2*c*d*e*f+d^2*e^2+f^2)*dilog((I*f-f*(d*x+c))/(d*e+I*f-c*f))-1/2*I*d*b^2
/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(I+d*x+c)*ln(1/2*I*(d*x+c-I))+I*d*b^2/(c
^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(f*(d*x+c)-c*f+d*e)*ln((I*f-f*(d*x+c))/(d*e
+I*f-c*f))-1/2*I*d*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(d*x+c-I)*ln(1+(d*
x+c)^2)+1/2*I*d*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(d*x+c-I)*ln(-1/2*I*(
I+d*x+c))-I*d*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(f*(d*x+c)-c*f+d*e)*ln(
(I*f+f*(d*x+c))/(I*f+c*f-d*e))-1/4*I*d*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*
ln(I+d*x+c)^2+1/2*I*d*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(I+d*x+c)*ln(1+
(d*x+c)^2)-2*d*a*b/(d*f*x+d*e)/f*arctan(d*x+c)+2*d*a*b/(c^2*f^2-2*c*d*e*f+d
^2*e^2+f^2)*ln(f*(d*x+c)-c*f+d*e)-d*a*b/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(
1+(d*x+c)^2)-2*d*a*b/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)*c+2*d^2*
a*b/f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)*e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(d\left(\frac{2\left(d^2e-cdf\right)\arctan\left(\frac{d^2x+cd}{d}\right)}{\left(d^2e^2f-2\,cdef^2+\left(c^2+1\right)f^3\right)d}-\frac{\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^2e^2-2\,cdef+\left(c^2+1\right)f^2}+\frac{2\,\log\left(fx+e\right)}{d^2e^2-2\,cdef+\left(c^2+1\right)f^2}\right)-\frac{2\,\arctan\left(dx\right.}{f^2x+ef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] (d*(2*(d^2*e - c*d*f)*arctan((d^2*x + c*d)/d)/((d^2*e^2*f - 2*c*d*e*f^2 + (
c^2 + 1)*f^3)*d) - log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*e^2 - 2*c*d*e*f +
(c^2 + 1)*f^2) + 2*log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 + 1)*f^2)) - 2*
arctan(d*x + c)/(f^2*x + e*f))*a*b - 1/16*(4*arctan(d*x + c)^2 - 16*(f^2*x
+ e*f)*integrate(1/16*(12*(d^2*f*x^2 + 2*c*d*f*x + (c^2 + 1)*f)*arctan(d*x
+ c)^2 + (d^2*f*x^2 + 2*c*d*f*x + (c^2 + 1)*f)*log(d^2*x^2 + 2*c*d*x + c^2
+ 1)^2 + 8*(d*f*x + d*e)*arctan(d*x + c) - 4*(d^2*f*x^2 + c*d*e + (d^2*e +
c*d*f)*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^2*f^3*x^4 + (c^2 + 1)*e^2*f
+ 2*(d^2*e*f^2 + c*d*f^3)*x^3 + (d^2*e^2*f + 4*c*d*e*f^2 + (c^2 + 1)*f^3)*x
^2 + 2*(c*d*e^2*f + (c^2 + 1)*e*f^2)*x), x) - log(d^2*x^2 + 2*c*d*x + c^2 +
 1)^2)*b^2/(f^2*x + e*f) - a^2/(f^2*x + e*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atan}\left(c+d\,x\right)\right)}^2}{{\left(e+f\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))^2/(e + f*x)^2,x)
```

```
[Out] int((a + b*atan(c + d*x))^2/(e + f*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**2/(f*x+e)**2,x)
```

```
[Out] Timed out
```

3.36 $\int (e+fx)^2 \left(a+b\tan^{-1}(c+dx)\right)^3\,dx$

Optimal. Leaf size=564

$$\frac{ib^2\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)\text{Li}_2\left(1-\frac{2}{i(c+dx)+1}\right)\left(a+b\tan^{-1}(c+dx)\right)}{d^3}-\frac{6b^2f(de-cf)\log\left(\frac{2}{1+i(c+dx)}\right)\left(a\right.}{d^3}$$

```
[Out] a*b^2*f^2*x/d^2+b^3*f^2*(d*x+c)*arctan(d*x+c)/d^3-1/2*b*f^2*(a+b*arctan(d*x
+c))^2/d^3-3*I*b*f*(-c*f+d*e)*(a+b*arctan(d*x+c))^2/d^3-3*b*f*(-c*f+d*e)*(d
*x+c)*(a+b*arctan(d*x+c))^2/d^3-1/2*b*f^2*(d*x+c)^2*(a+b*arctan(d*x+c))^2/d
^3+1/3*I*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*(a+b*arctan(d*x+c))^3/d^3-1/3
*(-c*f+d*e)*(d^2*e^2-2*c*d*e*f-(-c^2+3)*f^2)*(a+b*arctan(d*x+c))^3/d^3/f+1/
3*(f*x+e)^3*(a+b*arctan(d*x+c))^3/f-6*b^2*f*(-c*f+d*e)*(a+b*arctan(d*x+c))*
ln(2/(1+I*(d*x+c)))/d^3+b*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*(a+b*arctan(
d*x+c))^2*ln(2/(1+I*(d*x+c)))/d^3-1/2*b^3*f^2*ln(1+(d*x+c)^2)/d^3-3*I*b^3*f
*(-c*f+d*e)*polylog(2,1-2/(1+I*(d*x+c)))/d^3+I*b^2*(3*d^2*e^2-6*c*d*e*f-(-3
*c^2+1)*f^2)*(a+b*arctan(d*x+c))*polylog(2,1-2/(1+I*(d*x+c)))/d^3+1/2*b^3*(
3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*polylog(3,1-2/(1+I*(d*x+c)))/d^3
```

Rubi [A] time = 0.94, antiderivative size = 564, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5047, 4864, 4846, 4920, 4854, 2402, 2315, 4852, 4916, 260, 4884, 4984, 4994, 6610}

$$\frac{ib^2\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{d^3}+b^3\left(-\left(1-3c^2\right)f^2-6cd\right.$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] (a*b^2*f^2*x)/d^2 + (b^3*f^2*(c + d*x)*ArcTan[c + d*x])/d^3 - (b*f^2*(a + b
*ArcTan[c + d*x])^2)/(2*d^3) - ((3*I)*b*f*(d*e - c*f)*(a + b*ArcTan[c + d*x
])^2)/d^3 - (3*b*f*(d*e - c*f)*(c + d*x)*(a + b*ArcTan[c + d*x])^2)/d^3 - (
b*f^2*(c + d*x)^2*(a + b*ArcTan[c + d*x])^2)/(2*d^3) + ((I/3)*(3*d^2*e^2 -
6*c*d*e*f - (1 - 3*c^2)*f^2)*(a + b*ArcTan[c + d*x])^3)/d^3 - ((d*e - c*f)*
(d^2*e^2 - 2*c*d*e*f - (3 - c^2)*f^2)*(a + b*ArcTan[c + d*x])^3)/(3*d^3*f)
+ ((e + f*x)^3*(a + b*ArcTan[c + d*x])^3)/(3*f) - (6*b^2*f*(d*e - c*f)*(a +
 b*ArcTan[c + d*x])*Log[2/(1 + I*(c + d*x))])/d^3 + (b*(3*d^2*e^2 - 6*c*d*e
*f - (1 - 3*c^2)*f^2)*(a + b*ArcTan[c + d*x])^2*Log[2/(1 + I*(c + d*x))])/d
^3 - (b^3*f^2*Log[1 + (c + d*x)^2])/(2*d^3) - ((3*I)*b^3*f*(d*e - c*f)*Poly
Log[2, 1 - 2/(1 + I*(c + d*x))])/d^3 + (I*b^2*(3*d^2*e^2 - 6*c*d*e*f - (1 -
 3*c^2)*f^2)*(a + b*ArcTan[c + d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d
^3 + (b^3*(3*d^2*e^2 - 6*c*d*e*f - (1 - 3*c^2)*f^2)*PolyLog[3, 1 - 2/(1 + I
*(c + d*x))])/(2*d^3)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4984

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
```

```
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (e+fx)^2 \left(a+b\tan^{-1}(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2 \left(a+b\tan^{-1}(x)\right)^3 dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^3 \left(a+b\tan^{-1}(c+dx)\right)^3}{3f} - \frac{b\,\text{Subst}\left(\int \left(\frac{3f^2(de-cf)\left(a+b\tan^{-1}(x)\right)^2}{d^3}\right.\right.}{}$$

$$= \frac{(e+fx)^3 \left(a+b\tan^{-1}(c+dx)\right)^3}{3f} - \frac{b\,\text{Subst}\left(\int \frac{\left((de-cf)\left(d^2e^2-2cdef-3f^2+c^2\right.\right.}{}\right.}{}$$

$$= -\frac{3bf(de-cf)(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{d^3} - \frac{bf^2(c+dx)^2\left(a+b\,\text{ta}\right.}{2d^3}$$

$$= -\frac{3ibf(de-cf)\left(a+b\tan^{-1}(c+dx)\right)^2}{d^3} - \frac{3bf(de-cf)(c+dx)\left(a+b\,\text{t}\right.}{d^3}$$

$$= \frac{ab^2f^2x}{d^2} - \frac{bf^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^3} - \frac{3ibf(de-cf)\left(a+b\tan^{-1}(c+\right.}{d^3}$$

$$= \frac{ab^2f^2x}{d^2} + \frac{b^3f^2(c+dx)\tan^{-1}(c+dx)}{d^3} - \frac{bf^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^3} -$$

$$= \frac{ab^2f^2x}{d^2} + \frac{b^3f^2(c+dx)\tan^{-1}(c+dx)}{d^3} - \frac{bf^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^3} -$$

$$= \frac{ab^2f^2x}{d^2} + \frac{b^3f^2(c+dx)\tan^{-1}(c+dx)}{d^3} - \frac{bf^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^3} -$$

Mathematica [B] time = 9.98, size = 1844, normalized size = 3.27

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)^2*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] (a^2*(a*d^2*e^2 - 3*b*d*e*f + 2*b*c*f^2)*x)/d^2 - (a^2*f*(-2*a*d*e + b*f)*x
^2)/(2*d) + (a^3*f^2*x^3)/3 + ((3*a^2*b*c*d^2*e^2 + 3*a^2*b*d*e*f - 3*a^2*b
*c^2*d*e*f - 3*a^2*b*c*f^2 + a^2*b*c^3*f^2)*ArcTan[c + d*x])/d^3 + a^2*b*x*
(3*e^2 + 3*e*f*x + f^2*x^2)*ArcTan[c + d*x] + ((-3*a^2*b*d^2*e^2 + 6*a^2*b*
c*d*e*f + a^2*b*f^2 - 3*a^2*b*c^2*f^2)*Log[1 + c^2 + 2*c*d*x + d^2*x^2])/(2
*d^3) + (3*a*b^2*e^2*((-I)*ArcTan[c + d*x]^2 + (c + d*x)*ArcTan[c + d*x]^2
+ 2*ArcTan[c + d*x]*Log[1 + E^((2*I)*ArcTan[c + d*x])] - I*PolyLog[2, -E^((
2*I)*ArcTan[c + d*x])]))/d + 6*a*b^2*e*f*(-(((c + d*x)*ArcTan[c + d*x])/d^2
) + (I*c*ArcTan[c + d*x]^2)/d^2 - (c*(c + d*x)*ArcTan[c + d*x]^2)/d^2 + ((1
 + (c + d*x)^2)*ArcTan[c + d*x]^2)/(2*d^2) - (2*c*ArcTan[c + d*x]*Log[1 + E
^((2*I)*ArcTan[c + d*x])])/d^2 - Log[1/Sqrt[1 + (c + d*x)^2]]/d^2 + (I*c*Po
lyLog[2, -E^((2*I)*ArcTan[c + d*x])])/d^2) + (b^3*e^2*((-I)*ArcTan[c + d*x]
^3 + (c + d*x)*ArcTan[c + d*x]^3 + 3*ArcTan[c + d*x]^2*Log[1 + E^((2*I)*Arc
Tan[c + d*x])] - (3*I)*ArcTan[c + d*x]*PolyLog[2, -E^((2*I)*ArcTan[c + d*x]
)] + (3*PolyLog[3, -E^((2*I)*ArcTan[c + d*x])])/2))/d + (b^3*e*f*(ArcTan[c
+ d*x]*((3*I)*ArcTan[c + d*x] + (2*I)*c*ArcTan[c + d*x]^2 + (1 + (c + d*x)^
2)*ArcTan[c + d*x]^2 - (c + d*x)*ArcTan[c + d*x]*(3 + 2*c*ArcTan[c + d*x])
- 6*Log[1 + E^((2*I)*ArcTan[c + d*x])] - 6*c*ArcTan[c + d*x]*Log[1 + E^((2*
I)*ArcTan[c + d*x])]) + (3*I)*(1 + 2*c*ArcTan[c + d*x])*PolyLog[2, -E^((2*I
)*ArcTan[c + d*x])] - 3*c*PolyLog[3, -E^((2*I)*ArcTan[c + d*x])]))/d^2 + (a
*b^2*f^2*(1 + (c + d*x)^2)^(3/2)*((c + d*x)/Sqrt[1 + (c + d*x)^2] + (6*c*(c
 + d*x)*ArcTan[c + d*x])/Sqrt[1 + (c + d*x)^2] + (3*(c + d*x)*ArcTan[c + d*
x]^2)/Sqrt[1 + (c + d*x)^2] + (3*c^2*(c + d*x)*ArcTan[c + d*x]^2)/Sqrt[1 +
(c + d*x)^2] + I*ArcTan[c + d*x]^2*Cos[3*ArcTan[c + d*x]] - (3*I)*c^2*ArcTa
n[c + d*x]^2*Cos[3*ArcTan[c + d*x]] - 2*ArcTan[c + d*x]*Cos[3*ArcTan[c + d*
x]]*Log[1 + E^((2*I)*ArcTan[c + d*x])] + 6*c^2*ArcTan[c + d*x]*Cos[3*ArcTan
[c + d*x]]*Log[1 + E^((2*I)*ArcTan[c + d*x])] + 6*c*Cos[3*ArcTan[c + d*x]]*
Log[1/Sqrt[1 + (c + d*x)^2]] + (ArcTan[c + d*x]*(-4 + (3*I - 12*c - (9*I)*c
^2)*ArcTan[c + d*x]) + 6*(-1 + 3*c^2)*ArcTan[c + d*x]*Log[1 + E^((2*I)*ArcT
an[c + d*x])] + 18*c*Log[1/Sqrt[1 + (c + d*x)^2]])/Sqrt[1 + (c + d*x)^2] -
((4*I)*(-1 + 3*c^2)*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])])/(1 + (c + d*x)^
2)^(3/2) + Sin[3*ArcTan[c + d*x]] + 6*c*ArcTan[c + d*x]*Sin[3*ArcTan[c + d*
x]] - ArcTan[c + d*x]^2*Sin[3*ArcTan[c + d*x]] + 3*c^2*ArcTan[c + d*x]^2*Si
n[3*ArcTan[c + d*x]]))/(4*d^3) + (b^3*f^2*((-I)*(3*c - ArcTan[c + d*x] + 3*
c^2*ArcTan[c + d*x])*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])] + ((1 + (c + d*
x)^2)^(3/2)*((3*(c + d*x)*ArcTan[c + d*x])/Sqrt[1 + (c + d*x)^2] + (9*c*(c
+ d*x)*ArcTan[c + d*x]^2)/Sqrt[1 + (c + d*x)^2] + (3*(c + d*x)*ArcTan[c + d
*x]^3)/Sqrt[1 + (c + d*x)^2] + (3*c^2*(c + d*x)*ArcTan[c + d*x]^3)/Sqrt[1 +
 (c + d*x)^2] - (9*I)*c*ArcTan[c + d*x]^2*Cos[3*ArcTan[c + d*x]] + I*ArcTan
[c + d*x]^3*Cos[3*ArcTan[c + d*x]] - (3*I)*c^2*ArcTan[c + d*x]^3*Cos[3*ArcT
an[c + d*x]] + 18*c*ArcTan[c + d*x]*Cos[3*ArcTan[c + d*x]]*Log[1 + E^((2*I)
*ArcTan[c + d*x])] - 3*ArcTan[c + d*x]^2*Cos[3*ArcTan[c + d*x]]*Log[1 + E^(
(2*I)*ArcTan[c + d*x])] + 9*c^2*ArcTan[c + d*x]^2*Cos[3*ArcTan[c + d*x]]*Lo
g[1 + E^((2*I)*ArcTan[c + d*x])] + 3*Cos[3*ArcTan[c + d*x]]*Log[1/Sqrt[1 +
(c + d*x)^2]] + (3*(ArcTan[c + d*x]^2*(-2 - (9*I)*c + I*ArcTan[c + d*x] - 4
*c*ArcTan[c + d*x] - (3*I)*c^2*ArcTan[c + d*x]) + 3*ArcTan[c + d*x]*(6*c -
ArcTan[c + d*x] + 3*c^2*ArcTan[c + d*x])*Log[1 + E^((2*I)*ArcTan[c + d*x])]
 + 3*Log[1/Sqrt[1 + (c + d*x)^2]]))/Sqrt[1 + (c + d*x)^2] + (6*(-1 + 3*c^2)
*PolyLog[3, -E^((2*I)*ArcTan[c + d*x])])/(1 + (c + d*x)^2)^(3/2) + 3*ArcTan
[c + d*x]*Sin[3*ArcTan[c + d*x]] + 9*c*ArcTan[c + d*x]^2*Sin[3*ArcTan[c + d
*x]] - ArcTan[c + d*x]^3*Sin[3*ArcTan[c + d*x]] + 3*c^2*ArcTan[c + d*x]^3*S
in[3*ArcTan[c + d*x]]))/12))/d^3
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(a^3f^2x^2+2\,a^3efx+a^3e^2+\left(b^3f^2x^2+2\,b^3efx+b^3e^2\right)\arctan\left(dx+c\right)^3+3\left(ab^2f^2x^2+2\,ab^2efx+ab\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*f^2*x^2 + 2*a^3*e*f*x + a^3*e^2 + (b^3*f^2*x^2 + 2*b^3*e*f*x +
 b^3*e^2)*arctan(d*x + c)^3 + 3*(a*b^2*f^2*x^2 + 2*a*b^2*e*f*x + a*b^2*e^2)
*arctan(d*x + c)^2 + 3*(a^2*b*f^2*x^2 + 2*a^2*b*e*f*x + a^2*b*e^2)*arctan(d
*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [C] time = 10.28, size = 6682, normalized size = 11.85

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*(a+b*arctan(d*x+c))^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 7/8*b^3*c^2*e^2*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 3*a*b^2*c^2*e
^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - (3*arctan(d*x + c)*arctan(
(d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2*c^2*e^2 - 7/32*(6
*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)*arctan((
d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^3*c^2*e^2 + 1/3*a^3*f^
2*x^3 + 7/8*b^3*e^2*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 28*b^3*d^
2*f^2*integrate(1/32*x^4*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x
) + 3*b^3*d^2*f^2*integrate(1/32*x^4*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x
+ c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 96*a*b^2*d^2*f^2*integrate
(1/32*x^4*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 56*b^3*d^2*
e*f*integrate(1/32*x^3*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)
+ 56*b^3*c*d*f^2*integrate(1/32*x^3*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x +
c^2 + 1), x) + 4*b^3*d^2*f^2*integrate(1/32*x^4*arctan(d*x + c)*log(d^2*x^2
 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*d^2*e*f*int
egrate(1/32*x^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2
 + 2*c*d*x + c^2 + 1), x) + 6*b^3*c*d*f^2*integrate(1/32*x^3*arctan(d*x + c
)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 19
2*a*b^2*d^2*e*f*integrate(1/32*x^3*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c
^2 + 1), x) + 192*a*b^2*c*d*f^2*integrate(1/32*x^3*arctan(d*x + c)^2/(d^2*x
^2 + 2*c*d*x + c^2 + 1), x) + 28*b^3*d^2*e^2*integrate(1/32*x^2*arctan(d*x
+ c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 112*b^3*c*d*e*f*integrate(1/32*x
^2*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 28*b^3*c^2*f^2*int
```

```
egrate(1/32*x^2*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^
3*d^2*e*f*integrate(1/32*x^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 +
1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 4*b^3*c*d*f^2*integrate(1/32*x^3*arc
tan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1)
, x) + 3*b^3*d^2*e^2*integrate(1/32*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d
*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*c*d*e*f*integrat
e(1/32*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*
c*d*x + c^2 + 1), x) + 3*b^3*c^2*f^2*integrate(1/32*x^2*arctan(d*x + c)*log
(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 96*a*b^
2*d^2*e^2*integrate(1/32*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1
), x) + 384*a*b^2*c*d*e*f*integrate(1/32*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2
*c*d*x + c^2 + 1), x) + 96*a*b^2*c^2*f^2*integrate(1/32*x^2*arctan(d*x + c)
^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 56*b^3*c*d*e^2*integrate(1/32*x*arct
an(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 56*b^3*c^2*e*f*integrate(
1/32*x*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*d^2*e^2
*integrate(1/32*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x
^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*c*d*e*f*integrate(1/32*x^2*arctan(d*x
+ c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6
*b^3*c*d*e^2*integrate(1/32*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 +
 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*c^2*e*f*integrate(1/32*x*ar
ctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 +
 1), x) + 192*a*b^2*c*d*e^2*integrate(1/32*x*arctan(d*x + c)^2/(d^2*x^2 + 2
*c*d*x + c^2 + 1), x) + 192*a*b^2*c^2*e*f*integrate(1/32*x*arctan(d*x + c)^
2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*c*d*e^2*integrate(1/32*x*arcta
n(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1),
x) + 3*b^3*c^2*e^2*integrate(1/32*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c
^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + a^3*e*f*x^2 + 3*a*b^2*e^2*arc
tan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 4*b^3*d*f^2*integrate(1/32*x^3*a
rctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + b^3*d*f^2*integrate(1/
32*x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)
 - 12*b^3*d*e*f*integrate(1/32*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c
^2 + 1), x) + 3*b^3*d*e*f*integrate(1/32*x^2*log(d^2*x^2 + 2*c*d*x + c^2 +
1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 12*b^3*d*e^2*integrate(1/32*x*arct
an(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*d*e^2*integrate(1/3
2*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) -
(3*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/
d)*a*b^2*e^2 - 7/32*(6*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*ar
ctan(d*x + c)*arctan((d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^
3*e^2 + 3*(x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/
d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a^2*b*e*f + 1/2*(2*x^3*ar
ctan(d*x + c) - d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)
/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*a^2*b*f^2 + a^
3*e^2*x + 28*b^3*f^2*integrate(1/32*x^2*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*
x + c^2 + 1), x) + 3*b^3*f^2*integrate(1/32*x^2*arctan(d*x + c)*log(d^2*x^2
 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 96*a*b^2*f^2*in
tegrate(1/32*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 56*b
^3*e*f*integrate(1/32*x*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)
 + 6*b^3*e*f*integrate(1/32*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 +
 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*a*b^2*e*f*integrate(1/32*x*ar
ctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*e^2*integrate(1/3
2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c
^2 + 1), x) + 3/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*a^2*
b*e^2/d + 1/24*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^2*x)*arctan(d*x + c)^
3 - 1/32*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^2*x)*arctan(d*x + c)*log(d^
2*x^2 + 2*c*d*x + c^2 + 1)^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(e+f\,x\right)^2\,{\left(a+b\,\mathrm{atan}\left(c+d\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2*(a + b*atan(c + d*x))^3,x)
[Out] int((e + f*x)^2*(a + b*atan(c + d*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{atan}(c + dx))^3 \left(e + fx\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(a+b*atan(d*x+c))**3,x)
[Out] Integral((a + b*atan(c + d*x))**3*(e + f*x)**2, x)
```

3.37 $\int (e+fx)\left(a+b\tan^{-1}(c+dx)\right)^3\,dx$

Optimal. Leaf size=337

$$\frac{3ib^2(de-cf)\text{Li}_2\left(1-\frac{2}{i(c+dx)+1}\right)\left(a+b\tan^{-1}(c+dx)\right)}{d^2}-\frac{3b^2f\log\left(\frac{2}{1+i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{d^2}+\frac{i(de-cf)\left(a+\right.}{}$$

```
[Out] -3/2*I*b*f*(a+b*arctan(d*x+c))^2/d^2-3/2*b*f*(d*x+c)*(a+b*arctan(d*x+c))^2/
d^2+I*(-c*f+d*e)*(a+b*arctan(d*x+c))^3/d^2-1/2*(-c*f+d*e+f)*(d*e-(1+c)*f)*(
a+b*arctan(d*x+c))^3/d^2/f+1/2*(f*x+e)^2*(a+b*arctan(d*x+c))^3/f-3*b^2*f*(a
+b*arctan(d*x+c))*ln(2/(1+I*(d*x+c)))/d^2+3*b*(-c*f+d*e)*(a+b*arctan(d*x+c)
)^2*ln(2/(1+I*(d*x+c)))/d^2-3/2*I*b^3*f*polylog(2,1-2/(1+I*(d*x+c)))/d^2+3*
I*b^2*(-c*f+d*e)*(a+b*arctan(d*x+c))*polylog(2,1-2/(1+I*(d*x+c)))/d^2+3/2*b
^3*(-c*f+d*e)*polylog(3,1-2/(1+I*(d*x+c)))/d^2
```

Rubi [A] time = 0.63, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {5047, 4864, 4846, 4920, 4854, 2402, 2315, 4984, 4884, 4994, 6610}

$$\frac{3ib^2(de-cf)\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{d^2}+\frac{3b^3(de-cf)\text{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2d^2}-\frac{3ib^3f\text{Poly}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] (((-3*I)/2)*b*f*(a + b*ArcTan[c + d*x])^2)/d^2 - (3*b*f*(c + d*x)*(a + b*Ar
cTan[c + d*x])^2)/(2*d^2) + (I*(d*e - c*f)*(a + b*ArcTan[c + d*x])^3)/d^2 -
 ((d*e + f - c*f)*(d*e - (1 + c)*f)*(a + b*ArcTan[c + d*x])^3)/(2*d^2*f) +
((e + f*x)^2*(a + b*ArcTan[c + d*x])^3)/(2*f) - (3*b^2*f*(a + b*ArcTan[c +
d*x])*Log[2/(1 + I*(c + d*x))])/d^2 + (3*b*(d*e - c*f)*(a + b*ArcTan[c + d*
x])^2*Log[2/(1 + I*(c + d*x))])/d^2 - (((3*I)/2)*b^3*f*PolyLog[2, 1 - 2/(1
+ I*(c + d*x))])/d^2 + ((3*I)*b^2*(d*e - c*f)*(a + b*ArcTan[c + d*x])*PolyL
og[2, 1 - 2/(1 + I*(c + d*x))])/d^2 + (3*b^3*(d*e - c*f)*PolyLog[3, 1 - 2/(
1 + I*(c + d*x))])/(2*d^2)
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
```

```
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4984

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (e+fx)\left(a+b\tan^{-1}(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int\left(\frac{de-cf}{d}+\frac{fx}{d}\right)\left(a+b\tan^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$

$$= \frac{(e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)^3}{2f} - \frac{(3b)\,\text{Subst}\left(\int\left(\frac{f^2\left(a+b\tan^{-1}(x)\right)^2}{d^2}+\frac{((de-f-\ldots}{\ldots}\right.\right.}{\ldots}$$

$$= \frac{(e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)^3}{2f} - \frac{(3b)\,\text{Subst}\left(\int\frac{((de-f-cf)(de+f-cf)+2f(de-c\ldots}{1+x^2}\right.}{2d^2f}$$

$$= -\frac{3bf(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^2} + \frac{(e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)^3}{2f} - \ldots$$

$$= -\frac{3ibf\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^2} - \frac{3bf(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^2} + \frac{(e+\ldots}{\ldots}$$

$$= -\frac{3ibf\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^2} - \frac{3bf(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^2} + \frac{i(de\ldots}{\ldots}$$

$$= -\frac{3ibf\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^2} - \frac{3bf(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^2} + \frac{i(de\ldots}{\ldots}$$

$$= -\frac{3ibf\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^2} - \frac{3bf(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^2} + \frac{i(de\ldots}{\ldots}$$

$$= -\frac{3ibf\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^2} - \frac{3bf(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^2} + \frac{i(de\ldots}{\ldots}$$

Mathematica [A] time = 0.82, size = 592, normalized size = 1.76

$$\frac{a^3f(c+dx)^2+a^2(c+dx)(-2acf+2ade-3bf)-3a^2b(de-cf)\log\left((c+dx)^2+1\right)-3a^2b(c+dx)\tan^{-1}(c+dx)(\ldots}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] (a^2*(2*a*d*e - 3*b*f - 2*a*c*f)*(c + d*x) + a^3*f*(c + d*x)^2 + 3*a^2*b*f*
ArcTan[c + d*x] - 3*a^2*b*(c + d*x)*(c*f - d*(2*e + f*x))*ArcTan[c + d*x] +
 6*a*b^2*f*(-((c + d*x)*ArcTan[c + d*x]) + ((1 + (c + d*x)^2)*ArcTan[c + d*
x]^2)/2 - Log[1/Sqrt[1 + (c + d*x)^2]]) - 3*a^2*b*(d*e - c*f)*Log[1 + (c +
d*x)^2] + 6*a*b^2*d*e*(ArcTan[c + d*x]*((-I + c + d*x)*ArcTan[c + d*x] + 2*
Log[1 + E^((2*I)*ArcTan[c + d*x])]) - I*PolyLog[2, -E^((2*I)*ArcTan[c + d*x
])]) - 6*a*b^2*c*f*(ArcTan[c + d*x]*((-I + c + d*x)*ArcTan[c + d*x] + 2*Log
[1 + E^((2*I)*ArcTan[c + d*x])]) - I*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])]
) + b^3*f*(ArcTan[c + d*x]*((3*I)*ArcTan[c + d*x] - 3*(c + d*x)*ArcTan[c +
d*x] + (1 + (c + d*x)^2)*ArcTan[c + d*x]^2 - 6*Log[1 + E^((2*I)*ArcTan[c +
d*x])]) + (3*I)*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])]) + 2*b^3*d*e*(ArcTan
[c + d*x]^2*((-I + c + d*x)*ArcTan[c + d*x] + 3*Log[1 + E^((2*I)*ArcTan[c +
 d*x])]) - (3*I)*ArcTan[c + d*x]*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])] + (
3*PolyLog[3, -E^((2*I)*ArcTan[c + d*x])])/2) - 2*b^3*c*f*(ArcTan[c + d*x]^2
```

```
*((-I + c + d*x)*ArcTan[c + d*x] + 3*Log[1 + E^((2*I)*ArcTan[c + d*x])]) -
(3*I)*ArcTan[c + d*x]*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])] + (3*PolyLog[3
, -E^((2*I)*ArcTan[c + d*x])])/2))/(2*d^2)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(a^3fx + a^3e + \left(b^3fx + b^3e\right)\arctan\left(dx + c\right)^3 + 3\left(ab^2fx + ab^2e\right)\arctan\left(dx + c\right)^2 + 3\left(a^2bfx + a^2be\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*f*x + a^3*e + (b^3*f*x + b^3*e)*arctan(d*x + c)^3 + 3*(a*b^2*f
*x + a*b^2*e)*arctan(d*x + c)^2 + 3*(a^2*b*f*x + a^2*b*e)*arctan(d*x + c),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [C] time = 2.49, size = 16362, normalized size = 48.55

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(a+b*arctan(d*x+c))^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 7/8*b^3*c^2*e*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 3*a*b^2*c^2*e*a
rctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - (3*arctan(d*x + c)*arctan((d^2
*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2*c^2*e - 7/32*(6*arcta
n(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)*arctan((d^2*x
+ c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^3*c^2*e + 7/8*b^3*e*arctan(d
*x + c)^3*arctan((d^2*x + c*d)/d)/d + 56*b^3*d^2*f*integrate(1/64*x^3*arcta
n(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*d^2*f*integrate(1/64
*x^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x
+ c^2 + 1), x) + 192*a*b^2*d^2*f*integrate(1/64*x^3*arctan(d*x + c)^2/(d^2*
x^2 + 2*c*d*x + c^2 + 1), x) + 56*b^3*d^2*e*integrate(1/64*x^2*arctan(d*x +
 c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 112*b^3*c*d*f*integrate(1/64*x^2*
arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*d^2*f*integrat
e(1/64*x^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*
d*x + c^2 + 1), x) + 6*b^3*d^2*e*integrate(1/64*x^2*arctan(d*x + c)*log(d^2
*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*c*d*
f*integrate(1/64*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^
2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*a*b^2*d^2*e*integrate(1/64*x^2*arctan(
d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 384*a*b^2*c*d*f*integrate(1/
64*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 112*b^3*c*d*e*
```

```
integrate(1/64*x*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 56*b
^3*c^2*f*integrate(1/64*x*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1),
x) + 24*b^3*d^2*e*integrate(1/64*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x
+ c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*c*d*f*integrate(1/64*
x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c
^2 + 1), x) + 12*b^3*c*d*e*integrate(1/64*x*arctan(d*x + c)*log(d^2*x^2 + 2
*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*c^2*f*integra
te(1/64*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c
*d*x + c^2 + 1), x) + 384*a*b^2*c*d*e*integrate(1/64*x*arctan(d*x + c)^2/(d
^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*a*b^2*c^2*f*integrate(1/64*x*arctan(d
*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 24*b^3*c*d*e*integrate(1/64*x
*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2
+ 1), x) + 6*b^3*c^2*e*integrate(1/64*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x
 + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 1/2*a^3*f*x^2 + 3*a*b^2*e
*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 12*b^3*d*f*integrate(1/64*x^
2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*d*f*integrate
(1/64*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1),
 x) - 24*b^3*d*e*integrate(1/64*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^
2 + 1), x) + 6*b^3*d*e*integrate(1/64*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/
(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - (3*arctan(d*x + c)*arctan((d^2*x + c*d)
/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2*e - 7/32*(6*arctan(d*x + c)^2*
arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^3/d
 + arctan((d^2*x + c*d)/d)^4/d)*b^3*e + 3/2*(x^2*arctan(d*x + c) - d*(x/d^2
 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 +
1)/d^3))*a^2*b*f + a^3*e*x + 56*b^3*f*integrate(1/64*x*arctan(d*x + c)^3/(d
^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*f*integrate(1/64*x*arctan(d*x + c)*
log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*
a*b^2*f*integrate(1/64*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x
) + 6*b^3*e*integrate(1/64*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)
^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3/2*(2*(d*x + c)*arctan(d*x + c) - l
og((d*x + c)^2 + 1))*a^2*b*e/d + 1/16*(b^3*f*x^2 + 2*b^3*e*x)*arctan(d*x +
c)^3 - 3/64*(b^3*f*x^2 + 2*b^3*e*x)*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x +
 c^2 + 1)^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(e+f\,x\right)\,{\left(a+b\,\mathrm{atan}\left(c+d\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)*(a + b*atan(c + d*x))^3,x)
```

```
[Out] int((e + f*x)*(a + b*atan(c + d*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{atan}{\left(c + d x \right)}\right)^{3} \left(e + f x\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*atan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*atan(c + d*x))**3*(e + f*x), x)
```

3.38 $\int \left(a+b\tan^{-1}(c+dx)\right)^3\,dx$

Optimal. Leaf size=143

$$\frac{3ib^2\mathrm{Li}_2\left(1-\frac{2}{i(c+dx)+1}\right)\left(a+b\tan^{-1}(c+dx)\right)}{d}+\frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^3}{d}+\frac{i\left(a+b\tan^{-1}(c+dx)\right)^3}{d}+\frac{3b\log}{}$$

```
[Out] I*(a+b*arctan(d*x+c))^3/d+(d*x+c)*(a+b*arctan(d*x+c))^3/d+3*b*(a+b*arctan(d
*x+c))^2*ln(2/(1+I*(d*x+c)))/d+3*I*b^2*(a+b*arctan(d*x+c))*polylog(2,1-2/(1
+I*(d*x+c)))/d+3/2*b^3*polylog(3,1-2/(1+I*(d*x+c)))/d
```

Rubi [A] time = 0.21, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5039, 4846, 4920, 4854, 4884, 4994, 6610}

$$\frac{3ib^2\mathrm{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{d}+\frac{3b^3\mathrm{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2d}+\frac{(c+dx)\left(a+b\tan^{-1}(c+\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] (I*(a + b*ArcTan[c + d*x])^3)/d + ((c + d*x)*(a + b*ArcTan[c + d*x])^3)/d +
 (3*b*(a + b*ArcTan[c + d*x])^2*Log[2/(1 + I*(c + d*x))])/d + ((3*I)*b^2*(a
 + b*ArcTan[c + d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d + (3*b^3*PolyL
og[3, 1 - 2/(1 + I*(c + d*x))])/(2*d)
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
```

```
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 5039

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \left(a+b\tan^{-1}(c+dx)\right)^3\,dx=\frac{\mathrm{Subst}\left(\int \left(a+b\tan^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$

$$=\frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^3}{d}-\frac{(3b)\,\mathrm{Subst}\left(\int \frac{x\left(a+b\tan^{-1}(x)\right)^2}{1+x^2}\,dx,x,c+dx\right)}{d}$$

$$=\frac{i\left(a+b\tan^{-1}(c+dx)\right)^3}{d}+\frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^3}{d}+\frac{(3b)\,\mathrm{Subst}\left(\int \frac{\left(a+b\tan\right.}{i-}\right.}{d}$$

$$=\frac{i\left(a+b\tan^{-1}(c+dx)\right)^3}{d}+\frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^3}{d}+\frac{3b\left(a+b\tan^{-1}(c+d\right.}{d}$$

$$=\frac{i\left(a+b\tan^{-1}(c+dx)\right)^3}{d}+\frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^3}{d}+\frac{3b\left(a+b\tan^{-1}(c+d\right.}{d}$$

$$=\frac{i\left(a+b\tan^{-1}(c+dx)\right)^3}{d}+\frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^3}{d}+\frac{3b\left(a+b\tan^{-1}(c+d\right.}{d}$$

Mathematica [A] time = 0.15, size = 266, normalized size = 1.86

$$2a^3dx-3a^2b\log\left(c^2+2cdx+d^2x^2+1\right)+6a^2bc\tan^{-1}(c+dx)+6a^2bdx\tan^{-1}(c+dx)-6ib^2\mathrm{Li}_2\left(-e^{2i\tan^{-1}(c+dx)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] (2*a^3*d*x + 6*a^2*b*c*ArcTan[c + d*x] + 6*a^2*b*d*x*ArcTan[c + d*x] - (6*I
)*a*b^2*ArcTan[c + d*x]^2 + 6*a*b^2*c*ArcTan[c + d*x]^2 + 6*a*b^2*d*x*ArcTa
n[c + d*x]^2 - (2*I)*b^3*ArcTan[c + d*x]^3 + 2*b^3*c*ArcTan[c + d*x]^3 + 2*
b^3*d*x*ArcTan[c + d*x]^3 + 12*a*b^2*ArcTan[c + d*x]*Log[1 + E^((2*I)*ArcTa
n[c + d*x])] + 6*b^3*ArcTan[c + d*x]^2*Log[1 + E^((2*I)*ArcTan[c + d*x])] -
 3*a^2*b*Log[1 + c^2 + 2*c*d*x + d^2*x^2] - (6*I)*b^2*(a + b*ArcTan[c + d*x
])*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])] + 3*b^3*PolyLog[3, -E^((2*I)*ArcT
an[c + d*x])])/(2*d)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(b^3\arctan\left(dx+c\right)^3+3\,ab^2\arctan\left(dx+c\right)^2+3\,a^2b\arctan\left(dx+c\right)+a^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arctan
(d*x + c) + a^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.42, size = 359, normalized size = 2.51

$$x\,a^3+\frac{a^3 c}{d}-\frac{i b^3 \arctan\left(dx+c\right)^3}{d}+\arctan\left(dx+c\right)^3 x\,b^3+\frac{\arctan\left(dx+c\right)^3 b^3 c}{d}+\frac{3 b^3 \arctan\left(dx+c\right)^2 \ln\left(\frac{(1+i(dx+c))}{1+(dx+c)^2}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^3,x)
```

```
[Out] x*a^3+1/d*a^3*c-3*I/d*b^3*arctan(d*x+c)*polylog(2,-(1+I*(d*x+c))^2/(1+(d*x+
c)^2))+arctan(d*x+c)^3*x*b^3+1/d*arctan(d*x+c)^3*b^3*c+3/d*b^3*arctan(d*x+c
)^2*ln((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)-I/d*b^3*arctan(d*x+c)^3+3/2/d*b^3*p
olylog(3,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))-3*I/d*arctan(d*x+c)^2*a*b^2+3*arct
an(d*x+c)^2*x*a*b^2+3/d*arctan(d*x+c)^2*a*b^2*c+6/d*ln((1+I*(d*x+c))^2/(1+(
d*x+c)^2)+1)*arctan(d*x+c)*a*b^2-3*I/d*polylog(2,-(1+I*(d*x+c))^2/(1+(d*x+c
)^2))*a*b^2+3*arctan(d*x+c)*x*a^2*b+3/d*arctan(d*x+c)*a^2*b*c-3/2/d*a^2*b*l
n(1+(d*x+c)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 7/8*b^3*c^2*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 1/8*b^3*x*arctan(
d*x + c)^3 + 3*a*b^2*c^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 3/32
*b^3*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 - (3*arctan(d*x +
 c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2*c^2 -
7/32*(6*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)*a
rctan((d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^3*c^2 + 7/8*b^3
*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 28*b^3*d^2*integrate(1/32*x^
2*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*d^2*integrate
(1/32*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c
*d*x + c^2 + 1), x) + 96*a*b^2*d^2*integrate(1/32*x^2*arctan(d*x + c)^2/(d^
2*x^2 + 2*c*d*x + c^2 + 1), x) + 56*b^3*c*d*integrate(1/32*x*arctan(d*x + c
)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*d^2*integrate(1/32*x^2*arcta
n(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1),
x) + 6*b^3*c*d*integrate(1/32*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2
 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*a*b^2*c*d*integrate(1/32*x*
arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*c*d*integrate(
1/32*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x
+ c^2 + 1), x) + 3*b^3*c^2*integrate(1/32*arctan(d*x + c)*log(d^2*x^2 + 2*c
*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*a*b^2*arctan(d*x +
```

```
c)^2*arctan((d^2*x + c*d)/d)/d - 12*b^3*d*integrate(1/32*x*arctan(d*x + c)^
2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*d*integrate(1/32*x*log(d^2*x^2
+ 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - (3*arctan(d*x +
c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2 - 7/32*
(6*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)*arctan
((d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^3 + a^3*x + 3*b^3*in
tegrate(1/32*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 +
2*c*d*x + c^2 + 1), x) + 3/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2
 + 1))*a^2*b/d
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{atan}\,(c + d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))^3,x)
```

```
[Out] int((a + b*atan(c + d*x))^3, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{atan}\,(c + dx))^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*atan(c + d*x))**3, x)
```

3.39 $\int \frac{(a+b\tan^{-1}(c+dx))^3}{e+fx}\,dx$

Optimal. Leaf size=372

$$\frac{3b^2\left(a+b\tan^{-1}(c+dx)\right)\text{Li}_3\left(1-\frac{2d(e+fx)}{(de-cf+if)(1-i(c+dx))}\right)}{2f}-\frac{3b^2\text{Li}_3\left(1-\frac{2}{1-i(c+dx)}\right)\left(a+b\tan^{-1}(c+dx)\right)}{2f}-\frac{3ib\left(a+b\tan^{-1}\right.}{}$$

```
[Out] -(a+b*arctan(d*x+c))^3*ln(2/(1-I*(d*x+c)))/f+(a+b*arctan(d*x+c))^3*ln(2*d*(
f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f+3/2*I*b*(a+b*arctan(d*x+c))^2*polylog
(2,1-2/(1-I*(d*x+c)))/f-3/2*I*b*(a+b*arctan(d*x+c))^2*polylog(2,1-2*d*(f*x+
e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f-3/2*b^2*(a+b*arctan(d*x+c))*polylog(3,1-2
/(1-I*(d*x+c)))/f+3/2*b^2*(a+b*arctan(d*x+c))*polylog(3,1-2*d*(f*x+e)/(d*e+
I*f-c*f)/(1-I*(d*x+c)))/f-3/4*I*b^3*polylog(4,1-2/(1-I*(d*x+c)))/f+3/4*I*b^
3*polylog(4,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f
```

Rubi [A] time = 0.20, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5047, 4860}

$$\frac{3b^2\left(a+b\tan^{-1}(c+dx)\right)\text{PolyLog}\left(3,1-\frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{2f}-\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)\left(a+b\tan^{-1}(c+\right.}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^3/(e + f*x),x]
```

```
[Out] -(((a + b*ArcTan[c + d*x])^3*Log[2/(1 - I*(c + d*x))])/f) + ((a + b*ArcTan[
c + d*x])^3*Log[(2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f +
 (((3*I)/2)*b*(a + b*ArcTan[c + d*x])^2*PolyLog[2, 1 - 2/(1 - I*(c + d*x))]
)/f - (((3*I)/2)*b*(a + b*ArcTan[c + d*x])^2*PolyLog[2, 1 - (2*d*(e + f*x))
/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f - (3*b^2*(a + b*ArcTan[c + d*x])
*PolyLog[3, 1 - 2/(1 - I*(c + d*x))])/(2*f) + (3*b^2*(a + b*ArcTan[c + d*x]
)*PolyLog[3, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/(2
*f) - (((3*I)/4)*b^3*PolyLog[4, 1 - 2/(1 - I*(c + d*x))])/f + (((3*I)/4)*b^
3*PolyLog[4, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f
```

Rule 4860

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^3/((d_) + (e_.)*(x_)), x_Symbol] :>
-Simp[((a + b*ArcTan[c*x])^3*Log[2/(1 - I*c*x)])/e, x] + (Simp[((a + b*ArcT
an[c*x])^3*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x] + Simp[(3*
I*b*(a + b*ArcTan[c*x])^2*PolyLog[2, 1 - 2/(1 - I*c*x)])/(2*e), x] - Simp[(
3*I*b*(a + b*ArcTan[c*x])^2*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1
- I*c*x))])/(2*e), x] - Simp[(3*b^2*(a + b*ArcTan[c*x])*PolyLog[3, 1 - 2/(1
 - I*c*x)])/(2*e), x] + Simp[(3*b^2*(a + b*ArcTan[c*x])*PolyLog[3, 1 - (2*c
*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e), x] - Simp[(3*I*b^3*PolyLog[4
, 1 - 2/(1 - I*c*x)])/(4*e), x] + Simp[(3*I*b^3*PolyLog[4, 1 - (2*c*(d + e*
x))/((c*d + I*e)*(1 - I*c*x))])/(4*e), x]) /; FreeQ[{a, b, c, d, e}, x] &&
NeQ[c^2*d^2 + e^2, 0]
```

Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{e+fx}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^3}{\frac{de-cf}{d}+\frac{fx}{d}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^3\log\left(\frac{2}{1-i(c+dx)}\right)}{f} + \frac{\left(a+b\tan^{-1}(c+dx)\right)^3\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(}\right.}{f}$$

Mathematica [F] time = 8.95, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^3/(e + f*x),x]
```

```
[Out] Integrate[(a + b*ArcTan[c + d*x])^3/(e + f*x), x]
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\arctan\left(dx+c\right)^3+3\,ab^2\arctan\left(dx+c\right)^2+3\,a^2b\arctan\left(dx+c\right)+a^3}{fx+e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arcta
n(d*x + c) + a^3)/(f*x + e), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(f*x+e),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.69, size = 4389, normalized size = 11.80

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^3/(f*x+e),x)
```

```
[Out] -3*I*a*b^2*c/(I*f+c*f-d*e)*arctan(d*x+c)*polylog(2,(I*f+c*f-d*e)*(1+I*(d*x+
c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-3/2*I*a*b^2/f*Pi*csgn(I*(I*f*(1+I*(d*x+c
))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1
+(d*x+c)^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^3*arctan(d*x+c)
^2-3*I*d*b^3/f*e*polylog(4,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e
+I*f-c*f))/(4*I*f+4*c*f-4*d*e)-3*d*a*b^2/f*e/(I*f+c*f-d*e)*arctan(d*x+c)^2*
ln(1-(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-3/2*I*a*b^2
/f*Pi*csgn(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csgn(I*(I*f*(1+I*(d*x+c))^2
/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*
```

```
x+c)^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*arctan(d*x+c)^2-3
/2*I*a*b^2/f*Pi*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))
^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e))*csgn(I*(I*
f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*
(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*
arctan(d*x+c)^2+1/2*I*b^3/f*Pi*csgn(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*cs
gn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d
*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e))*csgn(I*(I*f*(1+I*(d*x+c))^2/
(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x
+c)^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*arctan(d*x+c)^3+3*I*
d*b^3/f*e*arctan(d*x+c)^2*polylog(2,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c
)^2)/(d*e+I*f-c*f))/(2*I*f+2*c*f-2*d*e)+3*a^2*b*ln(f*(d*x+c)-c*f+d*e)/f*arc
tan(d*x+c)+3*a*b^2*ln(f*(d*x+c)-c*f+d*e)/f*arctan(d*x+c)^2-3*a*b^2/f*arctan
(d*x+c)^2*ln(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+
c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)-3/4*b^3/(I*f+c*f-d*e)*
polylog(4,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+a^3*ln
(f*(d*x+c)-c*f+d*e)/f-b^3/f*arctan(d*x+c)^3*ln(I*f*(1+I*(d*x+c))^2/(1+(d*x+
c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I
*f+c*f-d*e)-3/2*b^3/f*arctan(d*x+c)*polylog(3,-(1+I*(d*x+c))^2/(1+(d*x+c)^2
))+3/2*b^3/(I*f+c*f-d*e)*arctan(d*x+c)^2*polylog(2,(I*f+c*f-d*e)*(1+I*(d*x+
c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+b^3*ln(f*(d*x+c)-c*f+d*e)/f*arctan(d*x+c
)^3-3/2*a*b^2/f*polylog(3,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))-3/4*I*b^3/f*polyl
og(4,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))-3/2*d*a*b^2/f*e/(I*f+c*f-d*e)*polylog(
3,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-d*b^3/f*e/(I*f
+c*f-d*e)*arctan(d*x+c)^3*ln(1-(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/
(d*e+I*f-c*f))-3/2*d*b^3/f*e/(I*f+c*f-d*e)*arctan(d*x+c)*polylog(3,(I*f+c*f
-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-1/2*I*b^3/f*Pi*csgn(I*(I
*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I
*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e))*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x
+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-
I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*arctan(d*x+c)^3-1/2*I*b^3
/f*Pi*csgn(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csgn(I*(I*f*(1+I*(d*x+c))^2
/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*
x+c)^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*arctan(d*x+c)^3+3
*I*a*b^2/f*Pi*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2
/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/((1+I*(d*x+c)
)^2/(1+(d*x+c)^2)+1))^2*arctan(d*x+c)^2+3*a*b^2*c/(I*f+c*f-d*e)*arctan(d*x+
c)^2*ln(1-(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-3/2*I*
b^3*c/(I*f+c*f-d*e)*arctan(d*x+c)^2*polylog(2,(I*f+c*f-d*e)*(1+I*(d*x+c))^2
/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3/2*I*a^2*b*ln(f*(d*x+c)-c*f+d*e)/f*ln((I*f-f
*(d*x+c))/(d*e+I*f-c*f))-3/2*I*a^2*b*ln(f*(d*x+c)-c*f+d*e)/f*ln((I*f+f*(d*x
+c))/(I*f+c*f-d*e))+3*I*a*b^2/(I*f+c*f-d*e)*arctan(d*x+c)^2*ln(1-(I*f+c*f-d
*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3*I*a*b^2/f*arctan(d*x+c)*
polylog(2,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))-3*I*a*b^2/f*Pi*arctan(d*x+c)^2-1/
2*I*b^3/f*Pi*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/
(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/((1+I*(d*x+c))
^2/(1+(d*x+c)^2)+1))^3*arctan(d*x+c)^3+6*I*d*a*b^2/f*e*arctan(d*x+c)*polylo
g(2,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))/(2*I*f+2*c*f
-2*d*e)+3/2*I*a*b^2/f*Pi*csgn(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csgn(I*(
I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+
I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e))*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*
x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)
-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*arctan(d*x+c)^2-3/2*I*a^2*
b/f*dilog((I*f+f*(d*x+c))/(I*f+c*f-d*e))+3/4*I*b^3*c/(I*f+c*f-d*e)*polylog(
4,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-I*b^3/f*Pi*arc
tan(d*x+c)^3+3/2*I*a*b^2/(I*f+c*f-d*e)*polylog(3,(I*f+c*f-d*e)*(1+I*(d*x+c)
)^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+b^3*c/(I*f+c*f-d*e)*arctan(d*x+c)^3*ln(1-(
I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3/2*b^3*c/(I*f+c*
f-d*e)*arctan(d*x+c)*polylog(3,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/
```

```
(d*e+I*f-c*f))+I*b^3/f*Pi*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+
I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/(
(1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*arctan(d*x+c)^3+3*a*b^2/(I*f+c*f-d*e)*a
rctan(d*x+c)*polylog(2,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f
-c*f))+I*b^3/(I*f+c*f-d*e)*arctan(d*x+c)^3*ln(1-(I*f+c*f-d*e)*(1+I*(d*x+c))
^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3/2*a*b^2*c/(I*f+c*f-d*e)*polylog(3,(I*f+c*
f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3/2*I*b^3/f*arctan(d*x+
c)^2*polylog(2,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))+3/2*I*b^3/(I*f+c*f-d*e)*arct
an(d*x+c)*polylog(3,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*
f))+3/2*I*a^2*b/f*dilog((I*f-f*(d*x+c))/(d*e+I*f-c*f))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^3\log\left(fx+e\right)}{f}+\int\frac{28\,b^3\arctan\left(dx+c\right)^3+3\,b^3\arctan\left(dx+c\right)\log\left(d^2x^2+2\,cdx+c^2+1\right)^2+96\,ab^2\arctan\left(dx\right.}{32\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(f*x+e),x, algorithm="maxima")
```

```
[Out] a^3*log(f*x + e)/f + integrate(1/32*(28*b^3*arctan(d*x + c)^3 + 3*b^3*arcta
n(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 96*a*b^2*arctan(d*x + c)^2
+ 96*a^2*b*arctan(d*x + c))/(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{{\left(a+b\,\mathrm{atan}\left(c+d\,x\right)\right)}^3}{e+f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))^3/(e + f*x),x)
```

```
[Out] int((a + b*atan(c + d*x))^3/(e + f*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\,\mathrm{atan}\left(c+dx\right)\right)^3}{e+fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**3/(f*x+e),x)
```

```
[Out] Integral((a + b*atan(c + d*x))**3/(e + f*x), x)
```

3.40 $\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx$

Optimal. Leaf size=1233

$$\frac{id\tan^{-1}(c+dx)^3b^3}{d^2e^2-2cdfe+\left(c^2+1\right)f^2}+\frac{d(de-cf)\tan^{-1}(c+dx)^3b^3}{f\left(d^2e^2-2cdfe+\left(c^2+1\right)f^2\right)}-\frac{3d\tan^{-1}(c+dx)^2\log\left(\frac{2}{1-i(c+dx)}\right)b^3}{d^2e^2-2cdfe+\left(c^2+1\right)f^2}+\frac{3d\tan^{-1}(c+\ldots}{d^2e\ldots}$$

```
[Out] 3*a^2*b*d*(-c*f+d*e)*arctan(d*x+c)/f/(f^2+(-c*f+d*e)^2)+3*I*b^3*d*arctan(d*
x+c)*polylog(2,1-2/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3*a*b^2*d
*(-c*f+d*e)*arctan(d*x+c)^2/f/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3*I*a*b^2*d*p
olylog(2,1-2/(1+I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+b^3*d*(-c*f+d*e
)*arctan(d*x+c)^3/f/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-(a+b*arctan(d*x+c))^3/f
/(f*x+e)+3*a^2*b*d*ln(f*x+e)/(f^2+(-c*f+d*e)^2)-6*a*b^2*d*arctan(d*x+c)*ln(
2/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-3*b^3*d*arctan(d*x+c)^2*ln
(2/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+6*a*b^2*d*arctan(d*x+c)*l
n(2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+
3*b^3*d*arctan(d*x+c)^2*ln(2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/(d^2*e^
2-2*c*d*e*f+(c^2+1)*f^2)+6*a*b^2*d*arctan(d*x+c)*ln(2/(1+I*(d*x+c)))/(d^2*e
^2-2*c*d*e*f+(c^2+1)*f^2)+3*b^3*d*arctan(d*x+c)^2*ln(2/(1+I*(d*x+c)))/(d^2*
e^2-2*c*d*e*f+(c^2+1)*f^2)-3/2*a^2*b*d*ln(1+(d*x+c)^2)/(f^2+(-c*f+d*e)^2)-3
*I*b^3*d*arctan(d*x+c)*polylog(2,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))
/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3*I*b^3*d*arctan(d*x+c)*polylog(2,1-2/(1+I
*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-3*I*a*b^2*d*polylog(2,1-2*d*(f*x
+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3*I*a*b^2*
d*polylog(2,1-2/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+I*b^3*d*arct
an(d*x+c)^3/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3*I*a*b^2*d*arctan(d*x+c)^2/(d^
2*e^2-2*c*d*e*f+(c^2+1)*f^2)-3/2*b^3*d*polylog(3,1-2/(1-I*(d*x+c)))/(d^2*e^
2-2*c*d*e*f+(c^2+1)*f^2)+3/2*b^3*d*polylog(3,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1
-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3/2*b^3*d*polylog(3,1-2/(1+I*(
d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)
```

Rubi [A] time = 2.31, antiderivative size = 1233, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 22, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {5045, 6741, 5057, 6688, 12, 6725, 706, 31, 635, 203, 260, 4856, 2402, 2315, 2447, 4984, 4884, 4920, 4854, 4858, 4994, 6610}

$$\frac{id\tan^{-1}(c+dx)^3b^3}{d^2e^2-2cdfe+\left(c^2+1\right)f^2}+\frac{d(de-cf)\tan^{-1}(c+dx)^3b^3}{f\left(d^2e^2-2cdfe+\left(c^2+1\right)f^2\right)}-\frac{3d\tan^{-1}(c+dx)^2\log\left(\frac{2}{1-i(c+dx)}\right)b^3}{d^2e^2-2cdfe+\left(c^2+1\right)f^2}+\frac{3d\tan^{-1}(c+\ldots}{d^2e\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^3/(e + f*x)^2,x]
```

```
[Out] (3*a^2*b*d*(d*e - c*f)*ArcTan[c + d*x])/(f*(f^2 + (d*e - c*f)^2)) + ((3*I)*
a*b^2*d*ArcTan[c + d*x]^2)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (3*a*b^2
*d*(d*e - c*f)*ArcTan[c + d*x]^2)/(f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2))
 + (I*b^3*d*ArcTan[c + d*x]^3)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (b^3
*d*(d*e - c*f)*ArcTan[c + d*x]^3)/(f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2))
 - (a + b*ArcTan[c + d*x])^3/(f*(e + f*x)) + (3*a^2*b*d*Log[e + f*x])/(f^2
+ (d*e - c*f)^2) - (6*a*b^2*d*ArcTan[c + d*x]*Log[2/(1 - I*(c + d*x))])/(d^
2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (3*b^3*d*ArcTan[c + d*x]^2*Log[2/(1 -
I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (6*a*b^2*d*ArcTan[c
+ d*x]*Log[(2*d*(e + f*x))/((d*e + (I - c)*f)*(1 - I*(c + d*x)))])/(d^2*e^2
 - 2*c*d*e*f + (1 + c^2)*f^2) + (3*b^3*d*ArcTan[c + d*x]^2*Log[(2*d*(e + f*
x))/((d*e + (I - c)*f)*(1 - I*(c + d*x)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2
)*f^2) + (6*a*b^2*d*ArcTan[c + d*x]*Log[2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*
```

```
c*d*e*f + (1 + c^2)*f^2) + (3*b^3*d*ArcTan[c + d*x]^2*Log[2/(1 + I*(c + d*x
))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (3*a^2*b*d*Log[1 + (c + d*x)^2
])/(2*(f^2 + (d*e - c*f)^2)) + ((3*I)*a*b^2*d*PolyLog[2, 1 - 2/(1 - I*(c +
d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + ((3*I)*b^3*d*ArcTan[c + d*x
]*PolyLog[2, 1 - 2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2
) - ((3*I)*a*b^2*d*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + (I - c)*f)*(1 - I
*(c + d*x)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - ((3*I)*b^3*d*ArcTan[
c + d*x]*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + (I - c)*f)*(1 - I*(c + d*x)
))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + ((3*I)*a*b^2*d*PolyLog[2, 1 -
2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + ((3*I)*b^3*d*
ArcTan[c + d*x]*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f +
 (1 + c^2)*f^2) - (3*b^3*d*PolyLog[3, 1 - 2/(1 - I*(c + d*x))])/(2*(d^2*e^2
 - 2*c*d*e*f + (1 + c^2)*f^2)) + (3*b^3*d*PolyLog[3, 1 - (2*d*(e + f*x))/((
d*e + (I - c)*f)*(1 - I*(c + d*x)))])/(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f
^2)) + (3*b^3*d*PolyLog[3, 1 - 2/(1 + I*(c + d*x))])/(2*(d^2*e^2 - 2*c*d*e*
f + (1 + c^2)*f^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4858

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
-Simp[((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e, x] + (Simp[((a + b*ArcT
an[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x] + Simp[(I*
b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/e, x] - Simp[(I*b*(a +
 b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/
e, x] - Simp[(b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e), x] + Simp[(b^2*Poly
Log[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e), x]) /; FreeQ[
{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4984

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
```

```
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 5045

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[((e + f*x)^(m + 1)*(a + b*ArcTan[c + d*x])^p)/(f*(m +
 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcTan[c
 + d*x])^(p - 1))/(1 + (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 5057

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst
[Int[((d*e - c*f)/d + (f*x)/d)^m*(C/d^2 + (C*x^2)/d^2)^q*(a + b*ArcTan[x])^
p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &&
 EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx = -\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{(3bd)\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{(e+fx)\left(1+(c+dx)^2\right)}\,dx}{f}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{(3bd)\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^2}{(e+fx)\left(1+c^2+2cdx+d^2x^2\right)}\,dx}{f}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{(3b)\,\mathrm{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)\left(1+x^2\right)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{(3b)\,\mathrm{Subst}\left(\int \frac{d\left(a+b\tan^{-1}(x)\right)^2}{(de-cf+fx)\left(1+x^2\right)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{(3bd)\,\mathrm{Subst}\left(\int \frac{\left(a+b\tan^{-1}(x)\right)^2}{(de-cf+fx)\left(1+x^2\right)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{(3bd)\,\mathrm{Subst}\left(\int \left(\frac{a^2}{(de-cf+fx)\left(1+x^2\right)} + \frac{2ab\tan^{-1}(x)}{(de-cf+fx)\left(1+x^2\right)} + \cdots\right.\right.}{f}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{\left(3a^2bd\right)\mathrm{Subst}\left(\int \frac{1}{(de-cf+fx)\left(1+x^2\right)}\,dx, x, c+dx\right)}{f} + \cdots$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{\left(6ab^2d\right)\mathrm{Subst}\left(\int \left(\frac{f^2\tan^{-1}(x)}{\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)(de-cf+fx)} + \cdots\right.\right.}{f}$$

$$= -\frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{3a^2bd\log(e+fx)}{f^2+(de-cf)^2} + \frac{\left(6ab^2d\right)\mathrm{Subst}\left(\int \frac{(de-cf-fx)\tan^{-1}\cdots}{1+x^2}\cdots\right.}{f\left(d^2e^2-2cdef+\left(1+\cdots\right.\right.}$$

$$= \frac{3a^2bd(de-cf)\tan^{-1}(c+dx)}{f\left(f^2+(de-cf)^2\right)} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{3a^2bd\log(e+fx)}{f^2+(de-cf)^2} - \cdots$$

$$= \frac{3a^2bd(de-cf)\tan^{-1}(c+dx)}{f\left(f^2+(de-cf)^2\right)} - \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{3a^2bd\log(e+fx)}{f^2+(de-cf)^2} - \cdots$$

$$= \frac{3a^2bd(de-cf)\tan^{-1}(c+dx)}{f\left(f^2+(de-cf)^2\right)} + \frac{3iab^2d\tan^{-1}(c+dx)^2}{d^2e^2-2cdef+\left(1+c^2\right)f^2} + \frac{3ab^2d(de-cf)\,\mathrm{t}\cdots}{f\left(d^2e^2-2cdef\cdots\right.}$$

$$= \frac{3a^2bd(de-cf)\tan^{-1}(c+dx)}{f\left(f^2+(de-cf)^2\right)} + \frac{3iab^2d\tan^{-1}(c+dx)^2}{d^2e^2-2cdef+\left(1+c^2\right)f^2} + \frac{3ab^2d(de-cf)\,\mathrm{t}\cdots}{f\left(d^2e^2-2cdef\cdots\right.}$$

$$= \frac{3a^2bd(de-cf)\tan^{-1}(c+dx)}{f\left(f^2+(de-cf)^2\right)} + \frac{3iab^2d\tan^{-1}(c+dx)^2}{d^2e^2-2cdef+\left(1+c^2\right)f^2} + \frac{3ab^2d(de-cf)\,\mathrm{t}\cdots}{f\left(d^2e^2-2cdef\cdots\right.}$$

$$= \frac{3a^2bd(de-cf)\tan^{-1}(c+dx)}{f\left(f^2+(de-cf)^2\right)} + \frac{3iab^2d\tan^{-1}(c+dx)^2}{d^2e^2-2cdef+\left(1+c^2\right)f^2} + \frac{3ab^2d(de-cf)\,\mathrm{t}\cdots}{f\left(d^2e^2-2cdef\cdots\right.}$$

Mathematica [F] time = 17.43, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\tan^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^3/(e + f*x)^2,x]
```

```
[Out] Integrate[(a + b*ArcTan[c + d*x])^3/(e + f*x)^2, x]
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\arctan\left(dx+c\right)^3+3\,ab^2\arctan\left(dx+c\right)^2+3\,a^2b\arctan\left(dx+c\right)+a^3}{f^2x^2+2\,efx+e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arcta
n(d*x + c) + a^3)/(f^2*x^2 + 2*e*f*x + e^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 1.80, size = 4764, normalized size = 3.86

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^3/(f*x+e)^2,x)
```

```
[Out] -3/2*I*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*Pi*csgn(I*(I*f
*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(
d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e))*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c
)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*
f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2+3/4*I*d*b^3/(c^2*f^2-2*c*d*
e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1
))^2*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)-3/2*I*d*b^3/(c^2*f^2-2*c*d
*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+
1))*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^2+3*d*b^3/(c^2*f^2-2*c*d*e*
f+d^2*e^2+f^2)*arctan(d*x+c)^2*ln(2)+3*d*a^2*b/(c^2*f^2-2*c*d*e*f+d^2*e^2+f
^2)*ln(f*(d*x+c)-c*f+d*e)-3/2*d*a^2*b/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(1+
(d*x+c)^2)+3*d*b^3*arctan(d*x+c)^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(f*(d*
x+c)-c*f+d*e)-d*b^3*arctan(d*x+c)^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*c-3*d*b
^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*ln(I*f*(1+I*(d*x+c))^2/(
1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+
c)^2)-I*f+c*f-d*e)+3*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*
ln((1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))-3/2*d*b^3*arctan(d*x+c)^2/(c^2*f^2-2*
c*d*e*f+d^2*e^2+f^2)*ln(1+(d*x+c)^2)-d*b^3/(d*f*x+d*e)/f*arctan(d*x+c)^3-I*
d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^3-d*a^3/(d*f*x+d*e)/f+3
*d*b^3*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*c/(I*f+c*f-d*e)*arctan(d*x+c)^2*ln
(1-(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3*I*d^2*b^3/(
```

c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*e/(I*f+c*f-d*e)*arctan(d*x+c)*polylog(2,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3*I*d*b^3*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)/(I*f+c*f-d*e)*arctan(d*x+c)^2*ln(1-(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3/4*I*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*Pi*csgn(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^2+3/4*I*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^2-3/4*I*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))^2*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))-3/2*I*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*Pi*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*csgn(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))+3/2*I*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2-3*I*d*b^3*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*c/(I*f+c*f-d*e)*arctan(d*x+c)*polylog(2,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3/2*I*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*Pi*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e))*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csgn(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))-3/4*I*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*Pi*csgn(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)+3*I*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*Pi*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2-3/4*I*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^3-3/2*I*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*Pi*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^3+3/2*I*d*b^3*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)/(I*f+c*f-d*e)*polylog(3,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-3/4*I*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))^3-3*I*d*a*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(f*(d*x+c)-c*f+d*e)*ln((I*f+f*(d*x+c))/(I*f+c*f-d*e))-3*d^2*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*e/(I*f+c*f-d*e)*arctan(d*x+c)^2*ln(1-(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3*d*b^3*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)/(I*f+c*f-d*e)*arctan(d*x+c)*polylog(2,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3/2*d*b^3*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*c/(I*f+c*f-d*e)*polylog(3,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3*d^2*a*b^2/f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*e+3*d^2*a^2*b/f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)*e+3/2*I*d*a*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(d*x+c-I)*ln(-1/2*I*(I+d*x+c))-3/2*I*d*a*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(I+d*x+c)*ln(1/2*I*(d*x+c-I))+3/2*I*d*a*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(I+d*x+c)*ln(1+(d*x+c)^2)+3*I*d*a*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(f*(d*x+c)-c*f+d*e)*ln((I*f-f*(d*x+c))/(d*e+I*f-c*f))-3/2*I*d*a*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(d*x+c-I)*ln(1+(d*x+c)^2)+3/4*I*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^3-3*d*a*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*c-3*d*a^2*b/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)*c+d^2*b^3/f*arctan(d*x+c)^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*e-3/2*d^2*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*e/(I*f+c*f-d*e)*polylog(3,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-3*d*a*b^2/(d*f*x+d*e)/f*arctan(d*x+c)^2-3*d*a^2*b/(d*f*x+d*e)/f*arctan(d*x+c)+6*d*a*b^2*arctan(d*x+c)/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(f*(d*x+c)-c*f+d*e)-3*d*a*b^2*arctan(d*x+c)/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(1+(d*x+c)^

```
2)+3*I*d*a*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*dilog((I*f-f*(d*x+c))/(d*e+I
*f-c*f))-3/2*I*d*a*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*dilog(1/2*I*(d*x+c-I
))-3/4*I*d*a*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(I+d*x+c)^2+3/2*I*d*a*b^
2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*dilog(-1/2*I*(I+d*x+c))-3*I*d*b^3/(c^2*f^
2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*Pi-3*I*d*a*b^2/(c^2*f^2-2*c*d*e*f+
d^2*e^2+f^2)*dilog((I*f+f*(d*x+c))/(I*f+c*f-d*e))+3/4*I*d*a*b^2/(c^2*f^2-2*
c*d*e*f+d^2*e^2+f^2)*ln(d*x+c-I)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3}{2}\left(d\left(\frac{2\left(d^2e-cdf\right)\arctan\left(\frac{d^2x+cd}{d}\right)}{\left(d^2e^2f-2\,cdef^2+\left(c^2+1\right)f^3\right)d}-\frac{\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^2e^2-2\,cdef+\left(c^2+1\right)f^2}+\frac{2\,\log\left(fx+e\right)}{d^2e^2-2\,cdef+\left(c^2+1\right)f^2}\right)-\frac{2\,\arctan\left(d\right.}{f^2x+e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] 3/2*(d*(2*(d^2*e - c*d*f)*arctan((d^2*x + c*d)/d)/((d^2*e^2*f - 2*c*d*e*f^2
 + (c^2 + 1)*f^3)*d) - log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*e^2 - 2*c*d*e*
f + (c^2 + 1)*f^2) + 2*log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 + 1)*f^2))
- 2*arctan(d*x + c)/(f^2*x + e*f))*a^2*b - a^3/(f^2*x + e*f) - 1/32*(4*b^3*
arctan(d*x + c)^3 - 3*b^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^
2 - 32*(f^2*x + e*f)*integrate(1/32*(28*(b^3*d^2*f*x^2 + 2*b^3*c*d*f*x + (b
^3*c^2 + b^3)*f)*arctan(d*x + c)^3 + 12*(8*a*b^2*d^2*f*x^2 + b^3*d*e + (16*
a*b^2*c + b^3)*d*f*x + 8*(a*b^2*c^2 + a*b^2)*f)*arctan(d*x + c)^2 - 12*(b^3
*d^2*f*x^2 + b^3*c*d*e + (b^3*d^2*e + b^3*c*d*f)*x)*arctan(d*x + c)*log(d^2
*x^2 + 2*c*d*x + c^2 + 1) - 3*(b^3*d*f*x + b^3*d*e - (b^3*d^2*f*x^2 + 2*b^3
*c*d*f*x + (b^3*c^2 + b^3)*f)*arctan(d*x + c))*log(d^2*x^2 + 2*c*d*x + c^2
+ 1)^2)/(d^2*f^3*x^4 + (c^2 + 1)*e^2*f + 2*(d^2*e*f^2 + c*d*f^3)*x^3 + (d^2
*e^2*f + 4*c*d*e*f^2 + (c^2 + 1)*f^3)*x^2 + 2*(c*d*e^2*f + (c^2 + 1)*e*f^2)
*x), x))/(f^2*x + e*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}\,(c+d\,x))^3}{\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))^3/(e + f*x)^2,x)
```

```
[Out] int((a + b*atan(c + d*x))^3/(e + f*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**3/(f*x+e)**2,x)
```

```
[Out] Timed out
```

3.41 $\int (e+fx)^m \left(a+b\tan^{-1}(c+dx)\right) dx$

Optimal. Leaf size=177

$$\frac{(e+fx)^{m+1}\left(a+b\tan^{-1}(c+dx)\right)}{f(m+1)}-\frac{ibd(e+fx)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{d(e+fx)}{de-cf+if}\right)}{2f(m+1)(m+2)(de+(-c+i)f)}+\frac{ibd(e+fx)^{m+2}\,{}_2F_1\left(1,m+\right.}{2f(m+1)(m+2)(de}$$

```
[Out] (f*x+e)^(1+m)*(a+b*arctan(d*x+c))/f/(1+m)-1/2*I*b*d*(f*x+e)^(2+m)*hypergeom
([1, 2+m],[3+m],d*(f*x+e)/(d*e+I*f-c*f))/f/(d*e+(I-c)*f)/(1+m)/(2+m)+1/2*I*
b*d*(f*x+e)^(2+m)*hypergeom([1, 2+m],[3+m],d*(f*x+e)/(d*e-(I+c)*f))/f/(d*e-
(I+c)*f)/(1+m)/(2+m)
```

Rubi [A] time = 0.25, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5047, 4862, 712, 68}

$$-\frac{ibd(e+fx)^{m+2}\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{d(e+fx)}{-cf+de+if}\right)}{2f(m+1)(m+2)(de+(-c+i)f)}+\frac{ibd(e+fx)^{m+2}\text{Hypergeometric2F1}\left(1,m\right.}{2f(m+1)(m+2)(de-(c}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^m*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] ((e + f*x)^(1 + m)*(a + b*ArcTan[c + d*x]))/(f*(1 + m)) - ((I/2)*b*d*(e + f
*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e + I*f - c
*f)])/(f*(d*e + (I - c)*f)*(1 + m)*(2 + m)) + ((I/2)*b*d*(e + f*x)^(2 + m)*
Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e - (I + c)*f)])/(f*(d*
e - (I + c)*f)*(1 + m)*(2 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 712

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\int (e+fx)^m \left(a+b\tan^{-1}(c+dx)\right) dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^m \left(a+b\tan^{-1}(x)\right) dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^{1+m}\left(a+b\tan^{-1}(c+dx)\right)}{f(1+m)} - \frac{b\,\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^{1+m}}{1+x^2} dx, x, c+dx\right.}{f(1+m)}$$

$$= \frac{(e+fx)^{1+m}\left(a+b\tan^{-1}(c+dx)\right)}{f(1+m)} - \frac{b\,\text{Subst}\left(\int \left(\frac{i\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^{1+m}}{2(i-x)} + \frac{i\left(\frac{de-cf}{d}+\frac{f}{}\right.}{2(i+x}\right.\right.}{f(1+m)}$$

$$= \frac{(e+fx)^{1+m}\left(a+b\tan^{-1}(c+dx)\right)}{f(1+m)} - \frac{(ib)\,\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^{1+m}}{i-x} dx, x, c+\right.}{2f(1+m)}$$

$$= \frac{(e+fx)^{1+m}\left(a+b\tan^{-1}(c+dx)\right)}{f(1+m)} - \frac{ibd(e+fx)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\right.}{2f(de+(i-c)f)(1+m)(2+}$$

Mathematica [A] time = 0.39, size = 162, normalized size = 0.92

$$\frac{(e+fx)^{m+1}\left(2\left(a+b\tan^{-1}(c+dx)\right)+\frac{bd(e+fx)\left((de-(c+i)f)\,{}_2F_1\left(1,m+2;m+3;\frac{d(e+fx)}{de-(c-i)f}\right)+(-de+(c-i)f)\,{}_2F_1\left(1,m+2;m+3;\frac{d(e+fx)}{de-(c+i)f}\right)\right)}{(m+2)(-icf+ide+f)(de-(c-i)f)}\right)}{2f(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^m*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] ((e + f*x)^(1 + m)*(2*(a + b*ArcTan[c + d*x]) + (b*d*(e + f*x)*((d*e - (I +
 c)*f)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e - (-I + c)*f)]
 + (-(d*e) + (-I + c)*f)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(
d*e - (I + c)*f)]))/((I*d*e + f - I*c*f)*(d*e - (-I + c)*f)*(2 + m))))/(2*f
*(1 + m))
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left((b\arctan(dx+c)+a)\left(fx+e\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(d*x + c) + a)*(f*x + e)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [F] time = 1.96, size = 0, normalized size = 0.00

$$\int \left(fx+e\right)^{m} \left(a+b\arctan \left(dx+c\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*(a+b*arctan(d*x+c)),x)
[Out] int((f*x+e)^m*(a+b*arctan(d*x+c)),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c)),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(e+f\,x\right)}^m\,\left(a+b\,\mathrm{atan}\left(c+d\,x\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^m*(a + b*atan(c + d*x)),x)
[Out] int((e + f*x)^m*(a + b*atan(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*(a+b*atan(d*x+c)),x)
[Out] Timed out
```

3.42 $\int (e+fx)^m \left(a+b\tan^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=23

$$\text{Int}\left((e+fx)^m \left(a+b\tan^{-1}(c+dx)\right)^2, x\right)$$

```
[Out] Unintegrable((f*x+e)^m*(a+b*arctan(d*x+c))^2,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (e+fx)^m \left(a+b\tan^{-1}(c+dx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(e + f*x)^m*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] Defer[Subst][Defer[Int][((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcTan[x])^2, x]
, x, c + d*x]/d
```

Rubi steps

$$\int (e+fx)^m \left(a+b\tan^{-1}(c+dx)\right)^2 \, dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^m \left(a+b\tan^{-1}(x)\right)^2 \, dx, x, c+dx\right)}{d}$$

Mathematica [A] time = 5.00, size = 0, normalized size = 0.00

$$\int (e+fx)^m \left(a+b\tan^{-1}(c+dx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e + f*x)^m*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] Integrate[(e + f*x)^m*(a + b*ArcTan[c + d*x])^2, x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\arctan\left(dx+c\right)^2 + 2\,ab\arctan\left(dx+c\right) + a^2\right)\left(fx+e\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2)*(f*x + e)^m,
 x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 1.48, size = 0, normalized size = 0.00

$$\int \left(fx+e\right)^{m} \left(a+b\arctan\left(dx+c\right)\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*(a+b*arctan(d*x+c))^2,x)
[Out] int((f*x+e)^m*(a+b*arctan(d*x+c))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(fx+e\right)^{m+1}a^{2}}{f(m+1)}+\frac{7\left(b^{2}fx+b^{2}e\right)\left(fx+e\right)^{m}\arctan\left(dx+c\right)^{2}-\frac{3}{4}\left(b^{2}fx+b^{2}e\right)\left(fx+e\right)^{m}\log\left(d^{2}x^{2}+2\,cdx+c^{2}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c))^2,x, algorithm="maxima")
[Out] (f*x + e)^(m + 1)*a^2/(f*(m + 1)) + 1/16*(4*(b^2*f*x + b^2*e)*(f*x + e)^m*a
rctan(d*x + c)^2 - (b^2*f*x + b^2*e)*(f*x + e)^m*log(d^2*x^2 + 2*c*d*x + c^
2 + 1)^2 + 16*(f*m + f)*integrate(1/16*(12*((b^2*c^2 + b^2)*f*m + (b^2*d^2*
f*m + b^2*d^2*f)*x^2 + (b^2*c^2 + b^2)*f + 2*(b^2*c*d*f*m + b^2*c*d*f)*x)*(
f*x + e)^m*arctan(d*x + c)^2 + ((b^2*c^2 + b^2)*f*m + (b^2*d^2*f*m + b^2*d^
2*f)*x^2 + (b^2*c^2 + b^2)*f + 2*(b^2*c*d*f*m + b^2*c*d*f)*x)*(f*x + e)^m*l
og(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 - 8*(b^2*d*e - 4*(a*b*c^2 + a*b)*f*m - 4*
(a*b*d^2*f*m + a*b*d^2*f)*x^2 - 4*(a*b*c^2 + a*b)*f - (8*a*b*c*d*f*m + (8*a
*b*c - b^2)*d*f)*x)*(f*x + e)^m*arctan(d*x + c) + 4*(b^2*d^2*f*x^2 + b^2*c*
d*e + (b^2*d^2*e + b^2*c*d*f)*x)*(f*x + e)^m*log(d^2*x^2 + 2*c*d*x + c^2 +
1))/((c^2 + 1)*f*m + (d^2*f*m + d^2*f)*x^2 + (c^2 + 1)*f + 2*(c*d*f*m + c*d
*f)*x), x))/(f*m + f)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int {\left(e+f\,x\right)}^m\,{\left(a+b\,\mathrm{atan}\left(c+d\,x\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^m*(a + b*atan(c + d*x))^2,x)
[Out] int((e + f*x)^m*(a + b*atan(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*(a+b*atan(d*x+c))**2,x)
[Out] Timed out
```

3.43 $\int (e+fx)^m \left(a+b\tan^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=23

$$\mathrm{Int}\left((e+fx)^m \left(a+b\tan^{-1}(c+dx)\right)^3, x\right)$$

```
[Out] Unintegrable((f*x+e)^m*(a+b*arctan(d*x+c))^3,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (e+fx)^m \left(a+b\tan^{-1}(c+dx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(e + f*x)^m*(a + b*ArcTan[c + d*x])^3,x]
[Out] Defer[Subst][Defer[Int][((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcTan[x])^3, x]
, x, c + d*x]/d
```

Rubi steps

$$\int (e+fx)^m \left(a+b\tan^{-1}(c+dx)\right)^3 \, dx = \frac{\mathrm{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^m \left(a+b\tan^{-1}(x)\right)^3 \, dx, x, c+dx\right)}{d}$$

Mathematica [A] time = 0.51, size = 0, normalized size = 0.00

$$\int (e+fx)^m \left(a+b\tan^{-1}(c+dx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e + f*x)^m*(a + b*ArcTan[c + d*x])^3,x]
[Out] Integrate[(e + f*x)^m*(a + b*ArcTan[c + d*x])^3, x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^3\arctan(dx+c)^3 + 3\,ab^2\arctan(dx+c)^2 + 3\,a^2b\arctan(dx+c) + a^3\right)\left(fx+e\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c))^3,x, algorithm="fricas")
[Out] integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arcta
n(d*x + c) + a^3)*(f*x + e)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c))^3,x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 1.43, size = 0, normalized size = 0.00

$$\int \left(fx+e\right)^{m} \left(a+b\arctan\left(dx+c\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*(a+b*arctan(d*x+c))^3,x)
[Out] int((f*x+e)^m*(a+b*arctan(d*x+c))^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(fx+e\right)^{m+1}a^{3}}{f(m+1)}+\frac{4\left(b^{3}fx+b^{3}e\right)\left(fx+e\right)^{m}\arctan\left(dx+c\right)^{3}-3\left(b^{3}fx+b^{3}e\right)\left(fx+e\right)^{m}\arctan\left(dx+c\right)\log\left(d^{2}x^{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c))^3,x, algorithm="maxima")
[Out] (f*x + e)^(m + 1)*a^3/(f*(m + 1)) + 1/32*(4*(b^3*f*x + b^3*e)*(f*x + e)^m*a
rctan(d*x + c)^3 - 3*(b^3*f*x + b^3*e)*(f*x + e)^m*arctan(d*x + c)*log(d^2*
x^2 + 2*c*d*x + c^2 + 1)^2 + 32*(f*m + f)*integrate(1/32*(28*((b^3*c^2 + b^
3)*f*m + (b^3*d^2*f*m + b^3*d^2*f)*x^2 + (b^3*c^2 + b^3)*f + 2*(b^3*c*d*f*m
 + b^3*c*d*f)*x)*(f*x + e)^m*arctan(d*x + c)^3 - 12*(b^3*d*e - 8*(a*b^2*c^2
 + a*b^2)*f*m - 8*(a*b^2*d^2*f*m + a*b^2*d^2*f)*x^2 - 8*(a*b^2*c^2 + a*b^2)
*f - (16*a*b^2*c*d*f*m + (16*a*b^2*c - b^3)*d*f)*x)*(f*x + e)^m*arctan(d*x
+ c)^2 + 12*(b^3*d^2*f*x^2 + b^3*c*d*e + (b^3*d^2*e + b^3*c*d*f)*x)*(f*x +
e)^m*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 96*((a^2*b*c^2 + a^
2*b)*f*m + (a^2*b*d^2*f*m + a^2*b*d^2*f)*x^2 + (a^2*b*c^2 + a^2*b)*f + 2*(a
^2*b*c*d*f*m + a^2*b*c*d*f)*x)*(f*x + e)^m*arctan(d*x + c) + 3*(((b^3*c^2 +
 b^3)*f*m + (b^3*d^2*f*m + b^3*d^2*f)*x^2 + (b^3*c^2 + b^3)*f + 2*(b^3*c*d*
f*m + b^3*c*d*f)*x)*(f*x + e)^m*arctan(d*x + c) + (b^3*d*f*x + b^3*d*e)*(f*
x + e)^m)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2)/((c^2 + 1)*f*m + (d^2*f*m + d
^2*f)*x^2 + (c^2 + 1)*f + 2*(c*d*f*m + c*d*f)*x), x))/(f*m + f)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int {\left(e+f\,x\right)}^m\,{\left(a+b\,\mathrm{atan}\left(c+d\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^m*(a + b*atan(c + d*x))^3,x)
[Out] int((e + f*x)^m*(a + b*atan(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*(a+b*atan(d*x+c))**3,x)
[Out] Timed out
```

3.44 $\int x^3 \tan^{-1}(a+bx)\,dx$

Optimal. Leaf size=106

$$-\frac{a\left(1-a^2\right)\log\left((a+bx)^2+1\right)}{2b^4}+\frac{\left(1-6a^2\right)x}{4b^3}-\frac{\left(a^4-6a^2+1\right)\tan^{-1}(a+bx)}{4b^4}-\frac{(a+bx)^3}{12b^4}+\frac{a(a+bx)^2}{2b^4}+\frac{1}{4}x^4\tan^{-1}(a+$$

```
[Out] 1/4*(-6*a^2+1)*x/b^3+1/2*a*(b*x+a)^2/b^4-1/12*(b*x+a)^3/b^4-1/4*(a^4-6*a^2+
1)*arctan(b*x+a)/b^4+1/4*x^4*arctan(b*x+a)-1/2*a*(-a^2+1)*ln(1+(b*x+a)^2)/b
^4
```

Rubi [A] time = 0.11, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5047, 4862, 702, 635, 203, 260}

$$\frac{\left(1-6a^2\right)x}{4b^3}-\frac{a\left(1-a^2\right)\log\left((a+bx)^2+1\right)}{2b^4}-\frac{\left(a^4-6a^2+1\right)\tan^{-1}(a+bx)}{4b^4}-\frac{(a+bx)^3}{12b^4}+\frac{a(a+bx)^2}{2b^4}+\frac{1}{4}x^4\tan^{-1}(a+b$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTan[a + b*x],x]
```

```
[Out] ((1 - 6*a^2)*x)/(4*b^3) + (a*(a + b*x)^2)/(2*b^4) - (a + b*x)^3/(12*b^4) -
((1 - 6*a^2 + a^4)*ArcTan[a + b*x])/(4*b^4) + (x^4*ArcTan[a + b*x])/4 - (a*
(1 - a^2)*Log[1 + (a + b*x)^2])/(2*b^4)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
```

```
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^3\tan^{-1}(a+bx)\,dx &= \frac{\text{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)^3\tan^{-1}(x)\,dx,x,a+bx\right)}{b}\\
&= \frac{1}{4}x^4\tan^{-1}(a+bx)-\frac{1}{4}\text{Subst}\left(\int\frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^4}{1+x^2}\,dx,x,a+bx\right)\\
&= \frac{1}{4}x^4\tan^{-1}(a+bx)-\frac{1}{4}\text{Subst}\left(\int\left(-\frac{1-6a^2}{b^4}-\frac{4ax}{b^4}+\frac{x^2}{b^4}+\frac{1-6a^2+a^4+4a\left(1-a^2\right)}{b^4\left(1+x^2\right)}\right.\right.\\
&= \frac{\left(1-6a^2\right)x}{4b^3}+\frac{a(a+bx)^2}{2b^4}-\frac{(a+bx)^3}{12b^4}+\frac{1}{4}x^4\tan^{-1}(a+bx)-\frac{\text{Subst}\left(\int\frac{1-6a^2+a^4+4a\left(1-a\right.}{1+x^2}\right.}{4b^4}\\
&= \frac{\left(1-6a^2\right)x}{4b^3}+\frac{a(a+bx)^2}{2b^4}-\frac{(a+bx)^3}{12b^4}+\frac{1}{4}x^4\tan^{-1}(a+bx)-\frac{\left(a\left(1-a^2\right)\right)\text{Subst}\left(\int\frac{x}{1+}\right.}{b^4}\\
&= \frac{\left(1-6a^2\right)x}{4b^3}+\frac{a(a+bx)^2}{2b^4}-\frac{(a+bx)^3}{12b^4}-\frac{\left(1-6a^2+a^4\right)\tan^{-1}(a+bx)}{4b^4}+\frac{1}{4}x^4\tan^{-1}(a+
\end{aligned}$$

Mathematica [C] time = 0.08, size = 95, normalized size = 0.90

$$\frac{6\left(1-6a^2\right)bx+6b^4x^4\tan^{-1}(a+bx)-2(a+bx)^3+12a(a+bx)^2+3i(a-i)^4\log(-a-bx+i)-3i(a+i)^4\log(}{24b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTan[a + b*x],x]
```

```
[Out] (6*(1 - 6*a^2)*b*x + 12*a*(a + b*x)^2 - 2*(a + b*x)^3 + 6*b^4*x^4*ArcTan[a
+ b*x] + (3*I)*(-I + a)^4*Log[I - a - b*x] - (3*I)*(I + a)^4*Log[I + a + b*
x])/(24*b^4)
```

fricas [A] time = 0.41, size = 87, normalized size = 0.82

$$-\frac{b^3x^3-3\,ab^2x^2+3\left(3\,a^2-1\right)bx-3\left(b^4x^4-a^4+6\,a^2-1\right)\arctan\left(bx+a\right)-6\left(a^3-a\right)\log\left(b^2x^2+2\,abx+a^2\right.}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/12*(b^3*x^3 - 3*a*b^2*x^2 + 3*(3*a^2 - 1)*b*x - 3*(b^4*x^4 - a^4 + 6*a^2
 - 1)*arctan(b*x + a) - 6*(a^3 - a)*log(b^2*x^2 + 2*a*b*x + a^2 + 1))/b^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(b*x+a),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.04, size = 132, normalized size = 1.25

$$\frac{x^4\arctan(bx+a)}{4}-\frac{\arctan(bx+a)\,a^4}{4b^4}-\frac{x^3}{12b}+\frac{x^2a}{4b^2}-\frac{3x\,a^2}{4b^3}-\frac{13a^3}{12b^4}+\frac{x}{4b^3}+\frac{a}{4b^4}+\frac{\ln\left(1+(bx+a)^2\right)a^3}{2b^4}-\frac{\ln\left(1+(bx+a\right.}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(b*x+a),x)
```

```
[Out] 1/4*x^4*arctan(b*x+a)-1/4/b^4*arctan(b*x+a)*a^4-1/12/b*x^3+1/4/b^2*x^2*a-3/
4/b^3*x*a^2-13/12/b^4*a^3+1/4/b^3*x+1/4/b^4*a+1/2/b^4*ln(1+(b*x+a)^2)*a^3-1
/2/b^4*ln(1+(b*x+a)^2)*a+3/2/b^4*arctan(b*x+a)*a^2-1/4/b^4*arctan(b*x+a)
```

maxima [A] time = 0.41, size = 104, normalized size = 0.98

$$\frac{1}{4}x^4\arctan(bx+a)-\frac{1}{12}b\left(\frac{b^2x^3-3\,abx^2+3\left(3\,a^2-1\right)x}{b^4}+\frac{3\left(a^4-6\,a^2+1\right)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^5}-\frac{6\left(a^3-a\right)\log\left(b^2x\right.}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arctan(b*x + a) - 1/12*b*((b^2*x^3 - 3*a*b*x^2 + 3*(3*a^2 - 1)*x)/b
^4 + 3*(a^4 - 6*a^2 + 1)*arctan((b^2*x + a*b)/b)/b^5 - 6*(a^3 - a)*log(b^2*
x^2 + 2*a*b*x + a^2 + 1)/b^5)
```

mupad [B] time = 0.59, size = 133, normalized size = 1.25

$$\frac{x^4\,\mathrm{atan}\left(a+b\,x\right)}{4}-\frac{\mathrm{atan}\left(a+b\,x\right)}{4\,b^4}+\frac{x}{4\,b^3}-\frac{x^3}{12\,b}+\frac{a^3\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2\,b^4}+\frac{3\,a^2\,\mathrm{atan}\left(a+b\,x\right)}{2\,b^4}-\frac{a^4\,\mathrm{atan}\left(a+b\right.}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atan(a + b*x),x)
```

```
[Out] (x^4*atan(a + b*x))/4 - atan(a + b*x)/(4*b^4) + x/(4*b^3) - x^3/(12*b) + (a
^3*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/(2*b^4) + (3*a^2*atan(a + b*x))/(2*b^4
) - (a^4*atan(a + b*x))/(4*b^4) + (a*x^2)/(4*b^2) - (3*a^2*x)/(4*b^3) - (a*
log(a^2 + b^2*x^2 + 2*a*b*x + 1))/(2*b^4)
```

sympy [A] time = 1.58, size = 155, normalized size = 1.46

$$\begin{cases}-\frac{a^4\,\mathrm{atan}\,(a+bx)}{4b^4}+\frac{a^3\log\left(a^2+2abx+b^2x^2+1\right)}{2b^4}-\frac{3a^2x}{4b^3}+\frac{3a^2\,\mathrm{atan}\,(a+bx)}{2b^4}+\frac{ax^2}{4b^2}-\frac{a\log\left(a^2+2abx+b^2x^2+1\right)}{2b^4}+\frac{x^4\,\mathrm{atan}\,(a+bx)}{4}-\frac{x^3}{12b}+\frac{x}{4b^3} \\ \frac{x^4\,\mathrm{atan}\,(a)}{4}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(b*x+a),x)
```

```
[Out] Piecewise((-a**4*atan(a + b*x)/(4*b**4) + a**3*log(a**2 + 2*a*b*x + b**2*x*
*2 + 1)/(2*b**4) - 3*a**2*x/(4*b**3) + 3*a**2*atan(a + b*x)/(2*b**4) + a*x*
*2/(4*b**2) - a*log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(2*b**4) + x**4*atan(a
+ b*x)/4 - x**3/(12*b) + x/(4*b**3) - atan(a + b*x)/(4*b**4), Ne(b, 0)), (x
**4*atan(a)/4, True))
```

3.45 $\int x^2 \tan^{-1}(a + bx)\, dx$

Optimal. Leaf size=79

$$\frac{(1-3a^2)\log\left((a+bx)^2+1\right)}{6b^3}-\frac{a(3-a^2)\tan^{-1}(a+bx)}{3b^3}-\frac{(a+bx)^2}{6b^3}+\frac{ax}{b^2}+\frac{1}{3}x^3\tan^{-1}(a+bx)$$

```
[Out] a*x/b^2-1/6*(b*x+a)^2/b^3-1/3*a*(-a^2+3)*arctan(b*x+a)/b^3+1/3*x^3*arctan(b
*x+a)+1/6*(-3*a^2+1)*ln(1+(b*x+a)^2)/b^3
```

Rubi [A] time = 0.09, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5047, 4862, 702, 635, 203, 260}

$$\frac{(1-3a^2)\log\left((a+bx)^2+1\right)}{6b^3}-\frac{a(3-a^2)\tan^{-1}(a+bx)}{3b^3}+\frac{ax}{b^2}-\frac{(a+bx)^2}{6b^3}+\frac{1}{3}x^3\tan^{-1}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[a + b*x],x]
```

```
[Out] (a*x)/b^2 - (a + b*x)^2/(6*b^3) - (a*(3 - a^2)*ArcTan[a + b*x])/(3*b^3) + (
x^3*ArcTan[a + b*x])/3 + ((1 - 3*a^2)*Log[1 + (a + b*x)^2])/(6*b^3)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\int x^2\tan^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)^2\tan^{-1}(x)\,dx,x,a+bx\right)}{b}$$
$$=\frac{1}{3}x^3\tan^{-1}(a+bx)-\frac{1}{3}\text{Subst}\left(\int\frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}{1+x^2}\,dx,x,a+bx\right)$$
$$=\frac{1}{3}x^3\tan^{-1}(a+bx)-\frac{1}{3}\text{Subst}\left(\int\left(-\frac{3a}{b^3}+\frac{x}{b^3}+\frac{a\left(3-a^2\right)-\left(1-3a^2\right)x}{b^3\left(1+x^2\right)}\right)dx,x,a+bx\right)$$
$$=\frac{ax}{b^2}-\frac{(a+bx)^2}{6b^3}+\frac{1}{3}x^3\tan^{-1}(a+bx)-\frac{\text{Subst}\left(\int\frac{a\left(3-a^2\right)-\left(1-3a^2\right)x}{1+x^2}\,dx,x,a+bx\right)}{3b^3}$$
$$=\frac{ax}{b^2}-\frac{(a+bx)^2}{6b^3}+\frac{1}{3}x^3\tan^{-1}(a+bx)+\frac{\left(1-3a^2\right)\text{Subst}\left(\int\frac{x}{1+x^2}\,dx,x,a+bx\right)}{3b^3}-\frac{\left(a\left(3-a\right.\right.}{}$$
$$=\frac{ax}{b^2}-\frac{(a+bx)^2}{6b^3}-\frac{a\left(3-a^2\right)\tan^{-1}(a+bx)}{3b^3}+\frac{1}{3}x^3\tan^{-1}(a+bx)+\frac{\left(1-3a^2\right)\log\left(1+(a+\right.}{6b^3}$$

Mathematica [C] time = 0.06, size = 114, normalized size = 1.44

$$\frac{\frac{1}{3}b\left(\frac{a+bx}{b}-\frac{a}{b}\right)^3\tan^{-1}(a+bx)-\frac{1}{3}b\left(\frac{(a+bx)^2}{2b^3}-\frac{(1-ia)^3\log(a+bx+i)}{2b^3}-\frac{(1+ia)^3\log(-a-bx+i)}{2b^3}-\frac{3ax}{b^2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[a + b*x],x]
```

```
[Out] ((b*(-(a/b) + (a + b*x)/b)^3*ArcTan[a + b*x])/3 - (b*((-3*a*x)/b^2 + (a + b
*x)^2/(2*b^3) - ((1 + I*a)^3*Log[I - a - b*x])/(2*b^3) - ((1 - I*a)^3*Log[I
 + a + b*x])/(2*b^3)))/3)/b
```

fricas [A] time = 0.41, size = 66, normalized size = 0.84

$$-\frac{b^2x^2-4\,abx-2\left(b^3x^3+a^3-3\,a\right)\arctan\left(bx+a\right)+\left(3\,a^2-1\right)\log\left(b^2x^2+2\,abx+a^2+1\right)}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/6*(b^2*x^2 - 4*a*b*x - 2*(b^3*x^3 + a^3 - 3*a)*arctan(b*x + a) + (3*a^2
- 1)*log(b^2*x^2 + 2*a*b*x + a^2 + 1))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(b*x+a),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.04, size = 95, normalized size = 1.20

$$\frac{x^3\arctan\left(bx+a\right)}{3}+\frac{\arctan\left(bx+a\right)a^3}{3b^3}-\frac{x^2}{6b}+\frac{2ax}{3b^2}+\frac{5a^2}{6b^3}-\frac{\ln\left(1+(bx+a)^2\right)a^2}{2b^3}+\frac{\ln\left(1+(bx+a)^2\right)}{6b^3}-\frac{\arctan\left(bx+a\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(b*x+a),x)
```

```
[Out] 1/3*x^3*arctan(b*x+a)+1/3/b^3*arctan(b*x+a)*a^3-1/6/b*x^2+2/3*a*x/b^2+5/6/b
^3*a^2-1/2/b^3*ln(1+(b*x+a)^2)*a^2+1/6/b^3*ln(1+(b*x+a)^2)-1/b^3*arctan(b*x
+a)*a
```

maxima [A] time = 0.41, size = 85, normalized size = 1.08

$$\frac{1}{3}x^3 \arctan(bx+a) - \frac{1}{6}b\left(\frac{bx^2-4ax}{b^3} - \frac{2\left(a^3-3a\right)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^4} + \frac{\left(3a^2-1\right)\log\left(b^2x^2+2abx+a^2+1\right)}{b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(b*x + a) - 1/6*b*((b*x^2 - 4*a*x)/b^3 - 2*(a^3 - 3*a)*arctan
((b^2*x + a*b)/b)/b^4 + (3*a^2 - 1)*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^4)
```

mupad [B] time = 0.86, size = 102, normalized size = 1.29

$$\frac{\ln\left(a^2+2abx+b^2x^2+1\right)}{6b^3} + \frac{x^3\,\mathrm{atan}(a+bx)}{3} - \frac{x^2}{6b} - \frac{a^2\ln\left(a^2+2abx+b^2x^2+1\right)}{2b^3} + \frac{a^3\,\mathrm{atan}(a+bx)}{3b^3} - \frac{a\,\mathrm{atan}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a + b*x),x)
```

```
[Out] log(a^2 + b^2*x^2 + 2*a*b*x + 1)/(6*b^3) + (x^3*atan(a + b*x))/3 - x^2/(6*b
) - (a^2*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/(2*b^3) + (a^3*atan(a + b*x))/(3
*b^3) - (a*atan(a + b*x))/b^3 + (2*a*x)/(3*b^2)
```

sympy [A] time = 1.15, size = 117, normalized size = 1.48

$$\begin{cases} \frac{a^3\,\mathrm{atan}(a+bx)}{3b^3} - \frac{a^2\log\left(a^2+2abx+b^2x^2+1\right)}{2b^3} + \frac{2ax}{3b^2} - \frac{a\,\mathrm{atan}(a+bx)}{b^3} + \frac{x^3\,\mathrm{atan}(a+bx)}{3} - \frac{x^2}{6b} + \frac{\log\left(a^2+2abx+b^2x^2+1\right)}{6b^3} & \text{for } b \neq 0 \\ \frac{x^3\,\mathrm{atan}(a)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(b*x+a),x)
```

```
[Out] Piecewise((a**3*atan(a + b*x)/(3*b**3) - a**2*log(a**2 + 2*a*b*x + b**2*x**
2 + 1)/(2*b**3) + 2*a*x/(3*b**2) - a*atan(a + b*x)/b**3 + x**3*atan(a + b*x
)/3 - x**2/(6*b) + log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(6*b**3), Ne(b, 0)),
 (x**3*atan(a)/3, True))
```

3.46 $\int x \tan^{-1}(a + bx)\, dx$

Optimal. Leaf size=60

$$\frac{(1-a^2)\tan^{-1}(a+bx)}{2b^2}+\frac{a\log\left((a+bx)^2+1\right)}{2b^2}+\frac{1}{2}x^2\tan^{-1}(a+bx)-\frac{x}{2b}$$

```
[Out] -1/2*x/b+1/2*(-a^2+1)*arctan(b*x+a)/b^2+1/2*x^2*arctan(b*x+a)+1/2*a*ln(1+(b
*x+a)^2)/b^2
```

Rubi [A] time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5047, 4862, 702, 635, 203, 260}

$$\frac{(1-a^2)\tan^{-1}(a+bx)}{2b^2}+\frac{a\log\left((a+bx)^2+1\right)}{2b^2}+\frac{1}{2}x^2\tan^{-1}(a+bx)-\frac{x}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[a + b*x],x]
```

```
[Out] -x/(2*b) + ((1 - a^2)*ArcTan[a + b*x])/(2*b^2) + (x^2*ArcTan[a + b*x])/2 +
(a*Log[1 + (a + b*x)^2])/(2*b^2)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\int x\tan^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)\tan^{-1}(x)\,dx, x, a+bx\right)}{b}$$
$$= \frac{1}{2}x^2\tan^{-1}(a+bx) - \frac{1}{2}\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}{1+x^2}\,dx, x, a+bx\right)$$
$$= \frac{1}{2}x^2\tan^{-1}(a+bx) - \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{b^2} - \frac{1-a^2+2ax}{b^2\left(1+x^2\right)}\right)dx, x, a+bx\right)$$
$$= -\frac{x}{2b} + \frac{1}{2}x^2\tan^{-1}(a+bx) + \frac{\text{Subst}\left(\int \frac{1-a^2+2ax}{1+x^2}\,dx, x, a+bx\right)}{2b^2}$$
$$= -\frac{x}{2b} + \frac{1}{2}x^2\tan^{-1}(a+bx) + \frac{a\,\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, a+bx\right)}{b^2} + \frac{\left(1-a^2\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx\right.}{2b^2}$$
$$= -\frac{x}{2b} + \frac{\left(1-a^2\right)\tan^{-1}(a+bx)}{2b^2} + \frac{1}{2}x^2\tan^{-1}(a+bx) + \frac{a\log\left(1+(a+bx)^2\right)}{2b^2}$$

Mathematica [C] time = 0.04, size = 90, normalized size = 1.50

$$\frac{-ia^2\log(a+bx+i)+2b^2x^2\tan^{-1}(a+bx)+2a\log(a+bx+i)+i(a-i)^2\log(-a-bx+i)+i\log(a+bx+i)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[a + b*x],x]
```

```
[Out] (-2*b*x + 2*b^2*x^2*ArcTan[a + b*x] + I*(-I + a)^2*Log[I - a - b*x] + I*Log
[I + a + b*x] + 2*a*Log[I + a + b*x] - I*a^2*Log[I + a + b*x])/(4*b^2)
```

fricas [A] time = 0.41, size = 52, normalized size = 0.87

$$-\frac{bx - \left(b^2x^2 - a^2 + 1\right)\arctan\left(bx+a\right) - a\log\left(b^2x^2 + 2\,abx + a^2 + 1\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x - (b^2*x^2 - a^2 + 1)*arctan(b*x + a) - a*log(b^2*x^2 + 2*a*b*x +
 a^2 + 1))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(b*x+a),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.04, size = 66, normalized size = 1.10

$$\frac{x^2\arctan\left(bx+a\right)}{2} - \frac{\arctan\left(bx+a\right)a^2}{2b^2} - \frac{x}{2b} - \frac{a}{2b^2} + \frac{a\ln\left(1+\left(bx+a\right)^2\right)}{2b^2} + \frac{\arctan\left(bx+a\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(b*x+a),x)
```

```
[Out] 1/2*x^2*arctan(b*x+a)-1/2/b^2*arctan(b*x+a)*a^2-1/2*x/b-1/2/b^2*a+1/2*a*ln(
1+(b*x+a)^2)/b^2+1/2/b^2*arctan(b*x+a)
```

maxima [A] time = 0.41, size = 68, normalized size = 1.13

$$\frac{1}{2}x^2\arctan(bx+a) - \frac{1}{2}b\left(\frac{x}{b^2} + \frac{(a^2-1)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^3} - \frac{a\log\left(b^2x^2+2abx+a^2+1\right)}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(b*x + a) - 1/2*b*(x/b^2 + (a^2 - 1)*arctan((b^2*x + a*b)/b)/
b^3 - a*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^3)
```

mupad [B] time = 0.97, size = 61, normalized size = 1.02

$$\frac{x^2\,\mathrm{atan}\left(a+b\,x\right)}{2} + \frac{\frac{\mathrm{atan}(a+b\,x)}{2} - \frac{b\,x}{2} - \frac{a^2\,\mathrm{atan}(a+b\,x)}{2} + \frac{a\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a + b*x),x)
```

```
[Out] (x^2*atan(a + b*x))/2 + (atan(a + b*x)/2 - (b*x)/2 - (a^2*atan(a + b*x))/2
+ (a*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/2)/b^2
```

sympy [A] time = 0.64, size = 78, normalized size = 1.30

$$\begin{cases} -\frac{a^2\,\mathrm{atan}\,(a+bx)}{2b^2} + \frac{a\log\left(a^2+2abx+b^2x^2+1\right)}{2b^2} + \frac{x^2\,\mathrm{atan}\,(a+bx)}{2} - \frac{x}{2b} + \frac{\mathrm{atan}\,(a+bx)}{2b^2} & \text{for } b \neq 0 \\ \frac{x^2\,\mathrm{atan}\,(a)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(b*x+a),x)
```

```
[Out] Piecewise((-a**2*atan(a + b*x)/(2*b**2) + a*log(a**2 + 2*a*b*x + b**2*x**2
+ 1)/(2*b**2) + x**2*atan(a + b*x)/2 - x/(2*b) + atan(a + b*x)/(2*b**2), Ne
(b, 0)), (x**2*atan(a)/2, True))
```

3.47 $\int \tan^{-1}(a+bx)\,dx$

Optimal. Leaf size=33

$$\frac{(a+bx)\tan^{-1}(a+bx)}{b}-\frac{\log\left((a+bx)^2+1\right)}{2b}$$

```
[Out] (b*x+a)*arctan(b*x+a)/b-1/2*ln(1+(b*x+a)^2)/b
```

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5039, 4846, 260}

$$\frac{(a+bx)\tan^{-1}(a+bx)}{b}-\frac{\log\left((a+bx)^2+1\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x],x]
```

```
[Out] ((a + b*x)*ArcTan[a + b*x])/b - Log[1 + (a + b*x)^2]/(2*b)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5039

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \tan^{-1}(a+bx)\,dx &= \frac{\text{Subst}\left(\int \tan^{-1}(x)\,dx,x,a+bx\right)}{b}\\ &= \frac{(a+bx)\tan^{-1}(a+bx)}{b}-\frac{\text{Subst}\left(\int \frac{x}{1+x^2}\,dx,x,a+bx\right)}{b}\\ &= \frac{(a+bx)\tan^{-1}(a+bx)}{b}-\frac{\log\left(1+(a+bx)^2\right)}{2b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 1.18

$$-\frac{\log\left(a^2+2abx+b^2x^2+1\right)-2(a+bx)\tan^{-1}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x],x]
```

```
[Out] -1/2*(-2*(a + b*x)*ArcTan[a + b*x] + Log[1 + a^2 + 2*a*b*x + b^2*x^2])/b
```

fricas [A] time = 0.40, size = 39, normalized size = 1.18

$$\frac{2\,(bx+a)\arctan\left(bx+a\right)-\log\left(b^2x^2+2\,abx+a^2+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(b*x + a)*arctan(b*x + a) - log(b^2*x^2 + 2*a*b*x + a^2 + 1))/b
```

giac [A] time = 0.13, size = 31, normalized size = 0.94

$$\frac{2\,(bx+a)\arctan\left(bx+a\right)-\log\left((bx+a)^2+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(2*(b*x + a)*arctan(b*x + a) - log((b*x + a)^2 + 1))/b
```

maple [A] time = 0.04, size = 36, normalized size = 1.09

$$x\arctan\left(bx+a\right)+\frac{\arctan\left(bx+a\right)a}{b}-\frac{\ln\left(1+(bx+a)^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a),x)
```

```
[Out] x*arctan(b*x+a)+1/b*arctan(b*x+a)*a-1/2*ln(1+(b*x+a)^2)/b
```

maxima [A] time = 0.31, size = 31, normalized size = 0.94

$$\frac{2\,(bx+a)\arctan\left(bx+a\right)-\log\left((bx+a)^2+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(b*x + a)*arctan(b*x + a) - log((b*x + a)^2 + 1))/b
```

mupad [B] time = 0.45, size = 42, normalized size = 1.27

$$x\,\mathrm{atan}\left(a+b\,x\right)-\frac{\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)-2\,a\,\mathrm{atan}\left(a+b\,x\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x),x)
```

```
[Out] x*atan(a + b*x) - (log(a^2 + b^2*x^2 + 2*a*b*x + 1) - 2*a*atan(a + b*x))/(2
*b)
```

sympy [A] time = 0.39, size = 46, normalized size = 1.39

$$\begin{cases} \frac{a\,\mathrm{atan}\,(a+bx)}{b}+x\,\mathrm{atan}\,(a+bx)-\frac{\log\left(a^2+2abx+b^2x^2+1\right)}{2b} & \text{for}\ b\neq 0 \\ x\,\mathrm{atan}\,(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a),x)
```

```
[Out] Piecewise((a*atan(a + b*x)/b + x*atan(a + b*x) - log(a**2 + 2*a*b*x + b**2*
x**2 + 1)/(2*b), Ne(b, 0)), (x*atan(a), True))
```

3.48 $\int \frac{\tan^{-1}(a+bx)}{x}\,dx$

Optimal. Leaf size=120

$$\frac{1}{2}i\text{Li}_2\left(1-\frac{2}{1-i(a+bx)}\right)-\frac{1}{2}i\text{Li}_2\left(1-\frac{2bx}{(i-a)(1-i(a+bx))}\right)+\log\left(\frac{2}{1-i(a+bx)}\right)\left(-\tan^{-1}(a+bx)\right)+\log\left(\frac{}{(-a+i)}\right.$$

```
[Out] -arctan(b*x+a)*ln(2/(1-I*(b*x+a)))+arctan(b*x+a)*ln(2*b*x/(I-a)/(1-I*(b*x+a
)))+1/2*I*polylog(2,1-2/(1-I*(b*x+a)))-1/2*I*polylog(2,1-2*b*x/(I-a)/(1-I*(
b*x+a)))
```

Rubi [A] time = 0.11, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5047, 4856, 2402, 2315, 2447}

$$\frac{1}{2}i\text{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)-\frac{1}{2}i\text{PolyLog}\left(2,1-\frac{2bx}{(-a+i)(1-i(a+bx))}\right)+\log\left(\frac{2}{1-i(a+bx)}\right)\left(-\tan^{-1}(a+b\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/x,x]
```

```
[Out] -(ArcTan[a + b*x]*Log[2/(1 - I*(a + b*x))]) + ArcTan[a + b*x]*Log[(2*b*x)/(
(I - a)*(1 - I*(a + b*x)))] + (I/2)*PolyLog[2, 1 - 2/(1 - I*(a + b*x))] - (
I/2)*PolyLog[2, 1 - (2*b*x)/((I - a)*(1 - I*(a + b*x)))]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\tan^{-1}(x)}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bx\right)}{b}$$

$$= -\tan^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right) + \tan^{-1}(a+bx)\log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right) + \text{Subs}$$

$$= -\tan^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right) + \tan^{-1}(a+bx)\log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right) - \frac{1}{2}i\text{Li}_2$$

$$= -\tan^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right) + \tan^{-1}(a+bx)\log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right) + \frac{1}{2}i\text{Li}_2$$

Mathematica [A] time = 0.01, size = 171, normalized size = 1.42

$$\frac{1}{2}i\text{Li}_2\left(\frac{i(1-i(a+bx))}{a+i}\right)-\frac{1}{2}i\text{Li}_2\left(-\frac{i(i(a+bx)+1)}{a-i}\right)-\frac{1}{2}i\log(1+i(a+bx))\log\left(\frac{i\left(\frac{a+bx}{b}-\frac{a}{b}\right)}{-\frac{1}{b}-\frac{ia}{b}}\right)+\frac{1}{2}i\log(1-i(a+bx))\,\mathrm{l}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a + b*x]/x,x]
```

```
[Out] (-1/2*I)*Log[1 + I*(a + b*x)]*Log[(I*(-(a/b) + (a + b*x)/b))/(-b^(-1) - (I*
a)/b)] + (I/2)*Log[1 - I*(a + b*x)]*Log[((-I)*(-(a/b) + (a + b*x)/b))/(-b^(
-1) + (I*a)/b)] + (I/2)*PolyLog[2, (I*(1 - I*(a + b*x)))/(I + a)] - (I/2)*P
olyLog[2, ((-I)*(1 + I*(a + b*x)))/(-I + a)]
```

fricas [F] time = 0.39, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\arctan(bx+a)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(arctan(b*x + a)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.06, size = 103, normalized size = 0.86

$$\ln(bx)\arctan(bx+a)+\frac{i\ln(bx)\ln\left(\frac{-bx-a+i}{i-a}\right)}{2}-\frac{i\ln(bx)\ln\left(\frac{bx+a+i}{i+a}\right)}{2}+\frac{i\,\text{dilog}\left(\frac{-bx-a+i}{i-a}\right)}{2}-\frac{i\,\text{dilog}\left(\frac{bx+a+i}{i+a}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/x,x)
```

```
[Out] ln(b*x)*arctan(b*x+a)+1/2*I*ln(b*x)*ln((I-a-b*x)/(I-a))-1/2*I*ln(b*x)*ln((I
+a+b*x)/(I+a))+1/2*I*dilog((I-a-b*x)/(I-a))-1/2*I*dilog((I+a+b*x)/(I+a))
```

maxima [A] time = 0.47, size = 134, normalized size = 1.12

$$-\frac{1}{2}\arctan\left(\frac{bx}{a^2+1}, -\frac{abx}{a^2+1}\right)\log\left(b^2x^2+2\,abx+a^2+1\right)+\frac{1}{2}\arctan\left(bx+a\right)\log\left(\frac{b^2x^2}{a^2+1}\right)+\arctan\left(bx+a\right)\log(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] -1/2*arctan2(b*x/(a^2 + 1), -a*b*x/(a^2 + 1))*log(b^2*x^2 + 2*a*b*x + a^2 +
 1) + 1/2*arctan(b*x + a)*log(b^2*x^2/(a^2 + 1)) + arctan(b*x + a)*log(x) -
 arctan((b^2*x + a*b)/b)*log(x) - 1/2*I*dilog((I*b*x + I*a + 1)/(I*a + 1))
+ 1/2*I*dilog((I*b*x + I*a - 1)/(I*a - 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atan}\left(a+b\,x\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)/x,x)
```

```
[Out] int(atan(a + b*x)/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/x,x)
```

```
[Out] Timed out
```

3.49 $\int \frac{\tan^{-1}(a+bx)}{x^2}\,dx$

Optimal. Leaf size=62

$$\frac{b\log(x)}{a^2+1}-\frac{b\log\left((a+bx)^2+1\right)}{2\left(a^2+1\right)}-\frac{ab\tan^{-1}(a+bx)}{a^2+1}-\frac{\tan^{-1}(a+bx)}{x}$$

```
[Out] -a*b*arctan(b*x+a)/(a^2+1)-arctan(b*x+a)/x+b*ln(x)/(a^2+1)-1/2*b*ln(1+(b*x+
a)^2)/(a^2+1)
```

Rubi [A] time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5045, 371, 706, 31, 635, 203, 260}

$$\frac{b\log(x)}{a^2+1}-\frac{b\log\left((a+bx)^2+1\right)}{2\left(a^2+1\right)}-\frac{ab\tan^{-1}(a+bx)}{a^2+1}-\frac{\tan^{-1}(a+bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/x^2,x]
```

```
[Out] -((a*b*ArcTan[a + b*x])/(1 + a^2)) - ArcTan[a + b*x]/x + (b*Log[x])/(1 + a^
2) - (b*Log[1 + (a + b*x)^2])/(2*(1 + a^2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
```

```
0]
```

Rule 5045

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[((e + f*x)^(m + 1)*(a + b*ArcTan[c + d*x])^p)/(f*(m +
 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcTan[c
 + d*x])^(p - 1))/(1 + (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(a+bx)}{x^2}\,dx &= -\frac{\tan^{-1}(a+bx)}{x}+b\int \frac{1}{x\left(1+(a+bx)^2\right)}\,dx\\ &= -\frac{\tan^{-1}(a+bx)}{x}+b\,\text{Subst}\left(\int \frac{1}{(-a+x)\left(1+x^2\right)}\,dx,x,a+bx\right)\\ &= -\frac{\tan^{-1}(a+bx)}{x}+\frac{b\,\text{Subst}\left(\int \frac{1}{-a+x}\,dx,x,a+bx\right)}{1+a^2}+\frac{b\,\text{Subst}\left(\int \frac{-a-x}{1+x^2}\,dx,x,a+bx\right)}{1+a^2}\\ &= -\frac{\tan^{-1}(a+bx)}{x}+\frac{b\log(x)}{1+a^2}-\frac{b\,\text{Subst}\left(\int \frac{x}{1+x^2}\,dx,x,a+bx\right)}{1+a^2}-\frac{(ab)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,a+\right.}{1+a^2}\\ &= -\frac{ab\tan^{-1}(a+bx)}{1+a^2}-\frac{\tan^{-1}(a+bx)}{x}+\frac{b\log(x)}{1+a^2}-\frac{b\log\left(1+(a+bx)^2\right)}{2\left(1+a^2\right)}\end{aligned}$$

Mathematica [C] time = 0.06, size = 67, normalized size = 1.08

$$-\frac{\tan^{-1}(a+bx)}{x}+\frac{b(i(a+i)\log(-a-bx+i)+(-1-ia)\log(a+bx+i)+2\log(x))}{2\left(a^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x]/x^2,x]
```

```
[Out] -(ArcTan[a + b*x]/x) + (b*(2*Log[x] + I*(I + a)*Log[I - a - b*x] + (-1 - I*
a)*Log[I + a + b*x]))/(2*(1 + a^2))
```

fricas [A] time = 0.41, size = 57, normalized size = 0.92

$$-\frac{bx\log\left(b^2x^2+2\,abx+a^2+1\right)-2\,bx\log(x)+2\left(abx+a^2+1\right)\arctan(bx+a)}{2\left(a^2+1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b*x*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*b*x*log(x) + 2*(a*b*x + a^2
+ 1)*arctan(b*x + a))/((a^2 + 1)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.05, size = 63, normalized size = 1.02

$$-\frac{\arctan\left(bx+a\right)}{x}+\frac{b\ln\left(bx\right)}{a^2+1}-\frac{b\ln\left(1+\left(bx+a\right)^2\right)}{2\left(a^2+1\right)}-\frac{ab\arctan\left(bx+a\right)}{a^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/x^2,x)
```

```
[Out] -arctan(b*x+a)/x+b/(a^2+1)*ln(b*x)-1/2*b*ln(1+(b*x+a)^2)/(a^2+1)-a*b*arctan
(b*x+a)/(a^2+1)
```

maxima [A] time = 0.41, size = 77, normalized size = 1.24

$$-\frac{1}{2}b\left(\frac{2\,a\arctan\left(\frac{b^2x+ab}{b}\right)}{a^2+1}+\frac{\log\left(b^2x^2+2\,abx+a^2+1\right)}{a^2+1}-\frac{2\,\log(x)}{a^2+1}\right)-\frac{\arctan\left(bx+a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*b*(2*a*arctan((b^2*x + a*b)/b)/(a^2 + 1) + log(b^2*x^2 + 2*a*b*x + a^2
 + 1)/(a^2 + 1) - 2*log(x)/(a^2 + 1)) - arctan(b*x + a)/x
```

mupad [B] time = 1.04, size = 63, normalized size = 1.02

$$-\frac{\mathrm{atan}\left(a+b\,x\right)}{x}-\frac{\frac{b\,x\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2}-b\,x\,\ln(x)+a\,b\,x\,\mathrm{atan}\left(a+b\,x\right)}{x\,\left(a^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)/x^2,x)
```

```
[Out] - atan(a + b*x)/x - ((b*x*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/2 - b*x*log(x)
+ a*b*x*atan(a + b*x))/(x*(a^2 + 1))
```

sympy [B] time = 1.89, size = 168, normalized size = 2.71

$$\begin{cases} -\frac{ib\,\mathrm{atan}\left(bx-i\right)}{2}-\frac{\mathrm{atan}\left(bx-i\right)}{x}-\frac{i}{2x} & \text{for}\: a=-i \\ \frac{ib\,\mathrm{atan}\left(bx+i\right)}{2}-\frac{\mathrm{atan}\left(bx+i\right)}{x}+\frac{i}{2x} & \text{for}\: a=i \\ -\frac{2a^2\,\mathrm{atan}\left(a+bx\right)}{2a^2x+2x}-\frac{2abx\,\mathrm{atan}\left(a+bx\right)}{2a^2x+2x}+\frac{2bx\log\left(x\right)}{2a^2x+2x}-\frac{bx\log\left(a^2+2abx+b^2x^2+1\right)}{2a^2x+2x}-\frac{2\,\mathrm{atan}\left(a+bx\right)}{2a^2x+2x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/x**2,x)
```

```
[Out] Piecewise((-I*b*atan(b*x - I)/2 - atan(b*x - I)/x - I/(2*x), Eq(a, -I)), (I
*b*atan(b*x + I)/2 - atan(b*x + I)/x + I/(2*x), Eq(a, I)), (-2*a**2*atan(a
+ b*x)/(2*a**2*x + 2*x) - 2*a*b*x*atan(a + b*x)/(2*a**2*x + 2*x) + 2*b*x*lo
g(x)/(2*a**2*x + 2*x) - b*x*log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(2*a**2*x +
 2*x) - 2*atan(a + b*x)/(2*a**2*x + 2*x), True))
```

3.50 $\int \frac{\tan^{-1}(a+bx)}{x^3} \, dx$

Optimal. Leaf size=96

$$-\frac{ab^2 \log(x)}{\left(a^2+1\right)^2}+\frac{ab^2 \log\left((a+bx)^2+1\right)}{2\left(a^2+1\right)^2}-\frac{\left(1-a^2\right)b^2 \tan^{-1}(a+bx)}{2\left(a^2+1\right)^2}-\frac{b}{2\left(a^2+1\right)x}-\frac{\tan^{-1}(a+bx)}{2x^2}$$

```
[Out] -1/2*b/(a^2+1)/x-1/2*(-a^2+1)*b^2*arctan(b*x+a)/(a^2+1)^2-1/2*arctan(b*x+a)
/x^2-a*b^2*ln(x)/(a^2+1)^2+1/2*a*b^2*ln(1+(b*x+a)^2)/(a^2+1)^2
```

Rubi [A] time = 0.08, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5045, 371, 710, 801, 635, 203, 260}

$$-\frac{ab^2 \log(x)}{\left(a^2+1\right)^2}+\frac{ab^2 \log\left((a+bx)^2+1\right)}{2\left(a^2+1\right)^2}-\frac{\left(1-a^2\right)b^2 \tan^{-1}(a+bx)}{2\left(a^2+1\right)^2}-\frac{b}{2\left(a^2+1\right)x}-\frac{\tan^{-1}(a+bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/x^3,x]
```

```
[Out] -b/(2*(1 + a^2)*x) - ((1 - a^2)*b^2*ArcTan[a + b*x])/(2*(1 + a^2)^2) - ArcT
an[a + b*x]/(2*x^2) - (a*b^2*Log[x])/(1 + a^2)^2 + (a*b^2*Log[1 + (a + b*x)
^2])/(2*(1 + a^2)^2)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 5045

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[((e + f*x)^(m + 1)*(a + b*ArcTan[c + d*x])^p)/(f*(m +
 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcTan[c
 + d*x])^(p - 1))/(1 + (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(a+bx)}{x^3}\,dx &= -\frac{\tan^{-1}(a+bx)}{2x^2}+\frac{1}{2}b\int \frac{1}{x^2\left(1+(a+bx)^2\right)}\,dx\\
&= -\frac{\tan^{-1}(a+bx)}{2x^2}+\frac{1}{2}b^2\,\mathrm{Subst}\left(\int \frac{1}{(-a+x)^2\left(1+x^2\right)}\,dx,x,a+bx\right)\\
&= -\frac{b}{2\left(1+a^2\right)x}-\frac{\tan^{-1}(a+bx)}{2x^2}+\frac{b^2\,\mathrm{Subst}\left(\int \frac{-a-x}{(-a+x)\left(1+x^2\right)}\,dx,x,a+bx\right)}{2\left(1+a^2\right)}\\
&= -\frac{b}{2\left(1+a^2\right)x}-\frac{\tan^{-1}(a+bx)}{2x^2}+\frac{b^2\,\mathrm{Subst}\left(\int \left(\frac{2a}{\left(1+a^2\right)(a-x)}+\frac{-1+a^2+2ax}{\left(1+a^2\right)\left(1+x^2\right)}\right)dx,x,a+bx\right)}{2\left(1+a^2\right)}\\
&= -\frac{b}{2\left(1+a^2\right)x}-\frac{\tan^{-1}(a+bx)}{2x^2}-\frac{ab^2\log(x)}{\left(1+a^2\right)^2}+\frac{b^2\,\mathrm{Subst}\left(\int \frac{-1+a^2+2ax}{1+x^2}\,dx,x,a+bx\right)}{2\left(1+a^2\right)^2}\\
&= -\frac{b}{2\left(1+a^2\right)x}-\frac{\tan^{-1}(a+bx)}{2x^2}-\frac{ab^2\log(x)}{\left(1+a^2\right)^2}+\frac{\left(ab^2\right)\mathrm{Subst}\left(\int \frac{x}{1+x^2}\,dx,x,a+bx\right)}{\left(1+a^2\right)^2}-\frac{((1
\end{aligned}$$

$$= -\frac{b}{2\left(1+a^2\right)x}-\frac{\left(1-a^2\right)b^2\tan^{-1}(a+bx)}{2\left(1+a^2\right)^2}-\frac{\tan^{-1}(a+bx)}{2x^2}-\frac{ab^2\log(x)}{\left(1+a^2\right)^2}+\frac{ab^2\log\left(1+\right.}{2\left(1+\right.}$$

Mathematica [C] time = 0.11, size = 92, normalized size = 0.96

$$\frac{-2\tan^{-1}(a+bx)+\frac{bx\left(-i(a+i)^2bx\log(-a-bx+i)-4abx\log(x)+(a-i)((1+ia)bx\log(a+bx+i)-2(a+i))\right)}{\left(a^2+1\right)^2}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x]/x^3,x]
```

```
[Out] (-2*ArcTan[a + b*x] + (b*x*(-4*a*b*x*Log[x] - I*(I + a)^2*b*x*Log[I - a - b
*x] + (-I + a)*(-2*(I + a) + (1 + I*a)*b*x*Log[I + a + b*x])))/(1 + a^2)^2)
/(4*x^2)
```

fricas [A] time = 0.50, size = 95, normalized size = 0.99

$$\frac{ab^2x^2\log\left(b^2x^2+2\,abx+a^2+1\right)-2\,ab^2x^2\log(x)-\left(a^2+1\right)bx+\left(\left(a^2-1\right)b^2x^2-a^4-2\,a^2-1\right)\arctan\left(bx+a\right.}{2\left(a^4+2\,a^2+1\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a*b^2*x^2*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*a*b^2*x^2*log(x) - (a^2
 + 1)*b*x + ((a^2 - 1)*b^2*x^2 - a^4 - 2*a^2 - 1)*arctan(b*x + a))/((a^4 +
2*a^2 + 1)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.05, size = 105, normalized size = 1.09

$$-\frac{\arctan(bx+a)}{2x^2}-\frac{b}{2\left(a^2+1\right)x}-\frac{b^2a\ln(bx)}{\left(a^2+1\right)^2}+\frac{b^2\arctan(bx+a)\,a^2}{2\left(a^2+1\right)^2}+\frac{a\,b^2\ln\left(1+(bx+a)^2\right)}{2\left(a^2+1\right)^2}-\frac{b^2\arctan(bx+a)}{2\left(a^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/x^3,x)
```

```
[Out] -1/2*arctan(b*x+a)/x^2-1/2*b/(a^2+1)/x-b^2/(a^2+1)^2*a*ln(b*x)+1/2*b^2/(a^2
+1)^2*arctan(b*x+a)*a^2+1/2*a*b^2*ln(1+(b*x+a)^2)/(a^2+1)^2-1/2*b^2/(a^2+1)
^2*arctan(b*x+a)
```

maxima [A] time = 0.41, size = 112, normalized size = 1.17

$$\frac{1}{2}\left(\frac{\left(a^2-1\right)b\arctan\left(\frac{b^2x+ab}{b}\right)}{a^4+2\,a^2+1}+\frac{ab\log\left(b^2x^2+2\,abx+a^2+1\right)}{a^4+2\,a^2+1}-\frac{2\,ab\log(x)}{a^4+2\,a^2+1}-\frac{1}{\left(a^2+1\right)x}\right)b-\frac{\arctan(bx+a)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2 - 1)*b*arctan((b^2*x + a*b)/b)/(a^4 + 2*a^2 + 1) + a*b*log(b^2*x^
2 + 2*a*b*x + a^2 + 1)/(a^4 + 2*a^2 + 1) - 2*a*b*log(x)/(a^4 + 2*a^2 + 1) -
 1/((a^2 + 1)*x))*b - 1/2*arctan(b*x + a)/x^2
```

mupad [B] time = 1.22, size = 232, normalized size = 2.42

$$\frac{a\,b^2\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2\left(a^2+1\right)^2}-\frac{\frac{b\,x}{2}+\mathrm{atan}\left(a+b\,x\right)\left(\frac{a^2}{2}+\frac{1}{2}\right)+\frac{b^2\,x^2\,\mathrm{atan}(a+b\,x)}{2}+\frac{x^3\left(b^3-3\,a^2\,b^3\right)}{2\left(a^4+2\,a^2+1\right)}-\frac{a\,b^4\,x^4}{\left(a^2+1\right)^2}+a\,b\,x\,\mathrm{ata}}{a^2\,x^2+2\,a\,b\,x^3+b^2\,x^4+x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)/x^3,x)
```

```
[Out] (a*b^2*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/(2*(a^2 + 1)^2) - ((b*x)/2 + atan(
a + b*x)*(a^2/2 + 1/2) + (b^2*x^2*atan(a + b*x))/2 + (x^3*(b^3 - 3*a^2*b^3)
)/(2*(2*a^2 + a^4 + 1)) - (a*b^4*x^4)/(a^2 + 1)^2 + a*b*x*atan(a + b*x))/(x
^2 + a^2*x^2 + b^2*x^4 + 2*a*b*x^3) - (atan((2*a*b + 2*b^2*x)/(2*(b^2*(a^2
+ 1) - a^2*b^2)^(1/2)))*(b^3 - a^2*b^3))/((b^2)^(1/2)*(4*a^2 + 2*a^4 + 2))
- (a*b^2*log(x))/(a^2 + 1)^2
```

sympy [B] time = 2.92, size = 382, normalized size = 3.98

$$\begin{cases} -\frac{b^2 \operatorname{atan}(bx-i)}{8} - \frac{b}{8x} - \frac{\operatorname{atan}(bx-i)}{2x^2} - \frac{i}{8x^2} \\ -\frac{b^2 \operatorname{atan}(bx+i)}{8} - \frac{b}{8x} - \frac{\operatorname{atan}(bx+i)}{2x^2} + \frac{i}{8x^2} \\ -\frac{a^4 \operatorname{atan}(a+bx)}{2a^4x^2+4a^2x^2+2x^2} + \frac{a^2b^2x^2 \operatorname{atan}(a+bx)}{2a^4x^2+4a^2x^2+2x^2} - \frac{a^2bx}{2a^4x^2+4a^2x^2+2x^2} - \frac{2a^2 \operatorname{atan}(a+bx)}{2a^4x^2+4a^2x^2+2x^2} - \frac{2ab^2x^2 \log(x)}{2a^4x^2+4a^2x^2+2x^2} + \frac{ab^2x^2 \log\left(a^2+2abx+b^2x^2+1\right)}{2a^4x^2+4a^2x^2+2x^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/x**3,x)
```

```
[Out] Piecewise((-b**2*atan(b*x - I)/8 - b/(8*x) - atan(b*x - I)/(2*x**2) - I/(8*
x**2), Eq(a, -I)), (-b**2*atan(b*x + I)/8 - b/(8*x) - atan(b*x + I)/(2*x**2
) + I/(8*x**2), Eq(a, I)), (-a**4*atan(a + b*x)/(2*a**4*x**2 + 4*a**2*x**2
+ 2*x**2) + a**2*b**2*x**2*atan(a + b*x)/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**
2) - a**2*b*x/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**2) - 2*a**2*atan(a + b*x)/(
2*a**4*x**2 + 4*a**2*x**2 + 2*x**2) - 2*a*b**2*x**2*log(x)/(2*a**4*x**2 + 4
*a**2*x**2 + 2*x**2) + a*b**2*x**2*log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(2*a
**4*x**2 + 4*a**2*x**2 + 2*x**2) - b**2*x**2*atan(a + b*x)/(2*a**4*x**2 + 4
*a**2*x**2 + 2*x**2) - b*x/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**2) - atan(a +
b*x)/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**2), True))
```

3.51 $\int \frac{\tan^{-1}(a+bx)}{x^4} dx$

Optimal. Leaf size=129

$$-\frac{(1-3a^2)\, b^3 \log(x)}{3(a^2+1)^3}+\frac{(1-3a^2)\, b^3 \log\left((a+bx)^2+1\right)}{6(a^2+1)^3}+\frac{a(3-a^2)\, b^3 \tan^{-1}(a+bx)}{3(a^2+1)^3}+\frac{2ab^2}{3(a^2+1)^2 x}-\frac{b}{6(a^2+1)x^2}-$$

```
[Out] -1/6*b/(a^2+1)/x^2+2/3*a*b^2/(a^2+1)^2/x+1/3*a*(-a^2+3)*b^3*arctan(b*x+a)/(
a^2+1)^3-1/3*arctan(b*x+a)/x^3-1/3*(-3*a^2+1)*b^3*ln(x)/(a^2+1)^3+1/6*(-3*a
^2+1)*b^3*ln(1+(b*x+a)^2)/(a^2+1)^3
```

Rubi [A] time = 0.11, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5045, 371, 710, 801, 635, 203, 260}

$$\frac{2ab^2}{3(a^2+1)^2 x}-\frac{(1-3a^2)\, b^3 \log(x)}{3(a^2+1)^3}+\frac{(1-3a^2)\, b^3 \log\left((a+bx)^2+1\right)}{6(a^2+1)^3}+\frac{a(3-a^2)\, b^3 \tan^{-1}(a+bx)}{3(a^2+1)^3}-\frac{b}{6(a^2+1)x^2}-\frac{\text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/x^4,x]
```

```
[Out] -b/(6*(1 + a^2)*x^2) + (2*a*b^2)/(3*(1 + a^2)^2*x) + (a*(3 - a^2)*b^3*ArcTa
n[a + b*x])/(3*(1 + a^2)^3) - ArcTan[a + b*x]/(3*x^3) - ((1 - 3*a^2)*b^3*Lo
g[x])/(3*(1 + a^2)^3) + ((1 - 3*a^2)*b^3*Log[1 + (a + b*x)^2])/(6*(1 + a^2)
^3)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 5045

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[((e + f*x)^(m + 1)*(a + b*ArcTan[c + d*x])^p)/(f*(m +
 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcTan[c
 + d*x])^(p - 1))/(1 + (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{x^4}\,dx = -\frac{\tan^{-1}(a+bx)}{3x^3}+\frac{1}{3}b\int \frac{1}{x^3\left(1+(a+bx)^2\right)}\,dx$$

$$= -\frac{\tan^{-1}(a+bx)}{3x^3}+\frac{1}{3}b^3\,\text{Subst}\left(\int \frac{1}{(-a+x)^3\left(1+x^2\right)}\,dx,x,a+bx\right)$$

$$= -\frac{b}{6\left(1+a^2\right)x^2}-\frac{\tan^{-1}(a+bx)}{3x^3}+\frac{b^3\,\text{Subst}\left(\int \frac{-a-x}{(-a+x)^2\left(1+x^2\right)}\,dx,x,a+bx\right)}{3\left(1+a^2\right)}$$

$$= -\frac{b}{6\left(1+a^2\right)x^2}-\frac{\tan^{-1}(a+bx)}{3x^3}+\frac{b^3\,\text{Subst}\left(\int \left(-\frac{2a}{\left(1+a^2\right)(a-x)^2}+\frac{1-3a^2}{\left(1+a^2\right)^2(a-x)}+\frac{a\left(3-a^2\right)+\left(1-3\right.}{\left(1+a^2\right)^2\left(1+\right.}\right.\right.}{3\left(1+a^2\right)}$$

$$= -\frac{b}{6\left(1+a^2\right)x^2}+\frac{2ab^2}{3\left(1+a^2\right)^2x}-\frac{\tan^{-1}(a+bx)}{3x^3}-\frac{\left(1-3a^2\right)b^3\log(x)}{3\left(1+a^2\right)^3}+\frac{b^3\,\text{Subst}\left(\int \frac{a(3}{}\right.}{}$$

$$= -\frac{b}{6\left(1+a^2\right)x^2}+\frac{2ab^2}{3\left(1+a^2\right)^2x}-\frac{\tan^{-1}(a+bx)}{3x^3}-\frac{\left(1-3a^2\right)b^3\log(x)}{3\left(1+a^2\right)^3}+\frac{\left(\left(1-3a^2\right)b^3\right)\text{S}}{}$$

$$= -\frac{b}{6\left(1+a^2\right)x^2}+\frac{2ab^2}{3\left(1+a^2\right)^2x}+\frac{a\left(3-a^2\right)b^3\tan^{-1}(a+bx)}{3\left(1+a^2\right)^3}-\frac{\tan^{-1}(a+bx)}{3x^3}-\frac{\left(1-3a\right.}{3(1}$$

Mathematica [C] time = 0.15, size = 128, normalized size = 0.99

$$\frac{2\left(3a^2-1\right)b^3x^3\log(x)-(a-i)bx\left((a+i)\left(a^2-4abx+1\right)+i(a-i)^2b^2x^2\log(a+bx+i)\right)-2\left(a^2+1\right)^3\tan^{-1}(}{6\left(a^2+1\right)^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x]/x^4,x]
```

```
[Out] (-2*(1 + a^2)^3*ArcTan[a + b*x] + 2*(-1 + 3*a^2)*b^3*x^3*Log[x] + I*(I + a)
^3*b^3*x^3*Log[I - a - b*x] - (-I + a)*b*x*((I + a)*(1 + a^2 - 4*a*b*x) + I
*(-I + a)^2*b^2*x^2*Log[I + a + b*x]))/(6*(1 + a^2)^3*x^3)
```

fricas [A] time = 0.45, size = 135, normalized size = 1.05

$$-\frac{\left(3\,a^2-1\right)b^3x^3\log\left(b^2x^2+2\,abx+a^2+1\right)-2\left(3\,a^2-1\right)b^3x^3\log(x)-4\left(a^3+a\right)b^2x^2+\left(a^4+2\,a^2+1\right)bx+2}{6\left(a^6+3\,a^4+3\,a^2+1\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*((3*a^2 - 1)*b^3*x^3*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*(3*a^2 - 1)*
b^3*x^3*log(x) - 4*(a^3 + a)*b^2*x^2 + (a^4 + 2*a^2 + 1)*b*x + 2*((a^3 - 3*
a)*b^3*x^3 + a^6 + 3*a^4 + 3*a^2 + 1)*arctan(b*x + a))/((a^6 + 3*a^4 + 3*a^
2 + 1)*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.05, size = 162, normalized size = 1.26

$$-\frac{\arctan(bx+a)}{3x^3}-\frac{b}{6\left(a^2+1\right)x^2}+\frac{b^3\ln(bx)\,a^2}{\left(a^2+1\right)^3}-\frac{b^3\ln(bx)}{3\left(a^2+1\right)^3}+\frac{2a\,b^2}{3\left(a^2+1\right)^2x}-\frac{b^3\ln\left(1+(bx+a)^2\right)a^2}{2\left(a^2+1\right)^3}+\frac{b^3\ln\left(1+(bx\right.}{6\left(a^2+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/x^4,x)
```

```
[Out] -1/3*arctan(b*x+a)/x^3-1/6*b/(a^2+1)/x^2+b^3/(a^2+1)^3*ln(b*x)*a^2-1/3*b^3/
(a^2+1)^3*ln(b*x)+2/3*a*b^2/(a^2+1)^2/x-1/2*b^3/(a^2+1)^3*ln(1+(b*x+a)^2)*a
^2+1/6*b^3/(a^2+1)^3*ln(1+(b*x+a)^2)-1/3*b^3/(a^2+1)^3*arctan(b*x+a)*a^3+b^
3/(a^2+1)^3*arctan(b*x+a)*a
```

maxima [A] time = 0.41, size = 165, normalized size = 1.28

$$-\frac{1}{6}\left(\frac{2\left(a^3-3a\right)b^2\arctan\left(\frac{b^2x+ab}{b}\right)}{a^6+3a^4+3a^2+1}+\frac{\left(3a^2-1\right)b^2\log\left(b^2x^2+2abx+a^2+1\right)}{a^6+3a^4+3a^2+1}-\frac{2\left(3a^2-1\right)b^2\log(x)}{a^6+3a^4+3a^2+1}-\frac{4abx-}{\left(a^4+2a^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(a^3 - 3*a)*b^2*arctan((b^2*x + a*b)/b)/(a^6 + 3*a^4 + 3*a^2 + 1) +
 (3*a^2 - 1)*b^2*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/(a^6 + 3*a^4 + 3*a^2 + 1)
 - 2*(3*a^2 - 1)*b^2*log(x)/(a^6 + 3*a^4 + 3*a^2 + 1) - (4*a*b*x - a^2 - 1)
/((a^4 + 2*a^2 + 1)*x^2))*b - 1/3*arctan(b*x + a)/x^3
```

mupad [B] time = 1.05, size = 288, normalized size = 2.23

$$-\frac{\frac{bx}{6}+\mathrm{atan}(a+bx)\left(\frac{a^2}{3}+\frac{1}{3}\right)+\frac{b^2x^2\,\mathrm{atan}(a+bx)}{3}+\frac{x^3\left(b^3-7a^2b^3\right)}{6\left(a^4+2a^2+1\right)}-\frac{a\,b^2x^2}{3\left(a^2+1\right)}-\frac{2a\,b^4x^4}{3\left(a^2+1\right)^2}+\frac{2a\,b\,x\,\mathrm{atan}(a+bx)}{3}}{a^2x^3+2a\,b\,x^4+b^2x^5+x^3}-\frac{\ln(x)\left(\frac{b^3}{3}-\right.}{a^6+3a^4+3\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)/x^4,x)
```

```
[Out] - ((b*x)/6 + atan(a + b*x)*(a^2/3 + 1/3) + (b^2*x^2*atan(a + b*x))/3 + (x^3
*(b^3 - 7*a^2*b^3))/(6*(2*a^2 + a^4 + 1)) - (a*b^2*x^2)/(3*(a^2 + 1)) - (2*
a*b^4*x^4)/(3*(a^2 + 1)^2) + (2*a*b*x*atan(a + b*x))/3)/(x^3 + a^2*x^3 + b^
```

```
2*x^5 + 2*a*b*x^4) - (log(x)*(b^3/3 - a^2*b^3))/(3*a^2 + 3*a^4 + a^6 + 1) -
 (b^3*log(a^2 + b^2*x^2 + 2*a*b*x + 1)*(3*a^2 - 1))/(6*(3*a^2 + 3*a^4 + a^6
 + 1)) - (a*atan((2*a*b + 2*b^2*x)/(2*(b^2*(a^2 + 1) - a^2*b^2)^(1/2)))*(a^
2 - 3)*(b^2)^(3/2))/(3*(3*a^2 + 3*a^4 + a^6 + 1))
```

sympy [B] time = 4.66, size = 760, normalized size = 5.89

$$\begin{cases} \frac{ib^3 \operatorname{atan}(bx-i)}{24} + \frac{ib^2}{24x} - \frac{b}{24x^2} - \frac{\operatorname{atan}(bx-i)}{3x^3} - \frac{i}{18x^3} \\ -\frac{ib^3 \operatorname{atan}(bx+i)}{24} - \frac{ib^2}{24x} - \frac{b}{24x^2} - \frac{\operatorname{atan}(bx+i)}{3x^3} + \frac{i}{18x^3} \\ -\frac{2a^6 \operatorname{atan}(a+bx)}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3} - \frac{a^4bx}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3} - \frac{6a^4 \operatorname{atan}(a+bx)}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3} - \frac{2a^3b^3x^3 \operatorname{atan}(a+bx)}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3} + \frac{4a}{6a^6x^3+18a^4x}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/x**4,x)
```

```
[Out] Piecewise((I*b**3*atan(b*x - I)/24 + I*b**2/(24*x) - b/(24*x**2) - atan(b*x
 - I)/(3*x**3) - I/(18*x**3), Eq(a, -I)), (-I*b**3*atan(b*x + I)/24 - I*b**
2/(24*x) - b/(24*x**2) - atan(b*x + I)/(3*x**3) + I/(18*x**3), Eq(a, I)), (
-2*a**6*atan(a + b*x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3)
- a**4*b*x/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 6*a**4*at
an(a + b*x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 2*a**3*b
**3*x**3*atan(a + b*x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3)
 + 4*a**3*b**2*x**2/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) +
6*a**2*b**3*x**3*log(x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3
) - 3*a**2*b**3*x**3*log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(6*a**6*x**3 + 18*
a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 2*a**2*b*x/(6*a**6*x**3 + 18*a**4*x**3
 + 18*a**2*x**3 + 6*x**3) - 6*a**2*atan(a + b*x)/(6*a**6*x**3 + 18*a**4*x**
3 + 18*a**2*x**3 + 6*x**3) + 6*a*b**3*x**3*atan(a + b*x)/(6*a**6*x**3 + 18*
a**4*x**3 + 18*a**2*x**3 + 6*x**3) + 4*a*b**2*x**2/(6*a**6*x**3 + 18*a**4*x
**3 + 18*a**2*x**3 + 6*x**3) - 2*b**3*x**3*log(x)/(6*a**6*x**3 + 18*a**4*x*
*3 + 18*a**2*x**3 + 6*x**3) + b**3*x**3*log(a**2 + 2*a*b*x + b**2*x**2 + 1)
/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - b*x/(6*a**6*x**3 +
18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 2*atan(a + b*x)/(6*a**6*x**3 + 18*a
**4*x**3 + 18*a**2*x**3 + 6*x**3), True))
```

3.52 $\int \frac{\tan^{-1}(a+bx)}{c+dx^3} dx$

Optimal. Leaf size=863

$$-\frac{i\log(ia+ibx+1)\log\left(\frac{b\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\sqrt[3]{d}(i-a)+b\sqrt[3]{c}}\right)}{6c^{2/3}\sqrt[3]{d}}+\frac{i\log(-ia-ibx+1)\log\left(\frac{b\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{b\sqrt[3]{c}-(a+i)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}}+\frac{\sqrt[6]{-1}\log(ia+ibx+1)\log\left(\frac{b\left(\sqrt[3]{c}-\right.}{b\sqrt[3]{c}-}\right.}{6c^{2/3}\sqrt[3]{d}}$$

```
[Out] -1/6*I*ln(1+I*a+I*b*x)*ln(b*(c^(1/3)+d^(1/3)*x)/(b*c^(1/3)+(I-a)*d^(1/3)))/
c^(2/3)/d^(1/3)+1/6*I*ln(1-I*a-I*b*x)*ln(b*(c^(1/3)+d^(1/3)*x)/(b*c^(1/3)-(
I+a)*d^(1/3)))/c^(2/3)/d^(1/3)+1/6*(-1)^(1/6)*ln(1+I*a+I*b*x)*ln(b*(c^(1/3)
-(-1)^(1/3)*d^(1/3)*x)/(b*c^(1/3)-(-1)^(1/3)*(I-a)*d^(1/3)))/c^(2/3)/d^(1/3
)-1/6*(-1)^(1/6)*ln(1-I*a-I*b*x)*ln(b*(c^(1/3)-(-1)^(1/3)*d^(1/3)*x)/(b*c^(
1/3)+(-1)^(1/3)*(I+a)*d^(1/3)))/c^(2/3)/d^(1/3)+1/6*(-1)^(5/6)*ln(1+I*a+I*b
*x)*ln(b*(c^(1/3)+(-1)^(2/3)*d^(1/3)*x)/(b*c^(1/3)+(-1)^(2/3)*(I-a)*d^(1/3)
))/c^(2/3)/d^(1/3)-1/6*(-1)^(5/6)*ln(1-I*a-I*b*x)*ln(b*(c^(1/3)+(-1)^(2/3)*
d^(1/3)*x)/(b*c^(1/3)+(-1)^(1/6)*(1-I*a)*d^(1/3)))/c^(2/3)/d^(1/3)-1/6*I*po
lylog(2,d^(1/3)*(I-a-b*x)/(b*c^(1/3)+(I-a)*d^(1/3)))/c^(2/3)/d^(1/3)+1/6*(-
1)^(5/6)*polylog(2,-(-1)^(1/6)*d^(1/3)*(I-a-b*x)/(I*b*c^(1/3)-(-1)^(1/6)*(I
-a)*d^(1/3)))/c^(2/3)/d^(1/3)+1/6*(-1)^(1/6)*polylog(2,-(-1)^(1/3)*d^(1/3)*
(I-a-b*x)/(b*c^(1/3)-(-1)^(1/3)*(I-a)*d^(1/3)))/c^(2/3)/d^(1/3)+1/6*I*polyl
og(2,-d^(1/3)*(I+a+b*x)/(b*c^(1/3)-(I+a)*d^(1/3)))/c^(2/3)/d^(1/3)-1/6*(-1)
^(1/6)*polylog(2,(-1)^(1/3)*d^(1/3)*(I+a+b*x)/(b*c^(1/3)+(-1)^(1/3)*(I+a)*d
^(1/3)))/c^(2/3)/d^(1/3)-1/6*(-1)^(5/6)*polylog(2,-(-1)^(2/3)*d^(1/3)*(I+a+
b*x)/(b*c^(1/3)-(-1)^(2/3)*(I+a)*d^(1/3)))/c^(2/3)/d^(1/3)
```

Rubi [A] time = 1.21, antiderivative size = 863, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5051, 2409, 2394, 2393, 2391}

$$-\frac{i\log(ia+ibx+1)\log\left(\frac{b\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\sqrt[3]{d}(i-a)+b\sqrt[3]{c}}\right)}{6c^{2/3}\sqrt[3]{d}}+\frac{i\log(-ia-ibx+1)\log\left(\frac{b\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{b\sqrt[3]{c}-(a+i)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}}+\frac{\sqrt[6]{-1}\log(ia+ibx+1)\log\left(\frac{b\left(\sqrt[3]{c}-\right.}{b\sqrt[3]{c}-}\right.}{6c^{2/3}\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/(c + d*x^3),x]
```

```
[Out] ((-I/6)*Log[1 + I*a + I*b*x]*Log[(b*(c^(1/3) + d^(1/3)*x))/(b*c^(1/3) + (I
- a)*d^(1/3))])/(c^(2/3)*d^(1/3)) + ((I/6)*Log[1 - I*a - I*b*x]*Log[(b*(c^(
1/3) + d^(1/3)*x))/(b*c^(1/3) - (I + a)*d^(1/3))])/(c^(2/3)*d^(1/3)) + ((-1
)^(1/6)*Log[1 + I*a + I*b*x]*Log[(b*(c^(1/3) - (-1)^(1/3)*d^(1/3)*x))/(b*c^
(1/3) - (-1)^(1/3)*(I - a)*d^(1/3))])/(6*c^(2/3)*d^(1/3)) - ((-1)^(1/6)*Log
[1 - I*a - I*b*x]*Log[(b*(c^(1/3) - (-1)^(1/3)*d^(1/3)*x))/(b*c^(1/3) + (-1
)^(1/3)*(I + a)*d^(1/3))])/(6*c^(2/3)*d^(1/3)) + ((-1)^(5/6)*Log[1 + I*a +
I*b*x]*Log[(b*(c^(1/3) + (-1)^(2/3)*d^(1/3)*x))/(b*c^(1/3) + (-1)^(2/3)*(I
- a)*d^(1/3))])/(6*c^(2/3)*d^(1/3)) - ((-1)^(5/6)*Log[1 - I*a - I*b*x]*Log[
(b*(c^(1/3) + (-1)^(2/3)*d^(1/3)*x))/(b*c^(1/3) + (-1)^(1/6)*(1 - I*a)*d^(1
/3))])/(6*c^(2/3)*d^(1/3)) - ((I/6)*PolyLog[2, (d^(1/3)*(I - a - b*x))/(b*c
^(1/3) + (I - a)*d^(1/3))])/(c^(2/3)*d^(1/3)) + ((-1)^(5/6)*PolyLog[2, -(((
-1)^(1/6)*d^(1/3)*(I - a - b*x))/(I*b*c^(1/3) - (-1)^(1/6)*(I - a)*d^(1/3))
)])/(6*c^(2/3)*d^(1/3)) + ((-1)^(1/6)*PolyLog[2, -(((-1)^(1/3)*d^(1/3)*(I -
 a - b*x))/(b*c^(1/3) - (-1)^(1/3)*(I - a)*d^(1/3)))])/(6*c^(2/3)*d^(1/3))
+ ((I/6)*PolyLog[2, -((d^(1/3)*(I + a + b*x))/(b*c^(1/3) - (I + a)*d^(1/3))
)])/(c^(2/3)*d^(1/3)) - ((-1)^(1/6)*PolyLog[2, ((-1)^(1/3)*d^(1/3)*(I + a +
 b*x))/(b*c^(1/3) + (-1)^(1/3)*(I + a)*d^(1/3))])/(6*c^(2/3)*d^(1/3)) - ((-
1)^(5/6)*PolyLog[2, -(((-1)^(2/3)*d^(1/3)*(I + a + b*x))/(b*c^(1/3) - (-1)^
(2/3)*(I + a)*d^(1/3)))])/(6*c^(2/3)*d^(1/3))
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 5051

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{c+dx^3}\,dx = \frac{1}{2}i\int \frac{\log(1-ia-ibx)}{c+dx^3}\,dx - \frac{1}{2}i\int \frac{\log(1+ia+ibx)}{c+dx^3}\,dx$$

$$= \frac{1}{2}i\int \left(-\frac{\log(1-ia-ibx)}{3c^{2/3}\left(-\sqrt[3]{c}-\sqrt[3]{d}\,x\right)} - \frac{\log(1-ia-ibx)}{3c^{2/3}\left(-\sqrt[3]{c}+\sqrt[3]{-1}\sqrt[3]{d}\,x\right)} - \frac{\log(1-ia-ibx)}{3c^{2/3}\left(-\sqrt[3]{c}-(-1)^{2/3}\sqrt[3]{d}\,x\right)}\right)$$

$$= -\frac{i\int \frac{\log(1-ia-ibx)}{-\sqrt[3]{c}-\sqrt[3]{d}\,x}\,dx}{6c^{2/3}} - \frac{i\int \frac{\log(1-ia-ibx)}{-\sqrt[3]{c}+\sqrt[3]{-1}\sqrt[3]{d}\,x}\,dx}{6c^{2/3}} - \frac{i\int \frac{\log(1-ia-ibx)}{-\sqrt[3]{c}-(-1)^{2/3}\sqrt[3]{d}\,x}\,dx}{6c^{2/3}} + \frac{i\int \frac{\log(1+ia+ibx)}{-\sqrt[3]{c}-\sqrt[3]{d}\,x}\,dx}{6c^{2/3}}$$

$$= -\frac{i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{b\sqrt[3]{c}+(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{i\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{b\sqrt[3]{c}-(i+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{\sqrt[6]{-1}\ \mathrm{lo}}{}$$

$$= -\frac{i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{b\sqrt[3]{c}+(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{i\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{b\sqrt[3]{c}-(i+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{\sqrt[6]{-1}\ \mathrm{lo}}{}$$

$$= -\frac{i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{b\sqrt[3]{c}+(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{i\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{b\sqrt[3]{c}-(i+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{\sqrt[6]{-1}\ \mathrm{lo}}{}$$

Mathematica [A] time = 0.91, size = 701, normalized size = 0.81

$$-i\text{Li}_2\left(\frac{\sqrt[3]{d}(a+bx-i)}{(a-i)\sqrt[3]{d}-b\sqrt[3]{c}}\right)+(-1)^{5/6}\text{Li}_2\left(\frac{\sqrt[6]{-1}\sqrt[3]{d}(a+bx-i)}{\sqrt[6]{-1}\sqrt[3]{d}(a-i)+ib\sqrt[3]{c}}\right)+\sqrt[6]{-1}\,\text{Li}_2\left(\frac{\sqrt[3]{-1}\sqrt[3]{d}(a+bx-i)}{\sqrt[3]{-1}\sqrt[3]{d}(a-i)+b\sqrt[3]{c}}\right)+i\text{Li}_2\left(\frac{\sqrt[3]{d}(a+bx+i)}{(a+i)\sqrt[3]{d}-b\sqrt[3]{c}}\right)-\sqrt[6]{-1}\,\text{Li}_2$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x]/(c + d*x^3),x]
```

```
[Out] ((-I)*Log[1 + I*a + I*b*x]*Log[(b*(c^(1/3) + d^(1/3)*x))/(b*c^(1/3) - (-I +
 a)*d^(1/3))] + I*Log[(-I)*(I + a + b*x)]*Log[(b*(c^(1/3) + d^(1/3)*x))/(b*
c^(1/3) - (I + a)*d^(1/3))] + (-1)^(1/6)*Log[1 + I*a + I*b*x]*Log[(b*(c^(1/
3) - (-1)^(1/3)*d^(1/3)*x))/(b*c^(1/3) + (-1)^(1/3)*(-I + a)*d^(1/3))] - (-
1)^(1/6)*Log[(-I)*(I + a + b*x)]*Log[(b*(c^(1/3) - (-1)^(1/3)*d^(1/3)*x))/(
b*c^(1/3) + (-1)^(1/3)*(I + a)*d^(1/3))] - (-1)^(5/6)*Log[(-I)*(I + a + b*x
)]*Log[(b*(c^(1/3) + (-1)^(2/3)*d^(1/3)*x))/(b*c^(1/3) + (-1)^(1/6)*(1 - I*
a)*d^(1/3))] + (-1)^(5/6)*Log[1 + I*a + I*b*x]*Log[(b*(c^(1/3) + (-1)^(2/3)
*d^(1/3)*x))/(b*c^(1/3) - (-1)^(2/3)*(-I + a)*d^(1/3))] - I*PolyLog[2, (d^(
1/3)*(-I + a + b*x))/(-(b*c^(1/3)) + (-I + a)*d^(1/3))] + (-1)^(5/6)*PolyLo
g[2, ((-1)^(1/6)*d^(1/3)*(-I + a + b*x))/(I*b*c^(1/3) + (-1)^(1/6)*(-I + a)
*d^(1/3))] + (-1)^(1/6)*PolyLog[2, ((-1)^(1/3)*d^(1/3)*(-I + a + b*x))/(b*c
^(1/3) + (-1)^(1/3)*(-I + a)*d^(1/3))] + I*PolyLog[2, (d^(1/3)*(I + a + b*x
))/(-(b*c^(1/3)) + (I + a)*d^(1/3))] - (-1)^(1/6)*PolyLog[2, ((-1)^(1/3)*d^
(1/3)*(I + a + b*x))/(b*c^(1/3) + (-1)^(1/3)*(I + a)*d^(1/3))] - (-1)^(5/6)
*PolyLog[2, ((-1)^(2/3)*d^(1/3)*(I + a + b*x))/(-(b*c^(1/3)) + (-1)^(2/3)*(
I + a)*d^(1/3))])/(6*c^(2/3)*d^(1/3))
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\arctan(bx+a)}{dx^3+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] integral(arctan(b*x + a)/(d*x^3 + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [C] time = 1.31, size = 631, normalized size = 0.73

$$\frac{2b^2\left(\sum_{\_R1=\text{RootOf}\left(\left(a^3d+3ia^2d-cb^3-3ad-id\right)\_Z^6+\left(3a^3d+3ia^2d-3cb^3+3ad+3id\right)\_Z^4+\left(3a^3d-3ia^2d-3cb^3+3ad-3id\right)\_Z^2-3ia^2d+a^3d-cb^3+id-3ad\right)}\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(d*x^3+c),x)
```

```
[Out] 2/3*b^2*sum(1/(a^3*d*_R1^4+3*I*a^2*d*_R1^4-b^3*c*_R1^4-3*a*d*_R1^4-I*d*_R1^
4+2*a^3*d*_R1^2+2*I*a^2*d*_R1^2-2*b^3*c*_R1^2+2*_R1^2*a*d+2*I*d*_R1^2+a^3*d
-I*a^2*d-c*b^3+a*d-I*d)*(I*arctan(b*x+a)*ln((_R1-(1+I*(b*x+a))/(1+(b*x+a)^2
)^(1/2))/_R1)+dilog((_R1-(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))/_R1)),_R1=RootO
f((3*I*a^2*d+a^3*d-c*b^3-I*d-3*a*d)*_Z^6+(3*I*a^2*d+3*a^3*d-3*c*b^3+3*I*d+3
*a*d)*_Z^4+(-3*I*a^2*d+3*a^3*d-3*c*b^3-3*I*d+3*a*d)*_Z^2-3*I*a^2*d+a^3*d-c*
b^3+I*d-3*a*d))+2/3*b^2*sum(_R1^2/(a^3*d*_R1^4+3*I*a^2*d*_R1^4-b^3*c*_R1^4-
3*a*d*_R1^4-I*d*_R1^4+2*a^3*d*_R1^2+2*I*a^2*d*_R1^2-2*b^3*c*_R1^2+2*_R1^2*a
*d+2*I*d*_R1^2+a^3*d-I*a^2*d-c*b^3+a*d-I*d)*(I*arctan(b*x+a)*ln((_R1-(1+I*(
b*x+a))/(1+(b*x+a)^2)^(1/2))/_R1)+dilog((_R1-(1+I*(b*x+a))/(1+(b*x+a)^2)^(1
/2))/_R1)),_R1=RootOf((3*I*a^2*d+a^3*d-c*b^3-I*d-3*a*d)*_Z^6+(3*I*a^2*d+3*a
^3*d-3*c*b^3+3*I*d+3*a*d)*_Z^4+(-3*I*a^2*d+3*a^3*d-3*c*b^3-3*I*d+3*a*d)*_Z^
2-3*I*a^2*d+a^3*d-c*b^3+I*d-3*a*d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(bx+a\right)}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(arctan(b*x + a)/(d*x^3 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}\left(a+b\,x\right)}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)/(c + d*x^3),x)
```

```
[Out] int(atan(a + b*x)/(c + d*x^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

3.53 $\int \frac{\tan^{-1}(a+bx)}{c+dx^2} \, dx$

Optimal. Leaf size=543

$$-\frac{i\text{Li}_2\left(-\frac{\sqrt{d}(-a-bx+i)}{b\sqrt{-c}-(i-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\frac{i\text{Li}_2\left(\frac{\sqrt{d}(-a-bx+i)}{\sqrt{d}(i-a)+b\sqrt{-c}}\right)}{4\sqrt{-c}\sqrt{d}}-\frac{i\text{Li}_2\left(-\frac{\sqrt{d}(a+bx+i)}{b\sqrt{-c}-(a+i)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\frac{i\text{Li}_2\left(\frac{\sqrt{d}(a+bx+i)}{\sqrt{d}(a+i)+b\sqrt{-c}}\right)}{4\sqrt{-c}\sqrt{d}}-\frac{i\log(ia+ibx+1)\log\left(\right.}{4\sqrt{-c}\sqrt{d}}$$

```
[Out] -1/4*I*ln(1+I*a+I*b*x)*ln(b*((-c)^(1/2)-x*d^(1/2))/(b*(-c)^(1/2)-(I-a)*d^(1
/2)))/(-c)^(1/2)/d^(1/2)+1/4*I*ln(1-I*a-I*b*x)*ln(b*((-c)^(1/2)-x*d^(1/2))/
(b*(-c)^(1/2)+(I+a)*d^(1/2)))/(-c)^(1/2)/d^(1/2)+1/4*I*ln(1+I*a+I*b*x)*ln(b
*((-c)^(1/2)+x*d^(1/2))/(b*(-c)^(1/2)+(I-a)*d^(1/2)))/(-c)^(1/2)/d^(1/2)-1/
4*I*ln(1-I*a-I*b*x)*ln(b*((-c)^(1/2)+x*d^(1/2))/(b*(-c)^(1/2)-(I+a)*d^(1/2)
))/(-c)^(1/2)/d^(1/2)-1/4*I*polylog(2,-(I-a-b*x)*d^(1/2)/(b*(-c)^(1/2)-(I-a
)*d^(1/2)))/(-c)^(1/2)/d^(1/2)+1/4*I*polylog(2,(I-a-b*x)*d^(1/2)/(b*(-c)^(1
/2)+(I-a)*d^(1/2)))/(-c)^(1/2)/d^(1/2)-1/4*I*polylog(2,-(I+a+b*x)*d^(1/2)/(
b*(-c)^(1/2)-(I+a)*d^(1/2)))/(-c)^(1/2)/d^(1/2)+1/4*I*polylog(2,(I+a+b*x)*d
^(1/2)/(b*(-c)^(1/2)+(I+a)*d^(1/2)))/(-c)^(1/2)/d^(1/2)
```

Rubi [A] time = 0.61, antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5051, 2409, 2394, 2393, 2391}

$$-\frac{i\text{PolyLog}\left(2,-\frac{\sqrt{d}(-a-bx+i)}{b\sqrt{-c}-(-a+i)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\frac{i\text{PolyLog}\left(2,\frac{\sqrt{d}(-a-bx+i)}{b\sqrt{-c}+(-a+i)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}-\frac{i\text{PolyLog}\left(2,-\frac{\sqrt{d}(a+bx+i)}{b\sqrt{-c}-(a+i)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\frac{i\text{PolyLog}\left(2,\frac{\sqrt{d}}{b\sqrt{-c}}\right.}{4\sqrt{-c}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/(c + d*x^2),x]
```

```
[Out] ((-I/4)*Log[1 + I*a + I*b*x]*Log[(b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] - (
I - a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d]) + ((I/4)*Log[1 - I*a - I*b*x]*Log[(b*(
Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] + (I + a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d])
+ ((I/4)*Log[1 + I*a + I*b*x]*Log[(b*(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c] +
(I - a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d]) - ((I/4)*Log[1 - I*a - I*b*x]*Log[(b*
(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c] - (I + a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d])
 - ((I/4)*PolyLog[2, -((Sqrt[d]*(I - a - b*x))/(b*Sqrt[-c] - (I - a)*Sqrt[d
]))])/(Sqrt[-c]*Sqrt[d]) + ((I/4)*PolyLog[2, (Sqrt[d]*(I - a - b*x))/(b*Sqr
t[-c] + (I - a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d]) - ((I/4)*PolyLog[2, -((Sqrt[d
]*(I + a + b*x))/(b*Sqrt[-c] - (I + a)*Sqrt[d]))])/(Sqrt[-c]*Sqrt[d]) + ((I
/4)*PolyLog[2, (Sqrt[d]*(I + a + b*x))/(b*Sqrt[-c] + (I + a)*Sqrt[d])])/(Sq
rt[-c]*Sqrt[d])
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
```

```
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 5051

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{c+dx^2}\,dx = \frac{1}{2}i\int \frac{\log(1-ia-ibx)}{c+dx^2}\,dx - \frac{1}{2}i\int \frac{\log(1+ia+ibx)}{c+dx^2}\,dx$$

$$= \frac{1}{2}i\int \left(\frac{\sqrt{-c}\,\log(1-ia-ibx)}{2c\left(\sqrt{-c}-\sqrt{d}\,x\right)} + \frac{\sqrt{-c}\,\log(1-ia-ibx)}{2c\left(\sqrt{-c}+\sqrt{d}\,x\right)}\right)dx - \frac{1}{2}i\int \left(\frac{\sqrt{-c}\,\log(1+ia+i}{2c\left(\sqrt{-c}-\sqrt{d}\,x\right.}\right.$$

$$= -\frac{i\int \frac{\log(1-ia-ibx)}{\sqrt{-c}-\sqrt{d}\,x}\,dx}{4\sqrt{-c}} - \frac{i\int \frac{\log(1-ia-ibx)}{\sqrt{-c}+\sqrt{d}\,x}\,dx}{4\sqrt{-c}} + \frac{i\int \frac{\log(1+ia+ibx)}{\sqrt{-c}-\sqrt{d}\,x}\,dx}{4\sqrt{-c}} + \frac{i\int \frac{\log(1+ia+ibx)}{\sqrt{-c}+\sqrt{d}\,x}\,dx}{4\sqrt{-c}}$$

$$= -\frac{i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}\,x\right)}{b\sqrt{-c}-(i-a)\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}} + \frac{i\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}\,x\right)}{b\sqrt{-c}+(i+a)\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}} + \frac{i\log}{}$$

$$= -\frac{i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}\,x\right)}{b\sqrt{-c}-(i-a)\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}} + \frac{i\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}\,x\right)}{b\sqrt{-c}+(i+a)\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}} + \frac{i\log}{}$$

$$= -\frac{i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}\,x\right)}{b\sqrt{-c}-(i-a)\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}} + \frac{i\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}\,x\right)}{b\sqrt{-c}+(i+a)\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}} + \frac{i\log}{}$$

Mathematica [A] time = 0.36, size = 409, normalized size = 0.75

$$-\frac{i\left(-\text{Li}_2\left(\frac{\sqrt{d}\,(a+bx-i)}{(a-i)\sqrt{d}-b\sqrt{-c}}\right)+\text{Li}_2\left(\frac{\sqrt{d}\,(a+bx-i)}{\sqrt{d}\,(a-i)+b\sqrt{-c}}\right)+\text{Li}_2\left(\frac{\sqrt{d}\,(a+bx+i)}{(a+i)\sqrt{d}-b\sqrt{-c}}\right)-\text{Li}_2\left(\frac{\sqrt{d}\,(a+bx+i)}{\sqrt{d}\,(a+i)+b\sqrt{-c}}\right)+\log(ia+ibx+1)\log\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x]/(c + d*x^2),x]
```

```
[Out] ((-1/4*I)*(Log[1 + I*a + I*b*x]*Log[(b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c]
+ (-I + a)*Sqrt[d])] - Log[(-I)*(I + a + b*x)]*Log[(b*(Sqrt[-c] - Sqrt[d]*x
))/(b*Sqrt[-c] + (I + a)*Sqrt[d])] - Log[1 + I*a + I*b*x]*Log[(b*(Sqrt[-c]
+ Sqrt[d]*x))/(b*Sqrt[-c] - (-I + a)*Sqrt[d])] + Log[(-I)*(I + a + b*x)]*Lo
g[(b*(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c] - (I + a)*Sqrt[d])] - PolyLog[2, (
Sqrt[d]*(-I + a + b*x))/(-(b*Sqrt[-c]) + (-I + a)*Sqrt[d])] + PolyLog[2, (S
qrt[d]*(-I + a + b*x))/(b*Sqrt[-c] + (-I + a)*Sqrt[d])] + PolyLog[2, (Sqrt[
```

```
d]*(I + a + b*x))/(-(b*Sqrt[-c]) + (I + a)*Sqrt[d])] - PolyLog[2, (Sqrt[d]*
(I + a + b*x))/(b*Sqrt[-c] + (I + a)*Sqrt[d])]))/(Sqrt[-c]*Sqrt[d])
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\arctan\left(bx+a\right)}{dx^2+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(arctan(b*x + a)/(d*x^2 + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 1.29, size = 2192, normalized size = 4.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(d*x^2+c),x)
```

```
[Out] -1/2*I*b/d*(b^2*c*d)^(1/2)/(a^2*d+c*b^2-2*(b^2*c*d)^(1/2)+d)*ln(1-(2*I*a*d+
a^2*d+c*b^2-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-c*b^2+2*(b^2*c*d)^(1/2
)-d))*arctan(b*x+a)+I*b/(a^2*d+c*b^2-2*(b^2*c*d)^(1/2)+d)*ln(1-(2*I*a*d+a^2
*d+c*b^2-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-c*b^2+2*(b^2*c*d)^(1/2)-d
))*arctan(b*x+a)+I*b/(a^2*d+c*b^2+2*(b^2*c*d)^(1/2)+d)*ln(1-(2*I*a*d+a^2*d+
c*b^2-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-c*b^2-2*(b^2*c*d)^(1/2)-d))*
arctan(b*x+a)-1/2*I/b*(b^2*c*d)^(1/2)/c/(a^2*d+c*b^2-2*(b^2*c*d)^(1/2)+d)*l
n(1-(2*I*a*d+a^2*d+c*b^2-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-c*b^2+2*(
b^2*c*d)^(1/2)-d))*arctan(b*x+a)+1/2*b/d*(b^2*c*d)^(1/2)/(a^2*d+c*b^2+2*(b^
2*c*d)^(1/2)+d)*arctan(b*x+a)^2+b/(a^2*d+c*b^2+2*(b^2*c*d)^(1/2)+d)*arctan(
b*x+a)^2+1/2/b*(b^2*c*d)^(1/2)/c/(a^2*d+c*b^2+2*(b^2*c*d)^(1/2)+d)*arctan(b
*x+a)^2+1/2/b*(b^2*c*d)^(1/2)/c/(a^2*d+c*b^2+2*(b^2*c*d)^(1/2)+d)*arctan(b*
x+a)^2*a^2+1/4*b/d*(b^2*c*d)^(1/2)/(a^2*d+c*b^2+2*(b^2*c*d)^(1/2)+d)*polylo
g(2,(2*I*a*d+a^2*d+c*b^2-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-c*b^2-2*(
b^2*c*d)^(1/2)-d))+1/2*b/(a^2*d+c*b^2+2*(b^2*c*d)^(1/2)+d)*polylog(2,(2*I*a
*d+a^2*d+c*b^2-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-c*b^2-2*(b^2*c*d)^(
1/2)-d))+1/4/b*(b^2*c*d)^(1/2)/c/(a^2*d+c*b^2+2*(b^2*c*d)^(1/2)+d)*polylog(
2,(2*I*a*d+a^2*d+c*b^2-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-c*b^2-2*(b^
2*c*d)^(1/2)-d))+1/4/b*(b^2*c*d)^(1/2)/c/(a^2*d+c*b^2+2*(b^2*c*d)^(1/2)+d)*
polylog(2,(2*I*a*d+a^2*d+c*b^2-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-c*b
^2-2*(b^2*c*d)^(1/2)-d))*a^2+1/2*I/b*(b^2*c*d)^(1/2)/c/(a^2*d+c*b^2+2*(b^2*
c*d)^(1/2)+d)*ln(1-(2*I*a*d+a^2*d+c*b^2-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-
a^2*d-c*b^2-2*(b^2*c*d)^(1/2)-d))*arctan(b*x+a)*a^2+1/2*I/b*(b^2*c*d)^(1/2)
/c/(a^2*d+c*b^2+2*(b^2*c*d)^(1/2)+d)*ln(1-(2*I*a*d+a^2*d+c*b^2-d)*(1+I*(b*x
+a))^2/(1+(b*x+a)^2)/(-a^2*d-c*b^2-2*(b^2*c*d)^(1/2)-d))*arctan(b*x+a)-1/2*
I/b*(b^2*c*d)^(1/2)/c/(a^2*d+c*b^2-2*(b^2*c*d)^(1/2)+d)*ln(1-(2*I*a*d+a^2*d
+c*b^2-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-c*b^2+2*(b^2*c*d)^(1/2)-d))
*arctan(b*x+a)*a^2+1/2*I*b/d*(b^2*c*d)^(1/2)/(a^2*d+c*b^2+2*(b^2*c*d)^(1/2)
+d)*ln(1-(2*I*a*d+a^2*d+c*b^2-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-c*b^
2-2*(b^2*c*d)^(1/2)-d))*arctan(b*x+a)-1/2*b/d*(b^2*c*d)^(1/2)/(a^2*d+c*b^2-
```

```
2*(b^2*c*d)^(1/2)+d)*arctan(b*x+a)^2+b/(a^2*d+c*b^2-2*(b^2*c*d)^(1/2)+d)*ar
ctan(b*x+a)^2-1/2/b*(b^2*c*d)^(1/2)/c/(a^2*d+c*b^2-2*(b^2*c*d)^(1/2)+d)*arc
tan(b*x+a)^2-1/2/b*(b^2*c*d)^(1/2)/c/(a^2*d+c*b^2-2*(b^2*c*d)^(1/2)+d)*arct
an(b*x+a)^2*a^2-1/4*b/d*(b^2*c*d)^(1/2)/(a^2*d+c*b^2-2*(b^2*c*d)^(1/2)+d)*p
olylog(2,(2*I*a*d+a^2*d+c*b^2-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-c*b^
2+2*(b^2*c*d)^(1/2)-d))+1/2*b/(a^2*d+c*b^2-2*(b^2*c*d)^(1/2)+d)*polylog(2,(
2*I*a*d+a^2*d+c*b^2-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-c*b^2+2*(b^2*c
*d)^(1/2)-d))-1/4/b*(b^2*c*d)^(1/2)/c/(a^2*d+c*b^2-2*(b^2*c*d)^(1/2)+d)*pol
ylog(2,(2*I*a*d+a^2*d+c*b^2-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-c*b^2+
2*(b^2*c*d)^(1/2)-d))-1/4/b*(b^2*c*d)^(1/2)/c/(a^2*d+c*b^2-2*(b^2*c*d)^(1/2
)+d)*polylog(2,(2*I*a*d+a^2*d+c*b^2-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*
d-c*b^2+2*(b^2*c*d)^(1/2)-d))*a^2
```

maxima [B] time = 5.93, size = 8520, normalized size = 15.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/8*b*(8*arctan(d*x/sqrt(c*d))*arctan((b^2*x + a*b)/b)/b - (4*arctan(sqrt(d
)*x/sqrt(c))*arctan2((2*a*b^2*c*d + (a*b^3*c + (a^3 + a)*b*d + (b^4*c + (a^
2 + 3)*b^2*d)*x)*sqrt(c)*sqrt(d) + (3*b^3*c*d + (a^2 + 1)*b*d^2)*x)/(b^4*c^
2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 + 4*(b^3*c + (a^2 + 1)*b*d)
*sqrt(c)*sqrt(d)), ((a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 + (2*a*b^2*d*
x + b^3*c + 3*(a^2 + 1)*b*d)*sqrt(c)*sqrt(d) + (a*b^3*c*d + (a^3 + a)*b*d^2
)*x)/(b^4*c^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 + 4*(b^3*c + (a
^2 + 1)*b*d)*sqrt(c)*sqrt(d))) + 4*arctan(sqrt(d)*x/sqrt(c))*arctan2((2*a*b
^2*c*d - (a*b^3*c + (a^3 + a)*b*d + (b^4*c + (a^2 + 3)*b^2*d)*x)*sqrt(c)*sq
rt(d) + (3*b^3*c*d + (a^2 + 1)*b*d^2)*x)/(b^4*c^2 + 2*(a^2 + 3)*b^2*c*d + (
a^4 + 2*a^2 + 1)*d^2 - 4*(b^3*c + (a^2 + 1)*b*d)*sqrt(c)*sqrt(d)), ((a^2 +
3)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 - (2*a*b^2*d*x + b^3*c + 3*(a^2 + 1)*b*d
)*sqrt(c)*sqrt(d) + (a*b^3*c*d + (a^3 + a)*b*d^2)*x)/(b^4*c^2 + 2*(a^2 + 3)
*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 - 4*(b^3*c + (a^2 + 1)*b*d)*sqrt(c)*sqrt(d
))) + log(d*x^2 + c)*log(((a^2 + 1)*b^22*c^11*d + 11*(a^4 + 22*a^2 + 21)*b^
20*c^10*d^2 + 55*(a^6 + 39*a^4 + 171*a^2 + 133)*b^18*c^9*d^3 + 33*(5*a^8 +
260*a^6 + 1870*a^4 + 3876*a^2 + 2261)*b^16*c^8*d^4 + 330*(a^10 + 61*a^8 + 5
70*a^6 + 1802*a^4 + 2261*a^2 + 969)*b^14*c^7*d^5 + 22*(21*a^12 + 1386*a^10
+ 15015*a^8 + 60060*a^6 + 109395*a^4 + 92378*a^2 + 29393)*b^12*c^6*d^6 + 22
*(21*a^14 + 1407*a^12 + 16401*a^10 + 75075*a^8 + 169455*a^6 + 201773*a^4 +
121771*a^2 + 29393)*b^10*c^5*d^7 + 330*(a^16 + 64*a^14 + 756*a^12 + 3696*a^
10 + 9438*a^8 + 13728*a^6 + 11492*a^4 + 5168*a^2 + 969)*b^8*c^4*d^8 + 33*(5
*a^18 + 285*a^16 + 3220*a^14 + 15876*a^12 + 42966*a^10 + 70070*a^8 + 70980*
a^6 + 43860*a^4 + 15181*a^2 + 2261)*b^6*c^3*d^9 + 55*(a^20 + 46*a^18 + 465*
a^16 + 2184*a^14 + 5922*a^12 + 10164*a^10 + 11466*a^8 + 8520*a^6 + 4029*a^4
 + 1102*a^2 + 133)*b^4*c^2*d^10 + 11*(a^22 + 31*a^20 + 255*a^18 + 1065*a^16
 + 2730*a^14 + 4662*a^12 + 5502*a^10 + 4530*a^8 + 2565*a^6 + 955*a^4 + 211*
a^2 + 21)*b^2*c*d^11 + (a^24 + 12*a^22 + 66*a^20 + 220*a^18 + 495*a^16 + 79
2*a^14 + 924*a^12 + 792*a^10 + 495*a^8 + 220*a^6 + 66*a^4 + 12*a^2 + 1)*d^1
2 + (b^24*c^11*d + 11*(a^2 + 21)*b^22*c^10*d^2 + 55*(a^4 + 38*a^2 + 133)*b^
20*c^9*d^3 + 33*(5*a^6 + 255*a^4 + 1615*a^2 + 2261)*b^18*c^8*d^4 + 330*(a^8
 + 60*a^6 + 510*a^4 + 1292*a^2 + 969)*b^16*c^7*d^5 + 22*(21*a^10 + 1365*a^8
 + 13650*a^6 + 46410*a^4 + 62985*a^2 + 29393)*b^14*c^6*d^6 + 22*(21*a^12 +
1386*a^10 + 15015*a^8 + 60060*a^6 + 109395*a^4 + 92378*a^2 + 29393)*b^12*c^
5*d^7 + 330*(a^14 + 63*a^12 + 693*a^10 + 3003*a^8 + 6435*a^6 + 7293*a^4 + 4
199*a^2 + 969)*b^10*c^4*d^8 + 33*(5*a^16 + 280*a^14 + 2940*a^12 + 12936*a^1
0 + 30030*a^8 + 40040*a^6 + 30940*a^4 + 12920*a^2 + 2261)*b^8*c^3*d^9 + 55*
(a^18 + 45*a^16 + 420*a^14 + 1764*a^12 + 4158*a^10 + 6006*a^8 + 5460*a^6 +
3060*a^4 + 969*a^2 + 133)*b^6*c^2*d^10 + 11*(a^20 + 30*a^18 + 225*a^16 + 84
0*a^14 + 1890*a^12 + 2772*a^10 + 2730*a^8 + 1800*a^6 + 765*a^4 + 190*a^2 +
```

21)*b^4*c*d^11 + (a^22 + 11*a^20 + 55*a^18 + 165*a^16 + 330*a^14 + 462*a^12
 + 462*a^10 + 330*a^8 + 165*a^6 + 55*a^4 + 11*a^2 + 1)*b^2*d^12)*x^2 + 2*(1
1*(a^2 + 1)*b^21*c^10*d + 110*(a^4 + 8*a^2 + 7)*b^19*c^9*d^2 + 33*(15*a^6 +
 205*a^4 + 589*a^2 + 399)*b^17*c^8*d^3 + 264*(5*a^8 + 90*a^6 + 408*a^4 + 64
6*a^2 + 323)*b^15*c^7*d^4 + 110*(21*a^10 + 441*a^8 + 2562*a^6 + 6018*a^4 +
6137*a^2 + 2261)*b^13*c^6*d^5 + 4*(693*a^12 + 15708*a^10 + 105105*a^8 + 308
880*a^6 + 449735*a^4 + 319124*a^2 + 88179)*b^11*c^5*d^6 + 110*(21*a^14 + 48
3*a^12 + 3465*a^10 + 11583*a^8 + 20735*a^6 + 20553*a^4 + 10659*a^2 + 2261)*
b^9*c^4*d^7 + 264*(5*a^16 + 110*a^14 + 798*a^12 + 2838*a^10 + 5720*a^8 + 68
90*a^6 + 4930*a^4 + 1938*a^2 + 323)*b^7*c^3*d^8 + 33*(15*a^18 + 295*a^16 +
2044*a^14 + 7308*a^12 + 15554*a^10 + 20930*a^8 + 18060*a^6 + 9724*a^4 + 298
3*a^2 + 399)*b^5*c^2*d^9 + 110*(a^20 + 16*a^18 + 99*a^16 + 336*a^14 + 714*a
^12 + 1008*a^10 + 966*a^8 + 624*a^6 + 261*a^4 + 64*a^2 + 7)*b^3*c*d^10 + 11
*(a^22 + 11*a^20 + 55*a^18 + 165*a^16 + 330*a^14 + 462*a^12 + 462*a^10 + 33
0*a^8 + 165*a^6 + 55*a^4 + 11*a^2 + 1)*b*d^11 + (11*b^23*c^10*d + 110*(a^2
+ 7)*b^21*c^9*d^2 + 33*(15*a^4 + 190*a^2 + 399)*b^19*c^8*d^3 + 264*(5*a^6 +
 85*a^4 + 323*a^2 + 323)*b^17*c^7*d^4 + 110*(21*a^8 + 420*a^6 + 2142*a^4 +
3876*a^2 + 2261)*b^15*c^6*d^5 + 4*(693*a^10 + 15015*a^8 + 90090*a^6 + 21879
0*a^4 + 230945*a^2 + 88179)*b^13*c^5*d^6 + 110*(21*a^12 + 462*a^10 + 3003*a
^8 + 8580*a^6 + 12155*a^4 + 8398*a^2 + 2261)*b^11*c^4*d^7 + 264*(5*a^14 + 1
05*a^12 + 693*a^10 + 2145*a^8 + 3575*a^6 + 3315*a^4 + 1615*a^2 + 323)*b^9*c
^3*d^8 + 33*(15*a^16 + 280*a^14 + 1764*a^12 + 5544*a^10 + 10010*a^8 + 10920
*a^6 + 7140*a^4 + 2584*a^2 + 399)*b^7*c^2*d^9 + 110*(a^18 + 15*a^16 + 84*a^
14 + 252*a^12 + 462*a^10 + 546*a^8 + 420*a^6 + 204*a^4 + 57*a^2 + 7)*b^5*c*
d^10 + 11*(a^20 + 10*a^18 + 45*a^16 + 120*a^14 + 210*a^12 + 252*a^10 + 210*
a^8 + 120*a^6 + 45*a^4 + 10*a^2 + 1)*b^3*d^11)*x^2 + 2*(11*a*b^22*c^10*d +
110*(a^3 + 7*a)*b^20*c^9*d^2 + 33*(15*a^5 + 190*a^3 + 399*a)*b^18*c^8*d^3 +
 264*(5*a^7 + 85*a^5 + 323*a^3 + 323*a)*b^16*c^7*d^4 + 110*(21*a^9 + 420*a^
7 + 2142*a^5 + 3876*a^3 + 2261*a)*b^14*c^6*d^5 + 4*(693*a^11 + 15015*a^9 +
90090*a^7 + 218790*a^5 + 230945*a^3 + 88179*a)*b^12*c^5*d^6 + 110*(21*a^13
+ 462*a^11 + 3003*a^9 + 8580*a^7 + 12155*a^5 + 8398*a^3 + 2261*a)*b^10*c^4*
d^7 + 264*(5*a^15 + 105*a^13 + 693*a^11 + 2145*a^9 + 3575*a^7 + 3315*a^5 +
1615*a^3 + 323*a)*b^8*c^3*d^8 + 33*(15*a^17 + 280*a^15 + 1764*a^13 + 5544*a
^11 + 10010*a^9 + 10920*a^7 + 7140*a^5 + 2584*a^3 + 399*a)*b^6*c^2*d^9 + 11
0*(a^19 + 15*a^17 + 84*a^15 + 252*a^13 + 462*a^11 + 546*a^9 + 420*a^7 + 204
*a^5 + 57*a^3 + 7*a)*b^4*c*d^10 + 11*(a^21 + 10*a^19 + 45*a^17 + 120*a^15 +
 210*a^13 + 252*a^11 + 210*a^9 + 120*a^7 + 45*a^5 + 10*a^3 + a)*b^2*d^11)*x
)*sqrt(c)*sqrt(d) + 2*(a*b^23*c^11*d + 11*(a^3 + 21*a)*b^21*c^10*d^2 + 55*(
a^5 + 38*a^3 + 133*a)*b^19*c^9*d^3 + 33*(5*a^7 + 255*a^5 + 1615*a^3 + 2261*
a)*b^17*c^8*d^4 + 330*(a^9 + 60*a^7 + 510*a^5 + 1292*a^3 + 969*a)*b^15*c^7*
d^5 + 22*(21*a^11 + 1365*a^9 + 13650*a^7 + 46410*a^5 + 62985*a^3 + 29393*a)
*b^13*c^6*d^6 + 22*(21*a^13 + 1386*a^11 + 15015*a^9 + 60060*a^7 + 109395*a^
5 + 92378*a^3 + 29393*a)*b^11*c^5*d^7 + 330*(a^15 + 63*a^13 + 693*a^11 + 30
03*a^9 + 6435*a^7 + 7293*a^5 + 4199*a^3 + 969*a)*b^9*c^4*d^8 + 33*(5*a^17 +
 280*a^15 + 2940*a^13 + 12936*a^11 + 30030*a^9 + 40040*a^7 + 30940*a^5 + 12
920*a^3 + 2261*a)*b^7*c^3*d^9 + 55*(a^19 + 45*a^17 + 420*a^15 + 1764*a^13 +
 4158*a^11 + 6006*a^9 + 5460*a^7 + 3060*a^5 + 969*a^3 + 133*a)*b^5*c^2*d^10
 + 11*(a^21 + 30*a^19 + 225*a^17 + 840*a^15 + 1890*a^13 + 2772*a^11 + 2730*
a^9 + 1800*a^7 + 765*a^5 + 190*a^3 + 21*a)*b^3*c*d^11 + (a^23 + 11*a^21 + 5
5*a^19 + 165*a^17 + 330*a^15 + 462*a^13 + 462*a^11 + 330*a^9 + 165*a^7 + 55
*a^5 + 11*a^3 + a)*b*d^12)*x)/(b^24*c^12 + 12*(a^2 + 23)*b^22*c^11*d + 66*(
a^4 + 42*a^2 + 161)*b^20*c^10*d^2 + 44*(5*a^6 + 285*a^4 + 1995*a^2 + 3059)*
b^18*c^9*d^3 + 99*(5*a^8 + 340*a^6 + 3230*a^4 + 9044*a^2 + 7429)*b^16*c^8*d
^4 + 264*(3*a^10 + 225*a^8 + 2550*a^6 + 9690*a^4 + 14535*a^2 + 7429)*b^14*c
^7*d^5 + 4*(231*a^12 + 18018*a^10 + 225225*a^8 + 1021020*a^6 + 2078505*a^4
+ 1939938*a^2 + 676039)*b^12*c^6*d^6 + 264*(3*a^14 + 231*a^12 + 3003*a^10 +
 15015*a^8 + 36465*a^6 + 46189*a^4 + 29393*a^2 + 7429)*b^10*c^5*d^7 + 99*(5
*a^16 + 360*a^14 + 4620*a^12 + 24024*a^10 + 64350*a^8 + 97240*a^6 + 83980*a
^4 + 38760*a^2 + 7429)*b^8*c^4*d^8 + 44*(5*a^18 + 315*a^16 + 3780*a^14 + 19

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404*a^12 + 54054*a^10 + 90090*a^8 + 92820*a^6 + 58140*a^4 + 20349*a^2 + 305
9)*b^6*c^3*d^9 + 66*(a^20 + 50*a^18 + 525*a^16 + 2520*a^14 + 6930*a^12 + 12
012*a^10 + 13650*a^8 + 10200*a^6 + 4845*a^4 + 1330*a^2 + 161)*b^4*c^2*d^10
 + 12*(a^22 + 33*a^20 + 275*a^18 + 1155*a^16 + 2970*a^14 + 5082*a^12 + 6006*
a^10 + 4950*a^8 + 2805*a^6 + 1045*a^4 + 231*a^2 + 23)*b^2*c*d^11 + (a^24 +
12*a^22 + 66*a^20 + 220*a^18 + 495*a^16 + 792*a^14 + 924*a^12 + 792*a^10 +
495*a^8 + 220*a^6 + 66*a^4 + 12*a^2 + 1)*d^12 + 8*(3*b^23*c^11 + 11*(3*a^2
 + 23)*b^21*c^10*d + 33*(5*a^4 + 70*a^2 + 161)*b^19*c^9*d^2 + 99*(5*a^6 + 95
*a^4 + 399*a^2 + 437)*b^17*c^8*d^3 + 22*(45*a^8 + 1020*a^6 + 5814*a^4 + 116
28*a^2 + 7429)*b^15*c^7*d^4 + 6*(231*a^10 + 5775*a^8 + 39270*a^6 + 106590*a
^4 + 124355*a^2 + 52003)*b^13*c^6*d^5 + 6*(231*a^12 + 6006*a^10 + 45045*a^8
 + 145860*a^6 + 230945*a^4 + 176358*a^2 + 52003)*b^11*c^5*d^6 + 22*(45*a^14
 + 1155*a^12 + 9009*a^10 + 32175*a^8 + 60775*a^6 + 62985*a^4 + 33915*a^2 +
7429)*b^9*c^4*d^7 + 99*(5*a^16 + 120*a^14 + 924*a^12 + 3432*a^10 + 7150*a^8
 + 8840*a^6 + 6460*a^4 + 2584*a^2 + 437)*b^7*c^3*d^8 + 33*(5*a^18 + 105*a^1
6 + 756*a^14 + 2772*a^12 + 6006*a^10 + 8190*a^8 + 7140*a^6 + 3876*a^4 + 119
7*a^2 + 161)*b^5*c^2*d^9 + 11*(3*a^20 + 50*a^18 + 315*a^16 + 1080*a^14 + 23
10*a^12 + 3276*a^10 + 3150*a^8 + 2040*a^6 + 855*a^4 + 210*a^2 + 23)*b^3*c*d
^10 + 3*(a^22 + 11*a^20 + 55*a^18 + 165*a^16 + 330*a^14 + 462*a^12 + 462*a^
10 + 330*a^8 + 165*a^6 + 55*a^4 + 11*a^2 + 1)*b*d^11)*sqrt(c)*sqrt(d))) - l
og(d*x^2 + c)*log(((a^2 + 1)*b^22*c^11*d + 11*(a^4 + 22*a^2 + 21)*b^20*c^10
*d^2 + 55*(a^6 + 39*a^4 + 171*a^2 + 133)*b^18*c^9*d^3 + 33*(5*a^8 + 260*a^6
 + 1870*a^4 + 3876*a^2 + 2261)*b^16*c^8*d^4 + 330*(a^10 + 61*a^8 + 570*a^6
+ 1802*a^4 + 2261*a^2 + 969)*b^14*c^7*d^5 + 22*(21*a^12 + 1386*a^10 + 15015
*a^8 + 60060*a^6 + 109395*a^4 + 92378*a^2 + 29393)*b^12*c^6*d^6 + 22*(21*a^
14 + 1407*a^12 + 16401*a^10 + 75075*a^8 + 169455*a^6 + 201773*a^4 + 121771*
a^2 + 29393)*b^10*c^5*d^7 + 330*(a^16 + 64*a^14 + 756*a^12 + 3696*a^10 + 94
38*a^8 + 13728*a^6 + 11492*a^4 + 5168*a^2 + 969)*b^8*c^4*d^8 + 33*(5*a^18 +
 285*a^16 + 3220*a^14 + 15876*a^12 + 42966*a^10 + 70070*a^8 + 70980*a^6 + 4
3860*a^4 + 15181*a^2 + 2261)*b^6*c^3*d^9 + 55*(a^20 + 46*a^18 + 465*a^16 +
2184*a^14 + 5922*a^12 + 10164*a^10 + 11466*a^8 + 8520*a^6 + 4029*a^4 + 1102
*a^2 + 133)*b^4*c^2*d^10 + 11*(a^22 + 31*a^20 + 255*a^18 + 1065*a^16 + 2730
*a^14 + 4662*a^12 + 5502*a^10 + 4530*a^8 + 2565*a^6 + 955*a^4 + 211*a^2 + 2
1)*b^2*c*d^11 + (a^24 + 12*a^22 + 66*a^20 + 220*a^18 + 495*a^16 + 792*a^14
+ 924*a^12 + 792*a^10 + 495*a^8 + 220*a^6 + 66*a^4 + 12*a^2 + 1)*d^12 + (b^
24*c^11*d + 11*(a^2 + 21)*b^22*c^10*d^2 + 55*(a^4 + 38*a^2 + 133)*b^20*c^9*
d^3 + 33*(5*a^6 + 255*a^4 + 1615*a^2 + 2261)*b^18*c^8*d^4 + 330*(a^8 + 60*a
^6 + 510*a^4 + 1292*a^2 + 969)*b^16*c^7*d^5 + 22*(21*a^10 + 1365*a^8 + 1365
0*a^6 + 46410*a^4 + 62985*a^2 + 29393)*b^14*c^6*d^6 + 22*(21*a^12 + 1386*a^
10 + 15015*a^8 + 60060*a^6 + 109395*a^4 + 92378*a^2 + 29393)*b^12*c^5*d^7 +
 330*(a^14 + 63*a^12 + 693*a^10 + 3003*a^8 + 6435*a^6 + 7293*a^4 + 4199*a^2
 + 969)*b^10*c^4*d^8 + 33*(5*a^16 + 280*a^14 + 2940*a^12 + 12936*a^10 + 300
30*a^8 + 40040*a^6 + 30940*a^4 + 12920*a^2 + 2261)*b^8*c^3*d^9 + 55*(a^18 +
 45*a^16 + 420*a^14 + 1764*a^12 + 4158*a^10 + 6006*a^8 + 5460*a^6 + 3060*a^
4 + 969*a^2 + 133)*b^6*c^2*d^10 + 11*(a^20 + 30*a^18 + 225*a^16 + 840*a^14
+ 1890*a^12 + 2772*a^10 + 2730*a^8 + 1800*a^6 + 765*a^4 + 190*a^2 + 21)*b^4
*c*d^11 + (a^22 + 11*a^20 + 55*a^18 + 165*a^16 + 330*a^14 + 462*a^12 + 462*
a^10 + 330*a^8 + 165*a^6 + 55*a^4 + 11*a^2 + 1)*b^2*d^12)*x^2 - 2*(11*(a^2
+ 1)*b^21*c^10*d + 110*(a^4 + 8*a^2 + 7)*b^19*c^9*d^2 + 33*(15*a^6 + 205*a^
4 + 589*a^2 + 399)*b^17*c^8*d^3 + 264*(5*a^8 + 90*a^6 + 408*a^4 + 646*a^2 +
 323)*b^15*c^7*d^4 + 110*(21*a^10 + 441*a^8 + 2562*a^6 + 6018*a^4 + 6137*a^
2 + 2261)*b^13*c^6*d^5 + 4*(693*a^12 + 15708*a^10 + 105105*a^8 + 308880*a^6
 + 449735*a^4 + 319124*a^2 + 88179)*b^11*c^5*d^6 + 110*(21*a^14 + 483*a^12
+ 3465*a^10 + 11583*a^8 + 20735*a^6 + 20553*a^4 + 10659*a^2 + 2261)*b^9*c^4
*d^7 + 264*(5*a^16 + 110*a^14 + 798*a^12 + 2838*a^10 + 5720*a^8 + 6890*a^6
+ 4930*a^4 + 1938*a^2 + 323)*b^7*c^3*d^8 + 33*(15*a^18 + 295*a^16 + 2044*a^
14 + 7308*a^12 + 15554*a^10 + 20930*a^8 + 18060*a^6 + 9724*a^4 + 2983*a^2 +
 399)*b^5*c^2*d^9 + 110*(a^20 + 16*a^18 + 99*a^16 + 336*a^14 + 714*a^12 + 1
008*a^10 + 966*a^8 + 624*a^6 + 261*a^4 + 64*a^2 + 7)*b^3*c*d^10 + 11*(a^22
```

```
+ 11*a^20 + 55*a^18 + 165*a^16 + 330*a^14 + 462*a^12 + 462*a^10 + 330*a^8 +
 165*a^6 + 55*a^4 + 11*a^2 + 1)*b*d^11 + (11*b^23*c^10*d + 110*(a^2 + 7)*b^
21*c^9*d^2 + 33*(15*a^4 + 190*a^2 + 399)*b^19*c^8*d^3 + 264*(5*a^6 + 85*a^4
 + 323*a^2 + 323)*b^17*c^7*d^4 + 110*(21*a^8 + 420*a^6 + 2142*a^4 + 3876*a^
2 + 2261)*b^15*c^6*d^5 + 4*(693*a^10 + 15015*a^8 + 90090*a^6 + 218790*a^4 +
 230945*a^2 + 88179)*b^13*c^5*d^6 + 110*(21*a^12 + 462*a^10 + 3003*a^8 + 85
80*a^6 + 12155*a^4 + 8398*a^2 + 2261)*b^11*c^4*d^7 + 264*(5*a^14 + 105*a^12
 + 693*a^10 + 2145*a^8 + 3575*a^6 + 3315*a^4 + 1615*a^2 + 323)*b^9*c^3*d^8
+ 33*(15*a^16 + 280*a^14 + 1764*a^12 + 5544*a^10 + 10010*a^8 + 10920*a^6 +
7140*a^4 + 2584*a^2 + 399)*b^7*c^2*d^9 + 110*(a^18 + 15*a^16 + 84*a^14 + 25
2*a^12 + 462*a^10 + 546*a^8 + 420*a^6 + 204*a^4 + 57*a^2 + 7)*b^5*c*d^10 +
11*(a^20 + 10*a^18 + 45*a^16 + 120*a^14 + 210*a^12 + 252*a^10 + 210*a^8 + 1
20*a^6 + 45*a^4 + 10*a^2 + 1)*b^3*d^11)*x^2 + 2*(11*a*b^22*c^10*d + 110*(a^
3 + 7*a)*b^20*c^9*d^2 + 33*(15*a^5 + 190*a^3 + 399*a)*b^18*c^8*d^3 + 264*(5
*a^7 + 85*a^5 + 323*a^3 + 323*a)*b^16*c^7*d^4 + 110*(21*a^9 + 420*a^7 + 214
2*a^5 + 3876*a^3 + 2261*a)*b^14*c^6*d^5 + 4*(693*a^11 + 15015*a^9 + 90090*a
^7 + 218790*a^5 + 230945*a^3 + 88179*a)*b^12*c^5*d^6 + 110*(21*a^13 + 462*a
^11 + 3003*a^9 + 8580*a^7 + 12155*a^5 + 8398*a^3 + 2261*a)*b^10*c^4*d^7 + 2
64*(5*a^15 + 105*a^13 + 693*a^11 + 2145*a^9 + 3575*a^7 + 3315*a^5 + 1615*a^
3 + 323*a)*b^8*c^3*d^8 + 33*(15*a^17 + 280*a^15 + 1764*a^13 + 5544*a^11 + 1
0010*a^9 + 10920*a^7 + 7140*a^5 + 2584*a^3 + 399*a)*b^6*c^2*d^9 + 110*(a^19
 + 15*a^17 + 84*a^15 + 252*a^13 + 462*a^11 + 546*a^9 + 420*a^7 + 204*a^5 +
57*a^3 + 7*a)*b^4*c*d^10 + 11*(a^21 + 10*a^19 + 45*a^17 + 120*a^15 + 210*a^
13 + 252*a^11 + 210*a^9 + 120*a^7 + 45*a^5 + 10*a^3 + a)*b^2*d^11)*x)*sqrt(
c)*sqrt(d) + 2*(a*b^23*c^11*d + 11*(a^3 + 21*a)*b^21*c^10*d^2 + 55*(a^5 + 3
8*a^3 + 133*a)*b^19*c^9*d^3 + 33*(5*a^7 + 255*a^5 + 1615*a^3 + 2261*a)*b^17
*c^8*d^4 + 330*(a^9 + 60*a^7 + 510*a^5 + 1292*a^3 + 969*a)*b^15*c^7*d^5 + 2
2*(21*a^11 + 1365*a^9 + 13650*a^7 + 46410*a^5 + 62985*a^3 + 29393*a)*b^13*c
^6*d^6 + 22*(21*a^13 + 1386*a^11 + 15015*a^9 + 60060*a^7 + 109395*a^5 + 923
78*a^3 + 29393*a)*b^11*c^5*d^7 + 330*(a^15 + 63*a^13 + 693*a^11 + 3003*a^9
+ 6435*a^7 + 7293*a^5 + 4199*a^3 + 969*a)*b^9*c^4*d^8 + 33*(5*a^17 + 280*a^
15 + 2940*a^13 + 12936*a^11 + 30030*a^9 + 40040*a^7 + 30940*a^5 + 12920*a^3
 + 2261*a)*b^7*c^3*d^9 + 55*(a^19 + 45*a^17 + 420*a^15 + 1764*a^13 + 4158*a
^11 + 6006*a^9 + 5460*a^7 + 3060*a^5 + 969*a^3 + 133*a)*b^5*c^2*d^10 + 11*(
a^21 + 30*a^19 + 225*a^17 + 840*a^15 + 1890*a^13 + 2772*a^11 + 2730*a^9 + 1
800*a^7 + 765*a^5 + 190*a^3 + 21*a)*b^3*c*d^11 + (a^23 + 11*a^21 + 55*a^19
+ 165*a^17 + 330*a^15 + 462*a^13 + 462*a^11 + 330*a^9 + 165*a^7 + 55*a^5 +
11*a^3 + a)*b*d^12)*x)/(b^24*c^12 + 12*(a^2 + 23)*b^22*c^11*d + 66*(a^4 + 4
2*a^2 + 161)*b^20*c^10*d^2 + 44*(5*a^6 + 285*a^4 + 1995*a^2 + 3059)*b^18*c^
9*d^3 + 99*(5*a^8 + 340*a^6 + 3230*a^4 + 9044*a^2 + 7429)*b^16*c^8*d^4 + 26
4*(3*a^10 + 225*a^8 + 2550*a^6 + 9690*a^4 + 14535*a^2 + 7429)*b^14*c^7*d^5
+ 4*(231*a^12 + 18018*a^10 + 225225*a^8 + 1021020*a^6 + 2078505*a^4 + 19399
38*a^2 + 676039)*b^12*c^6*d^6 + 264*(3*a^14 + 231*a^12 + 3003*a^10 + 15015*
a^8 + 36465*a^6 + 46189*a^4 + 29393*a^2 + 7429)*b^10*c^5*d^7 + 99*(5*a^16 +
 360*a^14 + 4620*a^12 + 24024*a^10 + 64350*a^8 + 97240*a^6 + 83980*a^4 + 38
760*a^2 + 7429)*b^8*c^4*d^8 + 44*(5*a^18 + 315*a^16 + 3780*a^14 + 19404*a^1
2 + 54054*a^10 + 90090*a^8 + 92820*a^6 + 58140*a^4 + 20349*a^2 + 3059)*b^6*
c^3*d^9 + 66*(a^20 + 50*a^18 + 525*a^16 + 2520*a^14 + 6930*a^12 + 12012*a^1
0 + 13650*a^8 + 10200*a^6 + 4845*a^4 + 1330*a^2 + 161)*b^4*c^2*d^10 + 12*(a
^22 + 33*a^20 + 275*a^18 + 1155*a^16 + 2970*a^14 + 5082*a^12 + 6006*a^10 +
4950*a^8 + 2805*a^6 + 1045*a^4 + 231*a^2 + 23)*b^2*c*d^11 + (a^24 + 12*a^22
 + 66*a^20 + 220*a^18 + 495*a^16 + 792*a^14 + 924*a^12 + 792*a^10 + 495*a^8
 + 220*a^6 + 66*a^4 + 12*a^2 + 1)*d^12 - 8*(3*b^23*c^11 + 11*(3*a^2 + 23)*b
^21*c^10*d + 33*(5*a^4 + 70*a^2 + 161)*b^19*c^9*d^2 + 99*(5*a^6 + 95*a^4 +
399*a^2 + 437)*b^17*c^8*d^3 + 22*(45*a^8 + 1020*a^6 + 5814*a^4 + 11628*a^2
+ 7429)*b^15*c^7*d^4 + 6*(231*a^10 + 5775*a^8 + 39270*a^6 + 106590*a^4 + 12
4355*a^2 + 52003)*b^13*c^6*d^5 + 6*(231*a^12 + 6006*a^10 + 45045*a^8 + 1458
60*a^6 + 230945*a^4 + 176358*a^2 + 52003)*b^11*c^5*d^6 + 22*(45*a^14 + 1155
*a^12 + 9009*a^10 + 32175*a^8 + 60775*a^6 + 62985*a^4 + 33915*a^2 + 7429)*b
```

```
^9*c^4*d^7 + 99*(5*a^16 + 120*a^14 + 924*a^12 + 3432*a^10 + 7150*a^8 + 8840
*a^6 + 6460*a^4 + 2584*a^2 + 437)*b^7*c^3*d^8 + 33*(5*a^18 + 105*a^16 + 756
*a^14 + 2772*a^12 + 6006*a^10 + 8190*a^8 + 7140*a^6 + 3876*a^4 + 1197*a^2 +
 161)*b^5*c^2*d^9 + 11*(3*a^20 + 50*a^18 + 315*a^16 + 1080*a^14 + 2310*a^12
 + 3276*a^10 + 3150*a^8 + 2040*a^6 + 855*a^4 + 210*a^2 + 23)*b^3*c*d^10 + 3
*(a^22 + 11*a^20 + 55*a^18 + 165*a^16 + 330*a^14 + 462*a^12 + 462*a^10 + 33
0*a^8 + 165*a^6 + 55*a^4 + 11*a^2 + 1)*b*d^11)*sqrt(c)*sqrt(d))) + 2*dilog(
((a + I)*b*d*x + b^2*c + (I*b^2*x + (-I*a + 1)*b)*sqrt(c)*sqrt(d))/(b^2*c +
 2*(-I*a + 1)*b*sqrt(c)*sqrt(d) - (a^2 + 2*I*a - 1)*d)) - 2*dilog(((a + I)*
b*d*x + b^2*c - (I*b^2*x + (-I*a + 1)*b)*sqrt(c)*sqrt(d))/(b^2*c - 2*(-I*a
+ 1)*b*sqrt(c)*sqrt(d) - (a^2 + 2*I*a - 1)*d)) - 2*dilog(((a - I)*b*d*x + b
^2*c + (I*b^2*x + (-I*a - 1)*b)*sqrt(c)*sqrt(d))/(b^2*c + 2*(-I*a - 1)*b*sq
rt(c)*sqrt(d) - (a^2 - 2*I*a - 1)*d)) + 2*dilog(((a - I)*b*d*x + b^2*c - (I
*b^2*x + (-I*a - 1)*b)*sqrt(c)*sqrt(d))/(b^2*c - 2*(-I*a - 1)*b*sqrt(c)*sqr
t(d) - (a^2 - 2*I*a - 1)*d)))/b)/sqrt(c*d) + arctan(b*x + a)*arctan(d*x/sqr
t(c*d))/sqrt(c*d) - arctan(d*x/sqrt(c*d))*arctan((b^2*x + a*b)/b)/sqrt(c*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a+b\,x)}{d\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)/(c + d*x^2),x)
```

```
[Out] int(atan(a + b*x)/(c + d*x^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

3.54 $\int \frac{\tan^{-1}(a+bx)}{c+dx} dx$

Optimal. Leaf size=152

$$-\frac{i\text{Li}_2\left(1-\frac{2b(c+dx)}{(bc-ad+id)(1-i(a+bx))}\right)}{2d}+\frac{\tan^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(1-i(a+bx))(-ad+bc+id)}\right)}{d}+\frac{i\text{Li}_2\left(1-\frac{2}{1-i(a+bx)}\right)}{2d}-\frac{\log\left(\frac{2}{1-i(a+bx)}\right)\tan^{-1}}{d}$$

```
[Out] -arctan(b*x+a)*ln(2/(1-I*(b*x+a)))/d+arctan(b*x+a)*ln(2*b*(d*x+c)/(b*c+I*d-
a*d)/(1-I*(b*x+a)))/d+1/2*I*polylog(2,1-2/(1-I*(b*x+a)))/d-1/2*I*polylog(2,
1-2*b*(d*x+c)/(b*c+I*d-a*d)/(1-I*(b*x+a)))/d
```

Rubi [A] time = 0.14, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5047, 4856, 2402, 2315, 2447}

$$-\frac{i\text{PolyLog}\left(2,1-\frac{2b(c+dx)}{(1-i(a+bx))(-ad+bc+id)}\right)}{2d}+\frac{i\text{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)}{2d}+\frac{\tan^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(1-i(a+bx))(-ad+bc+id)}\right)}{d}-\frac{\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/(c + d*x),x]
```

```
[Out] -((ArcTan[a + b*x]*Log[2/(1 - I*(a + b*x))])/d) + (ArcTan[a + b*x]*Log[(2*b
*(c + d*x))/((b*c + I*d - a*d)*(1 - I*(a + b*x)))])/d + ((I/2)*PolyLog[2, 1
 - 2/(1 - I*(a + b*x))])/d - ((I/2)*PolyLog[2, 1 - (2*b*(c + d*x))/((b*c +
I*d - a*d)*(1 - I*(a + b*x)))])/d
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
```

```
tQ[p, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{c+dx}\,dx = \frac{\text{Subst}\left(\int \frac{\tan^{-1}(x)}{\frac{bc-ad}{b}+\frac{dx}{b}}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\tan^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)}{d} + \frac{\tan^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{d} + \frac{\text{Subst}\left(\int \frac{\log}{1}\right.}{}$$

$$= -\frac{\tan^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)}{d} + \frac{\tan^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{d} - \frac{i\text{Li}_2\left(1-\frac{}{(bc+}\right.}{}$$

$$= -\frac{\tan^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)}{d} + \frac{\tan^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{d} + \frac{i\text{Li}_2\left(1-\frac{}{1-i(}\right.}{2d}$$

Mathematica [A] time = 0.02, size = 231, normalized size = 1.52

$$\frac{i\text{Li}_2\left(-\frac{id(1-i(a+bx))}{bc-ad-id}\right)}{2d} - \frac{i\text{Li}_2\left(\frac{id(i(a+bx)+1)}{bc-ad+id}\right)}{2d} + \frac{i\log(1-i(a+bx))\log\left(-\frac{i\left(\frac{bc-ad}{b}+\frac{d(a+bx)}{b}\right)}{-\frac{d}{b}-\frac{i(bc-ad)}{b}}\right)}{2d} - \frac{i\log(1+i(a+bx))\log\left(\frac{i\left(\frac{bc}{}\right.}{-}\right.}{2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a + b*x]/(c + d*x),x]
```

```
[Out] ((I/2)*Log[1 - I*(a + b*x)]*Log[((-I)*((b*c - a*d)/b + (d*(a + b*x))/b))/(-
(d/b) - (I*(b*c - a*d))/b)])/d - ((I/2)*Log[1 + I*(a + b*x)]*Log[(I*((b*c -
 a*d)/b + (d*(a + b*x))/b))/(-(d/b) + (I*(b*c - a*d))/b)])/d + ((I/2)*PolyL
og[2, ((-I)*d*(1 - I*(a + b*x)))/(b*c - I*d - a*d)])/d - ((I/2)*PolyLog[2,
(I*d*(1 + I*(a + b*x)))/(b*c + I*d - a*d)])/d
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\arctan(bx+a)}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(arctan(b*x + a)/(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.06, size = 198, normalized size = 1.30

$$\frac{\ln\left(d\left(bx+a\right)-ad+bc\right)\arctan\left(bx+a\right)}{d}+\frac{i\ln\left(d\left(bx+a\right)-ad+bc\right)\ln\left(\frac{id-d(bx+a)}{-ad+bc+id}\right)}{2d}-\frac{i\ln\left(d\left(bx+a\right)-ad+bc\right)\ln\left(\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(d*x+c),x)
```

```
[Out] ln(d*(b*x+a)-a*d+b*c)/d*arctan(b*x+a)+1/2*I*ln(d*(b*x+a)-a*d+b*c)/d*ln((I*d
-d*(b*x+a))/(b*c+I*d-a*d))-1/2*I*ln(d*(b*x+a)-a*d+b*c)/d*ln((I*d+d*(b*x+a))
/(I*d+a*d-b*c))+1/2*I/d*dilog((I*d-d*(b*x+a))/(b*c+I*d-a*d))-1/2*I/d*dilog(
(I*d+d*(b*x+a))/(I*d+a*d-b*c))
```

maxima [B] time = 0.53, size = 284, normalized size = 1.87

$$\frac{\arctan\left(bx+a\right)\log\left(dx+c\right)}{d}-\frac{\arctan\left(\frac{b^2x+ab}{b}\right)\log\left(dx+c\right)}{d}-\frac{\arctan\left(\frac{bd^2x+bcd}{b^2c^2-2\,abcd+\left(a^2+1\right)d^2},\frac{b^2c^2-abcd+\left(b^2cd-abd^2\right)x}{b^2c^2-2\,abcd+\left(a^2+1\right)d^2}\right)\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] arctan(b*x + a)*log(d*x + c)/d - arctan((b^2*x + a*b)/b)*log(d*x + c)/d - 1
/2*(arctan2((b*d^2*x + b*c*d)/(b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2), (b^2*c
^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x)/(b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)
)*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - arctan(b*x + a)*log((b^2*d^2*x^2 + 2*b
^2*c*d*x + b^2*c^2)/(b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)) + I*dilog((I*b*d
*x + (I*a + 1)*d)/(-I*b*c + (I*a + 1)*d)) - I*dilog((I*b*d*x + (I*a - 1)*d)
/(-I*b*c + (I*a - 1)*d)))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}\left(a+b\,x\right)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)/(c + d*x),x)
```

```
[Out] int(atan(a + b*x)/(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(atan(a + b*x)/(c + d*x), x)
```

3.55 $\int \frac{\tan^{-1}(a+bx)}{c+\frac{d}{x}} dx$

Optimal. Leaf size=244

$$\frac{id\text{Li}_2\left(\frac{c(-a-bx+i)}{-ac+ic+bd}\right)}{2c^2}-\frac{id\text{Li}_2\left(\frac{c(a+bx+i)}{(a+i)c-bd}\right)}{2c^2}+\frac{id\log(ia+ibx+1)\log\left(\frac{b(cx+d)}{bd+(-a+i)c}\right)}{2c^2}-\frac{id\log(-ia-ibx+1)\log\left(-\frac{b(cx+d)}{-bd+(a+i)c}\right)}{2c^2}$$

```
[Out] -1/2*(1+I*a+I*b*x)*ln(1+I*a+I*b*x)/b/c-1/2*(1-I*a-I*b*x)*ln(-I*(I+a+b*x))/b
/c-1/2*I*d*ln(1-I*a-I*b*x)*ln(-b*(c*x+d)/((I+a)*c-b*d))/c^2+1/2*I*d*ln(1+I*
a+I*b*x)*ln(b*(c*x+d)/((I-a)*c+b*d))/c^2+1/2*I*d*polylog(2,c*(I-a-b*x)/(I*c
-a*c+b*d))/c^2-1/2*I*d*polylog(2,c*(I+a+b*x)/((I+a)*c-b*d))/c^2
```

Rubi [A] time = 0.24, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5051, 2409, 2389, 2295, 2394, 2393, 2391}

$$\frac{id\text{PolyLog}\left(2,\frac{c(-a-bx+i)}{-ac+bd+ic}\right)}{2c^2}-\frac{id\text{PolyLog}\left(2,\frac{c(a+bx+i)}{-bd+(a+i)c}\right)}{2c^2}+\frac{id\log(ia+ibx+1)\log\left(\frac{b(cx+d)}{bd+(-a+i)c}\right)}{2c^2}-\frac{id\log(-ia-ibx+1)\ldots}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/(c + d/x),x]
```

```
[Out] -((1 + I*a + I*b*x)*Log[1 + I*a + I*b*x])/(2*b*c) - ((1 - I*a - I*b*x)*Log[
(-I)*(I + a + b*x)])/(2*b*c) - ((I/2)*d*Log[1 - I*a - I*b*x]*Log[-((b*(d +
c*x))/((I + a)*c - b*d))])/c^2 + ((I/2)*d*Log[1 + I*a + I*b*x]*Log[(b*(d +
c*x))/((I - a)*c + b*d)])/c^2 + ((I/2)*d*PolyLog[2, (c*(I - a - b*x))/(I*c
- a*c + b*d)])/c^2 - ((I/2)*d*PolyLog[2, (c*(I + a + b*x))/((I + a)*c - b*d
)])/c^2
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 5051

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{c+\frac{d}{x}}\,dx = \frac{1}{2}i\int \frac{\log(1-ia-ibx)}{c+\frac{d}{x}}\,dx - \frac{1}{2}i\int \frac{\log(1+ia+ibx)}{c+\frac{d}{x}}\,dx$$

$$= \frac{1}{2}i\int \left(\frac{\log(1-ia-ibx)}{c} - \frac{d\log(1-ia-ibx)}{c(d+cx)}\right)dx - \frac{1}{2}i\int \left(\frac{\log(1+ia+ibx)}{c} - \frac{d\log(1+i}{c(d+}\right.$$

$$= \frac{i\int \log(1-ia-ibx)\,dx}{2c} - \frac{i\int \log(1+ia+ibx)\,dx}{2c} - \frac{(id)\int \frac{\log(1-ia-ibx)}{d+cx}\,dx}{2c} + \frac{(id)\int \frac{\log(1+ia+}{d+cx}}{2c}$$

$$= -\frac{id\log(1-ia-ibx)\log\left(-\frac{b(d+cx)}{(i+a)c-bd}\right)}{2c^2} + \frac{id\log(1+ia+ibx)\log\left(\frac{b(d+cx)}{(i-a)c+bd}\right)}{2c^2} - \text{Subst}(\int \log($$

$$= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} - \frac{id\log(1-ia-ibx)}{2c^2}$$

$$= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} - \frac{id\log(1-ia-ibx)}{2c^2}$$

Mathematica [B] time = 11.80, size = 771, normalized size = 3.16

$$bcd\sqrt{a^2-\frac{2abd}{c}+\frac{b^2d^2}{c^2}+1}\,\tan^{-1}(a+bx)^2 e^{-i\tan^{-1}\left(a-\frac{bd}{c}\right)} - 2a^2c^2\tan^{-1}(a+bx) + 2b^2d^2\tan^{-1}\left(a-\frac{bd}{c}\right)\log\left(1-\exp\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a + b*x]/(c + d/x),x]
```

```
[Out] (-2*a^2*c^2*ArcTan[a + b*x] + 2*a*b*c*d*ArcTan[a + b*x] + I*a*b*c*d*Pi*ArcT
an[a + b*x] - I*b^2*d^2*Pi*ArcTan[a + b*x] - 2*a*b*c^2*x*ArcTan[a + b*x] +
2*b^2*c*d*x*ArcTan[a + b*x] + (2*I)*a*b*c*d*ArcTan[a - (b*d)/c]*ArcTan[a +
b*x] - (2*I)*b^2*d^2*ArcTan[a - (b*d)/c]*ArcTan[a + b*x] - b*c*d*ArcTan[a +
 b*x]^2 + I*a*b*c*d*ArcTan[a + b*x]^2 - I*b^2*d^2*ArcTan[a + b*x]^2 + (b*c*
d*Sqrt[1 + a^2 - (2*a*b*d)/c + (b^2*d^2)/c^2]*ArcTan[a + b*x]^2)/E^(I*ArcTa
n[a - (b*d)/c]) + a*b*c*d*Pi*Log[1 + E^((-2*I)*ArcTan[a + b*x])] - b^2*d^2*
Pi*Log[1 + E^((-2*I)*ArcTan[a + b*x])] - 2*a*b*c*d*ArcTan[a + b*x]*Log[1 +
E^((2*I)*ArcTan[a + b*x])] + 2*b^2*d^2*ArcTan[a + b*x]*Log[1 + E^((2*I)*Arc
Tan[a + b*x])] - 2*a*b*c*d*ArcTan[a - (b*d)/c]*Log[1 - E^((2*I)*(-ArcTan[a
- (b*d)/c] + ArcTan[a + b*x]))] + 2*b^2*d^2*ArcTan[a - (b*d)/c]*Log[1 - E^(
```

```
(2*I)*(-ArcTan[a - (b*d)/c] + ArcTan[a + b*x]))] + 2*a*b*c*d*ArcTan[a + b*x
]*Log[1 - E^((2*I)*(-ArcTan[a - (b*d)/c] + ArcTan[a + b*x]))] - 2*b^2*d^2*A
rcTan[a + b*x]*Log[1 - E^((2*I)*(-ArcTan[a - (b*d)/c] + ArcTan[a + b*x]))]
- 2*a*c^2*Log[1/Sqrt[1 + (a + b*x)^2]] + 2*b*c*d*Log[1/Sqrt[1 + (a + b*x)^2
]] - a*b*c*d*Pi*Log[1/Sqrt[1 + (a + b*x)^2]] + b^2*d^2*Pi*Log[1/Sqrt[1 + (a
 + b*x)^2]] + 2*a*b*c*d*ArcTan[a - (b*d)/c]*Log[-Sin[ArcTan[a - (b*d)/c] -
ArcTan[a + b*x]]] - 2*b^2*d^2*ArcTan[a - (b*d)/c]*Log[-Sin[ArcTan[a - (b*d)
/c] - ArcTan[a + b*x]]] + I*b*d*(a*c - b*d)*PolyLog[2, -E^((2*I)*ArcTan[a +
 b*x])] + I*b*d*(-(a*c) + b*d)*PolyLog[2, E^((2*I)*(-ArcTan[a - (b*d)/c] +
ArcTan[a + b*x]))])/(b*c^2*(-2*a*c + 2*b*d))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x\arctan(bx+a)}{cx+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x),x, algorithm="fricas")
```

```
[Out] integral(x*arctan(b*x + a)/(c*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.07, size = 317, normalized size = 1.30

$$\frac{\arctan(bx+a)\,x}{c}+\frac{\arctan(bx+a)\,a}{bc}-\frac{\arctan(bx+a)\,d\ln(c(bx+a)-ac+bd)}{c^2}-\frac{\ln\left(a^2c^2-2abcd+b^2d^2+2(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(c+1/x*d),x)
```

```
[Out] arctan(b*x+a)/c*x+1/b*arctan(b*x+a)/c*a-arctan(b*x+a)*d/c^2*ln(c*(b*x+a)-a*
c+b*d)-1/2/b/c*ln(a^2*c^2-2*a*b*c*d+b^2*d^2+2*(c*(b*x+a)-a*c+b*d)*a*c-2*(c*
(b*x+a)-a*c+b*d)*b*d+(c*(b*x+a)-a*c+b*d)^2+c^2)-1/2*I/c^2*d*ln(c*(b*x+a)-a*
c+b*d)*ln((I*c-c*(b*x+a))/(I*c-a*c+b*d))+1/2*I/c^2*d*ln(c*(b*x+a)-a*c+b*d)*
ln((I*c+c*(b*x+a))/(I*c+a*c-b*d))-1/2*I/c^2*d*dilog((I*c-c*(b*x+a))/(I*c-a*
c+b*d))+1/2*I/c^2*d*dilog((I*c+c*(b*x+a))/(I*c+a*c-b*d))
```

maxima [A] time = 0.55, size = 284, normalized size = 1.16

$$-\frac{bd\arctan(bx+a)\log\left(-\frac{b^2c^2x^2+2\,b^2cdx+b^2d^2}{2\,abcd-b^2d^2-\left(a^2+1\right)c^2}\right)+i\,bd\mathrm{Li}_2\left(-\frac{i\,bcx+(i\,a-1)c}{(-i\,a+1)c+i\,bd}\right)-i\,bd\mathrm{Li}_2\left(-\frac{i\,bcx+(i\,a+1)c}{(-i\,a-1)c+i\,bd}\right)-2\,(bcx+ac)\,a}{2\,bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x),x, algorithm="maxima")
```

```
[Out] -1/2*(b*d*arctan(b*x + a)*log(-(b^2*c^2*x^2 + 2*b^2*c*d*x + b^2*d^2)/(2*a*b
*c*d - b^2*d^2 - (a^2 + 1)*c^2)) + I*b*d*dilog(-(I*b*c*x + (I*a - 1)*c)/((-
I*a + 1)*c + I*b*d)) - I*b*d*dilog(-(I*b*c*x + (I*a + 1)*c)/((-I*a - 1)*c +
 I*b*d)) - 2*(b*c*x + a*c)*arctan(b*x + a) - (b*d*arctan2(-(b*c^2*x + b*c*d
)/(2*a*b*c*d - b^2*d^2 - (a^2 + 1)*c^2), (a*b*c*d - b^2*d^2 + (a*b*c^2 - b^
```

```
2*c*d)*x)/(2*a*b*c*d - b^2*d^2 - (a^2 + 1)*c^2)) - c)*log(b^2*x^2 + 2*a*b*x
 + a^2 + 1))/(b*c^2)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}\left(a+b\,x\right)}{c+\frac{d}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)/(c + d/x),x)
[Out] int(atan(a + b*x)/(c + d/x), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(c+d/x),x)
[Out] Timed out
```

3.56 $\int \frac{\tan^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx$

Optimal. Leaf size=668

$$\frac{i\sqrt{d}\,\mathrm{Li}_2\left(\frac{\sqrt{-c}\,(-a-bx+i)}{-\sqrt{-c}\,a+i\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}}-\frac{i\sqrt{d}\,\mathrm{Li}_2\left(\frac{\sqrt{-c}\,(ia+ibx+1)}{(ia+1)\sqrt{-c}-ib\sqrt{d}}\right)}{4(-c)^{3/2}}+\frac{i\sqrt{d}\,\mathrm{Li}_2\left(\frac{\sqrt{-c}\,(a+bx+i)}{\sqrt{-c}\,a+i\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}}-\frac{i\sqrt{d}\,\mathrm{Li}_2\left(\frac{\sqrt{-c}\,(a+bx+i)}{\sqrt{-c}\,a+i\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}}+\cdots$$

```
[Out] -1/2*(1+I*a+I*b*x)*ln(1+I*a+I*b*x)/b/c-1/2*(1-I*a-I*b*x)*ln(-I*(I+a+b*x))/b
/c+1/4*I*ln(1+I*a+I*b*x)*ln(-b*(-x*(-c)^(1/2)+d^(1/2))/(I*(-c)^(1/2)-a*(-c)
^(1/2)-b*d^(1/2)))*d^(1/2)/(-c)^(3/2)+1/4*I*ln(1-I*a-I*b*x)*ln(-b*(x*(-c)^(
1/2)+d^(1/2))/((I+a)*(-c)^(1/2)-b*d^(1/2)))*d^(1/2)/(-c)^(3/2)-1/4*I*ln(1+I
*a+I*b*x)*ln(b*(x*(-c)^(1/2)+d^(1/2))/(I*(-c)^(1/2)-a*(-c)^(1/2)+b*d^(1/2))
)*d^(1/2)/(-c)^(3/2)-1/4*I*ln(1-I*a-I*b*x)*ln(b*(-x*(-c)^(1/2)+d^(1/2))/(I*
(-c)^(1/2)+a*(-c)^(1/2)+b*d^(1/2)))*d^(1/2)/(-c)^(3/2)+1/4*I*polylog(2,(I-a
-b*x)*(-c)^(1/2)/(I*(-c)^(1/2)-a*(-c)^(1/2)-b*d^(1/2)))*d^(1/2)/(-c)^(3/2)+
1/4*I*polylog(2,(I+a+b*x)*(-c)^(1/2)/(I*(-c)^(1/2)+a*(-c)^(1/2)-b*d^(1/2)))
*d^(1/2)/(-c)^(3/2)-1/4*I*polylog(2,(1+I*a+I*b*x)*(-c)^(1/2)/((1+I*a)*(-c)^
(1/2)-I*b*d^(1/2)))*d^(1/2)/(-c)^(3/2)-1/4*I*polylog(2,(I+a+b*x)*(-c)^(1/2)
/(I*(-c)^(1/2)+a*(-c)^(1/2)+b*d^(1/2)))*d^(1/2)/(-c)^(3/2)
```

Rubi [A] time = 0.85, antiderivative size = 668, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5051, 2409, 2389, 2295, 2394, 2393, 2391}

$$\frac{i\sqrt{d}\,\text{PolyLog}\left(2,\frac{\sqrt{-c}\,(-a-bx+i)}{a\left(-\sqrt{-c}\right)-b\sqrt{d}+i\sqrt{-c}}\right)}{4(-c)^{3/2}}-\frac{i\sqrt{d}\,\text{PolyLog}\left(2,\frac{\sqrt{-c}\,(ia+ibx+1)}{(1+ia)\sqrt{-c}-ib\sqrt{d}}\right)}{4(-c)^{3/2}}+\frac{i\sqrt{d}\,\text{PolyLog}\left(2,\frac{\sqrt{-c}\,(a+bx+i)}{a\sqrt{-c}-b\sqrt{d}+i\sqrt{-c}}\right)}{4(-c)^{3/2}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/(c + d/x^2),x]
```

```
[Out] -((1 + I*a + I*b*x)*Log[1 + I*a + I*b*x])/(2*b*c) - ((1 - I*a - I*b*x)*Log[
(-I)*(I + a + b*x)])/(2*b*c) + ((I/4)*Sqrt[d]*Log[1 + I*a + I*b*x]*Log[-((b
*(Sqrt[d] - Sqrt[-c]*x))/(I*Sqrt[-c] - a*Sqrt[-c] - b*Sqrt[d]))])/(-c)^(3/2
) - ((I/4)*Sqrt[d]*Log[1 - I*a - I*b*x]*Log[(b*(Sqrt[d] - Sqrt[-c]*x))/(I*S
qrt[-c] + a*Sqrt[-c] + b*Sqrt[d])]])/(-c)^(3/2) + ((I/4)*Sqrt[d]*Log[1 - I*a
 - I*b*x]*Log[-((b*(Sqrt[d] + Sqrt[-c]*x))/((I + a)*Sqrt[-c] - b*Sqrt[d]))]
)/(-c)^(3/2) - ((I/4)*Sqrt[d]*Log[1 + I*a + I*b*x]*Log[(b*(Sqrt[d] + Sqrt[-
c]*x))/(I*Sqrt[-c] - a*Sqrt[-c] + b*Sqrt[d])]])/(-c)^(3/2) + ((I/4)*Sqrt[d]*
PolyLog[2, (Sqrt[-c]*(I - a - b*x))/(I*Sqrt[-c] - a*Sqrt[-c] - b*Sqrt[d])]])
/(-c)^(3/2) - ((I/4)*Sqrt[d]*PolyLog[2, (Sqrt[-c]*(1 + I*a + I*b*x))/((1 +
I*a)*Sqrt[-c] - I*b*Sqrt[d])]])/(-c)^(3/2) + ((I/4)*Sqrt[d]*PolyLog[2, (Sqrt
[-c]*(I + a + b*x))/(I*Sqrt[-c] + a*Sqrt[-c] - b*Sqrt[d])]])/(-c)^(3/2) - ((
I/4)*Sqrt[d]*PolyLog[2, (Sqrt[-c]*(I + a + b*x))/(I*Sqrt[-c] + a*Sqrt[-c] +
 b*Sqrt[d])]])/(-c)^(3/2)
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 5051

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx = \frac{1}{2}i\int \frac{\log(1-ia-ibx)}{c+\frac{d}{x^2}}\,dx - \frac{1}{2}i\int \frac{\log(1+ia+ibx)}{c+\frac{d}{x^2}}\,dx$$

$$= \frac{1}{2}i\int\left(\frac{\log(1-ia-ibx)}{c} - \frac{d\log(1-ia-ibx)}{c\left(d+cx^2\right)}\right)dx - \frac{1}{2}i\int\left(\frac{\log(1+ia+ibx)}{c} - \frac{d\log(1}{c\left(d\right.}\right.$$

$$= \frac{i\int\log(1-ia-ibx)\,dx}{2c} - \frac{i\int\log(1+ia+ibx)\,dx}{2c} - \frac{(id)\int\frac{\log(1-ia-ibx)}{d+cx^2}\,dx}{2c} + \frac{(id)\int\frac{\log(1+}{d+}}{2c}$$

$$= -\frac{\text{Subst}(\int\log(x)\,dx, x, 1-ia-ibx)}{2bc} - \frac{\text{Subst}(\int\log(x)\,dx, x, 1+ia+ibx)}{2bc} - \frac{(id)\int\left(\frac{\text{lo}}{2\sqrt{d}}\right.}{}$$

$$= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} - \frac{\left(i\sqrt{d}\right)\int\frac{\log(1-ia-}{\sqrt{d}-\sqrt{-}}}{4c}$$

$$= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} + \frac{i\sqrt{d}\,\log(1+ia}{}$$

$$= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} + \frac{i\sqrt{d}\,\log(1+ia}{}$$

$$= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} + \frac{i\sqrt{d}\,\log(1+ia}{}$$

Mathematica [B] time = 25.83, size = 1536, normalized size = 2.30

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a + b*x]/(c + d/x^2),x]
```

```
[Out] ((a + b*x)*ArcTan[a + b*x] + Log[1/Sqrt[1 + (a + b*x)^2]])/(b*c) - (Sqrt[d]
*(-2*Sqrt[c]*ArcTan[((-I + a)*Sqrt[c])/(b*Sqrt[d])]*ArcTan[(Sqrt[c]*x)/Sqrt
[d]] - 2*a^2*Sqrt[c]*ArcTan[((-I + a)*Sqrt[c])/(b*Sqrt[d])]*ArcTan[(Sqrt[c]
*x)/Sqrt[d]] + 2*Sqrt[c]*ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])]*ArcTan[(Sqrt
[c]*x)/Sqrt[d]] + 2*a^2*Sqrt[c]*ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])]*ArcTa
n[(Sqrt[c]*x)/Sqrt[d]] - 2*b*Sqrt[d]*ArcTan[(Sqrt[c]*x)/Sqrt[d]]^2 + (b*Sqr
t[d]*Sqrt[((-I + a)^2*c + b^2*d)/(b^2*d)]*ArcTan[(Sqrt[c]*x)/Sqrt[d]]^2)/E^
(I*ArcTan[((-I + a)*Sqrt[c])/(b*Sqrt[d])]) - (I*a*b*Sqrt[d]*Sqrt[((-I + a)^
2*c + b^2*d)/(b^2*d)]*ArcTan[(Sqrt[c]*x)/Sqrt[d]]^2)/E^(I*ArcTan[((-I + a)*
Sqrt[c])/(b*Sqrt[d])]) + (b*Sqrt[d]*Sqrt[((I + a)^2*c + b^2*d)/(b^2*d)]*Arc
Tan[(Sqrt[c]*x)/Sqrt[d]]^2)/E^(I*ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])]) + (
I*a*b*Sqrt[d]*Sqrt[((I + a)^2*c + b^2*d)/(b^2*d)]*ArcTan[(Sqrt[c]*x)/Sqrt[d
]]^2)/E^(I*ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])]) + 4*(1 + a^2)*Sqrt[c]*Arc
Tan[(Sqrt[c]*x)/Sqrt[d]]*ArcTan[a + b*x] + (2*I)*Sqrt[c]*ArcTan[((-I + a)*S
qrt[c])/(b*Sqrt[d])]*Log[1 - E^((-2*I)*(ArcTan[((-I + a)*Sqrt[c])/(b*Sqrt[d
])] + ArcTan[(Sqrt[c]*x)/Sqrt[d]]))] + (2*I)*a^2*Sqrt[c]*ArcTan[((-I + a)*S
qrt[c])/(b*Sqrt[d])]*Log[1 - E^((-2*I)*(ArcTan[((-I + a)*Sqrt[c])/(b*Sqrt[d
])] + ArcTan[(Sqrt[c]*x)/Sqrt[d]]))] + (2*I)*Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqr
t[d]]*Log[1 - E^((-2*I)*(ArcTan[((-I + a)*Sqrt[c])/(b*Sqrt[d])] + ArcTan[(S
qrt[c]*x)/Sqrt[d]]))] + (2*I)*a^2*Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[d]]*Log[1
 - E^((-2*I)*(ArcTan[((-I + a)*Sqrt[c])/(b*Sqrt[d])] + ArcTan[(Sqrt[c]*x)/S
qrt[d]]))] - (2*I)*Sqrt[c]*ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])]*Log[1 - E^
```

```
((-2*I)*(ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])] + ArcTan[(Sqrt[c]*x)/Sqrt[d]
]))] - (2*I)*a^2*Sqrt[c]*ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])]*Log[1 - E^((
-2*I)*(ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])] + ArcTan[(Sqrt[c]*x)/Sqrt[d]])
)] - (2*I)*Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[d]]*Log[1 - E^((-2*I)*(ArcTan[((
I + a)*Sqrt[c])/(b*Sqrt[d])] + ArcTan[(Sqrt[c]*x)/Sqrt[d]]))] - (2*I)*a^2*S
qrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[d]]*Log[1 - E^((-2*I)*(ArcTan[((I + a)*Sqrt[
c])/(b*Sqrt[d])] + ArcTan[(Sqrt[c]*x)/Sqrt[d]]))] - (2*I)*Sqrt[c]*ArcTan[((
-I + a)*Sqrt[c])/(b*Sqrt[d])]*Log[-Sin[ArcTan[((-I + a)*Sqrt[c])/(b*Sqrt[d]
)] + ArcTan[(Sqrt[c]*x)/Sqrt[d]]]] - (2*I)*a^2*Sqrt[c]*ArcTan[((-I + a)*Sqr
t[c])/(b*Sqrt[d])]*Log[-Sin[ArcTan[((-I + a)*Sqrt[c])/(b*Sqrt[d])] + ArcTan
[(Sqrt[c]*x)/Sqrt[d]]]] + (2*I)*Sqrt[c]*ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d]
)]*Log[-Sin[ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])] + ArcTan[(Sqrt[c]*x)/Sqrt
[d]]]] + (2*I)*a^2*Sqrt[c]*ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])]*Log[-Sin[A
rcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])] + ArcTan[(Sqrt[c]*x)/Sqrt[d]]]] - (1 +
 a^2)*Sqrt[c]*PolyLog[2, E^((-2*I)*(ArcTan[((-I + a)*Sqrt[c])/(b*Sqrt[d])]
+ ArcTan[(Sqrt[c]*x)/Sqrt[d]]))] + (1 + a^2)*Sqrt[c]*PolyLog[2, E^((-2*I)*(
ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])] + ArcTan[(Sqrt[c]*x)/Sqrt[d]]))]))/(4
*(1 + a^2)*c^2)
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2 \arctan(bx+a)}{cx^2+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x^2),x, algorithm="fricas")
```

```
[Out] integral(x^2*arctan(b*x + a)/(c*x^2 + d), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x^2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 2.70, size = 53434, normalized size = 79.99

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(c+d/x^2),x)
```

```
[Out] result too large to display
```

maxima [B] time = 1.35, size = 8518, normalized size = 12.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x^2),x, algorithm="maxima")
```

```
[Out] -(d*arctan(c*x/sqrt(c*d))/(sqrt(c*d)*c) - x/c)*arctan(b*x + a) + 1/8*(8*a*c
*arctan(b*x + a) + (4*b*arctan(sqrt(c)*x/sqrt(d))*arctan2((2*a*b^2*c*d + (a
*b^3*d + (a^3 + a)*b*c + (b^4*d + (a^2 + 3)*b^2*c)*x)*sqrt(c)*sqrt(d) + (3*
b^3*c*d + (a^2 + 1)*b*c^2)*x)/(b^4*d^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2
 + 1)*c^2 + 4*(b^3*d + (a^2 + 1)*b*c)*sqrt(c)*sqrt(d)), ((a^2 + 3)*b^2*c*d
```

```
+ (a^4 + 2*a^2 + 1)*c^2 + (2*a*b^2*c*x + b^3*d + 3*(a^2 + 1)*b*c)*sqrt(c)*s
qrt(d) + (a*b^3*c*d + (a^3 + a)*b*c^2)*x)/(b^4*d^2 + 2*(a^2 + 3)*b^2*c*d +
(a^4 + 2*a^2 + 1)*c^2 + 4*(b^3*d + (a^2 + 1)*b*c)*sqrt(c)*sqrt(d))) + 4*b*a
rctan(sqrt(c)*x/sqrt(d))*arctan2((2*a*b^2*c*d - (a*b^3*d + (a^3 + a)*b*c +
(b^4*d + (a^2 + 3)*b^2*c)*x)*sqrt(c)*sqrt(d) + (3*b^3*c*d + (a^2 + 1)*b*c^2
)*x)/(b^4*d^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*c^2 - 4*(b^3*d + (a
^2 + 1)*b*c)*sqrt(c)*sqrt(d)), ((a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*c^2 -
 (2*a*b^2*c*x + b^3*d + 3*(a^2 + 1)*b*c)*sqrt(c)*sqrt(d) + (a*b^3*c*d + (a^
3 + a)*b*c^2)*x)/(b^4*d^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*c^2 - 4
*(b^3*d + (a^2 + 1)*b*c)*sqrt(c)*sqrt(d))) + b*log(c*x^2 + d)*log(((a^2 + 1
)*b^22*c*d^11 + 11*(a^4 + 22*a^2 + 21)*b^20*c^2*d^10 + 55*(a^6 + 39*a^4 + 1
71*a^2 + 133)*b^18*c^3*d^9 + 33*(5*a^8 + 260*a^6 + 1870*a^4 + 3876*a^2 + 22
61)*b^16*c^4*d^8 + 330*(a^10 + 61*a^8 + 570*a^6 + 1802*a^4 + 2261*a^2 + 969
)*b^14*c^5*d^7 + 22*(21*a^12 + 1386*a^10 + 15015*a^8 + 60060*a^6 + 109395*a
^4 + 92378*a^2 + 29393)*b^12*c^6*d^6 + 22*(21*a^14 + 1407*a^12 + 16401*a^10
 + 75075*a^8 + 169455*a^6 + 201773*a^4 + 121771*a^2 + 29393)*b^10*c^7*d^5 +
 330*(a^16 + 64*a^14 + 756*a^12 + 3696*a^10 + 9438*a^8 + 13728*a^6 + 11492*
a^4 + 5168*a^2 + 969)*b^8*c^8*d^4 + 33*(5*a^18 + 285*a^16 + 3220*a^14 + 158
76*a^12 + 42966*a^10 + 70070*a^8 + 70980*a^6 + 43860*a^4 + 15181*a^2 + 2261
)*b^6*c^9*d^3 + 55*(a^20 + 46*a^18 + 465*a^16 + 2184*a^14 + 5922*a^12 + 101
64*a^10 + 11466*a^8 + 8520*a^6 + 4029*a^4 + 1102*a^2 + 133)*b^4*c^10*d^2 +
11*(a^22 + 31*a^20 + 255*a^18 + 1065*a^16 + 2730*a^14 + 4662*a^12 + 5502*a^
10 + 4530*a^8 + 2565*a^6 + 955*a^4 + 211*a^2 + 21)*b^2*c^11*d + (a^24 + 12*
a^22 + 66*a^20 + 220*a^18 + 495*a^16 + 792*a^14 + 924*a^12 + 792*a^10 + 495
*a^8 + 220*a^6 + 66*a^4 + 12*a^2 + 1)*c^12 + (b^24*c*d^11 + 11*(a^2 + 21)*b
^22*c^2*d^10 + 55*(a^4 + 38*a^2 + 133)*b^20*c^3*d^9 + 33*(5*a^6 + 255*a^4 +
 1615*a^2 + 2261)*b^18*c^4*d^8 + 330*(a^8 + 60*a^6 + 510*a^4 + 1292*a^2 + 9
69)*b^16*c^5*d^7 + 22*(21*a^10 + 1365*a^8 + 13650*a^6 + 46410*a^4 + 62985*a
^2 + 29393)*b^14*c^6*d^6 + 22*(21*a^12 + 1386*a^10 + 15015*a^8 + 60060*a^6
+ 109395*a^4 + 92378*a^2 + 29393)*b^12*c^7*d^5 + 330*(a^14 + 63*a^12 + 693*
a^10 + 3003*a^8 + 6435*a^6 + 7293*a^4 + 4199*a^2 + 969)*b^10*c^8*d^4 + 33*(
5*a^16 + 280*a^14 + 2940*a^12 + 12936*a^10 + 30030*a^8 + 40040*a^6 + 30940*
a^4 + 12920*a^2 + 2261)*b^8*c^9*d^3 + 55*(a^18 + 45*a^16 + 420*a^14 + 1764*
a^12 + 4158*a^10 + 6006*a^8 + 5460*a^6 + 3060*a^4 + 969*a^2 + 133)*b^6*c^10
*d^2 + 11*(a^20 + 30*a^18 + 225*a^16 + 840*a^14 + 1890*a^12 + 2772*a^10 + 2
730*a^8 + 1800*a^6 + 765*a^4 + 190*a^2 + 21)*b^4*c^11*d + (a^22 + 11*a^20 +
 55*a^18 + 165*a^16 + 330*a^14 + 462*a^12 + 462*a^10 + 330*a^8 + 165*a^6 +
55*a^4 + 11*a^2 + 1)*b^2*c^12)*x^2 + 2*(11*(a^2 + 1)*b^21*c*d^10 + 110*(a^4
 + 8*a^2 + 7)*b^19*c^2*d^9 + 33*(15*a^6 + 205*a^4 + 589*a^2 + 399)*b^17*c^3
*d^8 + 264*(5*a^8 + 90*a^6 + 408*a^4 + 646*a^2 + 323)*b^15*c^4*d^7 + 110*(2
1*a^10 + 441*a^8 + 2562*a^6 + 6018*a^4 + 6137*a^2 + 2261)*b^13*c^5*d^6 + 4*
(693*a^12 + 15708*a^10 + 105105*a^8 + 308880*a^6 + 449735*a^4 + 319124*a^2
+ 88179)*b^11*c^6*d^5 + 110*(21*a^14 + 483*a^12 + 3465*a^10 + 11583*a^8 + 2
0735*a^6 + 20553*a^4 + 10659*a^2 + 2261)*b^9*c^7*d^4 + 264*(5*a^16 + 110*a^
14 + 798*a^12 + 2838*a^10 + 5720*a^8 + 6890*a^6 + 4930*a^4 + 1938*a^2 + 323
)*b^7*c^8*d^3 + 33*(15*a^18 + 295*a^16 + 2044*a^14 + 7308*a^12 + 15554*a^10
 + 20930*a^8 + 18060*a^6 + 9724*a^4 + 2983*a^2 + 399)*b^5*c^9*d^2 + 110*(a^
20 + 16*a^18 + 99*a^16 + 336*a^14 + 714*a^12 + 1008*a^10 + 966*a^8 + 624*a^
6 + 261*a^4 + 64*a^2 + 7)*b^3*c^10*d + 11*(a^22 + 11*a^20 + 55*a^18 + 165*a
^16 + 330*a^14 + 462*a^12 + 462*a^10 + 330*a^8 + 165*a^6 + 55*a^4 + 11*a^2
+ 1)*b*c^11 + (11*b^23*c*d^10 + 110*(a^2 + 7)*b^21*c^2*d^9 + 33*(15*a^4 + 1
90*a^2 + 399)*b^19*c^3*d^8 + 264*(5*a^6 + 85*a^4 + 323*a^2 + 323)*b^17*c^4*
d^7 + 110*(21*a^8 + 420*a^6 + 2142*a^4 + 3876*a^2 + 2261)*b^15*c^5*d^6 + 4*
(693*a^10 + 15015*a^8 + 90090*a^6 + 218790*a^4 + 230945*a^2 + 88179)*b^13*c
^6*d^5 + 110*(21*a^12 + 462*a^10 + 3003*a^8 + 8580*a^6 + 12155*a^4 + 8398*a
^2 + 2261)*b^11*c^7*d^4 + 264*(5*a^14 + 105*a^12 + 693*a^10 + 2145*a^8 + 35
75*a^6 + 3315*a^4 + 1615*a^2 + 323)*b^9*c^8*d^3 + 33*(15*a^16 + 280*a^14 +
1764*a^12 + 5544*a^10 + 10010*a^8 + 10920*a^6 + 7140*a^4 + 2584*a^2 + 399)*
b^7*c^9*d^2 + 110*(a^18 + 15*a^16 + 84*a^14 + 252*a^12 + 462*a^10 + 546*a^8
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 + 420*a^6 + 204*a^4 + 57*a^2 + 7)*b^5*c^10*d + 11*(a^20 + 10*a^18 + 45*a^1
6 + 120*a^14 + 210*a^12 + 252*a^10 + 210*a^8 + 120*a^6 + 45*a^4 + 10*a^2 +
1)*b^3*c^11)*x^2 + 2*(11*a*b^22*c*d^10 + 110*(a^3 + 7*a)*b^20*c^2*d^9 + 33*
(15*a^5 + 190*a^3 + 399*a)*b^18*c^3*d^8 + 264*(5*a^7 + 85*a^5 + 323*a^3 + 3
23*a)*b^16*c^4*d^7 + 110*(21*a^9 + 420*a^7 + 2142*a^5 + 3876*a^3 + 2261*a)*
b^14*c^5*d^6 + 4*(693*a^11 + 15015*a^9 + 90090*a^7 + 218790*a^5 + 230945*a^
3 + 88179*a)*b^12*c^6*d^5 + 110*(21*a^13 + 462*a^11 + 3003*a^9 + 8580*a^7 +
 12155*a^5 + 8398*a^3 + 2261*a)*b^10*c^7*d^4 + 264*(5*a^15 + 105*a^13 + 693
*a^11 + 2145*a^9 + 3575*a^7 + 3315*a^5 + 1615*a^3 + 323*a)*b^8*c^8*d^3 + 33
*(15*a^17 + 280*a^15 + 1764*a^13 + 5544*a^11 + 10010*a^9 + 10920*a^7 + 7140
*a^5 + 2584*a^3 + 399*a)*b^6*c^9*d^2 + 110*(a^19 + 15*a^17 + 84*a^15 + 252*
a^13 + 462*a^11 + 546*a^9 + 420*a^7 + 204*a^5 + 57*a^3 + 7*a)*b^4*c^10*d +
11*(a^21 + 10*a^19 + 45*a^17 + 120*a^15 + 210*a^13 + 252*a^11 + 210*a^9 + 1
20*a^7 + 45*a^5 + 10*a^3 + a)*b^2*c^11)*x)*sqrt(c)*sqrt(d) + 2*(a*b^23*c*d^
11 + 11*(a^3 + 21*a)*b^21*c^2*d^10 + 55*(a^5 + 38*a^3 + 133*a)*b^19*c^3*d^9
 + 33*(5*a^7 + 255*a^5 + 1615*a^3 + 2261*a)*b^17*c^4*d^8 + 330*(a^9 + 60*a^
7 + 510*a^5 + 1292*a^3 + 969*a)*b^15*c^5*d^7 + 22*(21*a^11 + 1365*a^9 + 136
50*a^7 + 46410*a^5 + 62985*a^3 + 29393*a)*b^13*c^6*d^6 + 22*(21*a^13 + 1386
*a^11 + 15015*a^9 + 60060*a^7 + 109395*a^5 + 92378*a^3 + 29393*a)*b^11*c^7*
d^5 + 330*(a^15 + 63*a^13 + 693*a^11 + 3003*a^9 + 6435*a^7 + 7293*a^5 + 419
9*a^3 + 969*a)*b^9*c^8*d^4 + 33*(5*a^17 + 280*a^15 + 2940*a^13 + 12936*a^11
 + 30030*a^9 + 40040*a^7 + 30940*a^5 + 12920*a^3 + 2261*a)*b^7*c^9*d^3 + 55
*(a^19 + 45*a^17 + 420*a^15 + 1764*a^13 + 4158*a^11 + 6006*a^9 + 5460*a^7 +
 3060*a^5 + 969*a^3 + 133*a)*b^5*c^10*d^2 + 11*(a^21 + 30*a^19 + 225*a^17 +
 840*a^15 + 1890*a^13 + 2772*a^11 + 2730*a^9 + 1800*a^7 + 765*a^5 + 190*a^3
 + 21*a)*b^3*c^11*d + (a^23 + 11*a^21 + 55*a^19 + 165*a^17 + 330*a^15 + 462
*a^13 + 462*a^11 + 330*a^9 + 165*a^7 + 55*a^5 + 11*a^3 + a)*b*c^12)*x)/(b^2
4*d^12 + 12*(a^2 + 23)*b^22*c*d^11 + 66*(a^4 + 42*a^2 + 161)*b^20*c^2*d^10
+ 44*(5*a^6 + 285*a^4 + 1995*a^2 + 3059)*b^18*c^3*d^9 + 99*(5*a^8 + 340*a^6
 + 3230*a^4 + 9044*a^2 + 7429)*b^16*c^4*d^8 + 264*(3*a^10 + 225*a^8 + 2550*
a^6 + 9690*a^4 + 14535*a^2 + 7429)*b^14*c^5*d^7 + 4*(231*a^12 + 18018*a^10
+ 225225*a^8 + 1021020*a^6 + 2078505*a^4 + 1939938*a^2 + 676039)*b^12*c^6*d
^6 + 264*(3*a^14 + 231*a^12 + 3003*a^10 + 15015*a^8 + 36465*a^6 + 46189*a^4
 + 29393*a^2 + 7429)*b^10*c^7*d^5 + 99*(5*a^16 + 360*a^14 + 4620*a^12 + 240
24*a^10 + 64350*a^8 + 97240*a^6 + 83980*a^4 + 38760*a^2 + 7429)*b^8*c^8*d^4
 + 44*(5*a^18 + 315*a^16 + 3780*a^14 + 19404*a^12 + 54054*a^10 + 90090*a^8
+ 92820*a^6 + 58140*a^4 + 20349*a^2 + 3059)*b^6*c^9*d^3 + 66*(a^20 + 50*a^1
8 + 525*a^16 + 2520*a^14 + 6930*a^12 + 12012*a^10 + 13650*a^8 + 10200*a^6 +
 4845*a^4 + 1330*a^2 + 161)*b^4*c^10*d^2 + 12*(a^22 + 33*a^20 + 275*a^18 +
1155*a^16 + 2970*a^14 + 5082*a^12 + 6006*a^10 + 4950*a^8 + 2805*a^6 + 1045*
a^4 + 231*a^2 + 23)*b^2*c^11*d + (a^24 + 12*a^22 + 66*a^20 + 220*a^18 + 495
*a^16 + 792*a^14 + 924*a^12 + 792*a^10 + 495*a^8 + 220*a^6 + 66*a^4 + 12*a^
2 + 1)*c^12 + 8*(3*b^23*d^11 + 11*(3*a^2 + 23)*b^21*c*d^10 + 33*(5*a^4 + 70
*a^2 + 161)*b^19*c^2*d^9 + 99*(5*a^6 + 95*a^4 + 399*a^2 + 437)*b^17*c^3*d^8
 + 22*(45*a^8 + 1020*a^6 + 5814*a^4 + 11628*a^2 + 7429)*b^15*c^4*d^7 + 6*(2
31*a^10 + 5775*a^8 + 39270*a^6 + 106590*a^4 + 124355*a^2 + 52003)*b^13*c^5*
d^6 + 6*(231*a^12 + 6006*a^10 + 45045*a^8 + 145860*a^6 + 230945*a^4 + 17635
8*a^2 + 52003)*b^11*c^6*d^5 + 22*(45*a^14 + 1155*a^12 + 9009*a^10 + 32175*a
^8 + 60775*a^6 + 62985*a^4 + 33915*a^2 + 7429)*b^9*c^7*d^4 + 99*(5*a^16 + 1
20*a^14 + 924*a^12 + 3432*a^10 + 7150*a^8 + 8840*a^6 + 6460*a^4 + 2584*a^2
+ 437)*b^7*c^8*d^3 + 33*(5*a^18 + 105*a^16 + 756*a^14 + 2772*a^12 + 6006*a^
10 + 8190*a^8 + 7140*a^6 + 3876*a^4 + 1197*a^2 + 161)*b^5*c^9*d^2 + 11*(3*a
^20 + 50*a^18 + 315*a^16 + 1080*a^14 + 2310*a^12 + 3276*a^10 + 3150*a^8 + 2
040*a^6 + 855*a^4 + 210*a^2 + 23)*b^3*c^10*d + 3*(a^22 + 11*a^20 + 55*a^18
+ 165*a^16 + 330*a^14 + 462*a^12 + 462*a^10 + 330*a^8 + 165*a^6 + 55*a^4 +
11*a^2 + 1)*b*c^11)*sqrt(c)*sqrt(d))) - b*log(c*x^2 + d)*log(((a^2 + 1)*b^2
2*c*d^11 + 11*(a^4 + 22*a^2 + 21)*b^20*c^2*d^10 + 55*(a^6 + 39*a^4 + 171*a^
2 + 133)*b^18*c^3*d^9 + 33*(5*a^8 + 260*a^6 + 1870*a^4 + 3876*a^2 + 2261)*b
^16*c^4*d^8 + 330*(a^10 + 61*a^8 + 570*a^6 + 1802*a^4 + 2261*a^2 + 969)*b^1
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4*c^5*d^7 + 22*(21*a^12 + 1386*a^10 + 15015*a^8 + 60060*a^6 + 109395*a^4 +
92378*a^2 + 29393)*b^12*c^6*d^6 + 22*(21*a^14 + 1407*a^12 + 16401*a^10 + 75
075*a^8 + 169455*a^6 + 201773*a^4 + 121771*a^2 + 29393)*b^10*c^7*d^5 + 330*
(a^16 + 64*a^14 + 756*a^12 + 3696*a^10 + 9438*a^8 + 13728*a^6 + 11492*a^4 +
 5168*a^2 + 969)*b^8*c^8*d^4 + 33*(5*a^18 + 285*a^16 + 3220*a^14 + 15876*a^
12 + 42966*a^10 + 70070*a^8 + 70980*a^6 + 43860*a^4 + 15181*a^2 + 2261)*b^6
*c^9*d^3 + 55*(a^20 + 46*a^18 + 465*a^16 + 2184*a^14 + 5922*a^12 + 10164*a^
10 + 11466*a^8 + 8520*a^6 + 4029*a^4 + 1102*a^2 + 133)*b^4*c^10*d^2 + 11*(a
^22 + 31*a^20 + 255*a^18 + 1065*a^16 + 2730*a^14 + 4662*a^12 + 5502*a^10 +
4530*a^8 + 2565*a^6 + 955*a^4 + 211*a^2 + 21)*b^2*c^11*d + (a^24 + 12*a^22
+ 66*a^20 + 220*a^18 + 495*a^16 + 792*a^14 + 924*a^12 + 792*a^10 + 495*a^8
+ 220*a^6 + 66*a^4 + 12*a^2 + 1)*c^12 + (b^24*c*d^11 + 11*(a^2 + 21)*b^22*c
^2*d^10 + 55*(a^4 + 38*a^2 + 133)*b^20*c^3*d^9 + 33*(5*a^6 + 255*a^4 + 1615
*a^2 + 2261)*b^18*c^4*d^8 + 330*(a^8 + 60*a^6 + 510*a^4 + 1292*a^2 + 969)*b
^16*c^5*d^7 + 22*(21*a^10 + 1365*a^8 + 13650*a^6 + 46410*a^4 + 62985*a^2 +
29393)*b^14*c^6*d^6 + 22*(21*a^12 + 1386*a^10 + 15015*a^8 + 60060*a^6 + 109
395*a^4 + 92378*a^2 + 29393)*b^12*c^7*d^5 + 330*(a^14 + 63*a^12 + 693*a^10
+ 3003*a^8 + 6435*a^6 + 7293*a^4 + 4199*a^2 + 969)*b^10*c^8*d^4 + 33*(5*a^1
6 + 280*a^14 + 2940*a^12 + 12936*a^10 + 30030*a^8 + 40040*a^6 + 30940*a^4 +
 12920*a^2 + 2261)*b^8*c^9*d^3 + 55*(a^18 + 45*a^16 + 420*a^14 + 1764*a^12
+ 4158*a^10 + 6006*a^8 + 5460*a^6 + 3060*a^4 + 969*a^2 + 133)*b^6*c^10*d^2
+ 11*(a^20 + 30*a^18 + 225*a^16 + 840*a^14 + 1890*a^12 + 2772*a^10 + 2730*a
^8 + 1800*a^6 + 765*a^4 + 190*a^2 + 21)*b^4*c^11*d + (a^22 + 11*a^20 + 55*a
^18 + 165*a^16 + 330*a^14 + 462*a^12 + 462*a^10 + 330*a^8 + 165*a^6 + 55*a^
4 + 11*a^2 + 1)*b^2*c^12)*x^2 - 2*(11*(a^2 + 1)*b^21*c*d^10 + 110*(a^4 + 8*
a^2 + 7)*b^19*c^2*d^9 + 33*(15*a^6 + 205*a^4 + 589*a^2 + 399)*b^17*c^3*d^8
+ 264*(5*a^8 + 90*a^6 + 408*a^4 + 646*a^2 + 323)*b^15*c^4*d^7 + 110*(21*a^1
0 + 441*a^8 + 2562*a^6 + 6018*a^4 + 6137*a^2 + 2261)*b^13*c^5*d^6 + 4*(693*
a^12 + 15708*a^10 + 105105*a^8 + 308880*a^6 + 449735*a^4 + 319124*a^2 + 881
79)*b^11*c^6*d^5 + 110*(21*a^14 + 483*a^12 + 3465*a^10 + 11583*a^8 + 20735*
a^6 + 20553*a^4 + 10659*a^2 + 2261)*b^9*c^7*d^4 + 264*(5*a^16 + 110*a^14 +
798*a^12 + 2838*a^10 + 5720*a^8 + 6890*a^6 + 4930*a^4 + 1938*a^2 + 323)*b^7
*c^8*d^3 + 33*(15*a^18 + 295*a^16 + 2044*a^14 + 7308*a^12 + 15554*a^10 + 20
930*a^8 + 18060*a^6 + 9724*a^4 + 2983*a^2 + 399)*b^5*c^9*d^2 + 110*(a^20 +
16*a^18 + 99*a^16 + 336*a^14 + 714*a^12 + 1008*a^10 + 966*a^8 + 624*a^6 + 2
61*a^4 + 64*a^2 + 7)*b^3*c^10*d + 11*(a^22 + 11*a^20 + 55*a^18 + 165*a^16 +
 330*a^14 + 462*a^12 + 462*a^10 + 330*a^8 + 165*a^6 + 55*a^4 + 11*a^2 + 1)*
b*c^11 + (11*b^23*c*d^10 + 110*(a^2 + 7)*b^21*c^2*d^9 + 33*(15*a^4 + 190*a^
2 + 399)*b^19*c^3*d^8 + 264*(5*a^6 + 85*a^4 + 323*a^2 + 323)*b^17*c^4*d^7 +
 110*(21*a^8 + 420*a^6 + 2142*a^4 + 3876*a^2 + 2261)*b^15*c^5*d^6 + 4*(693*
a^10 + 15015*a^8 + 90090*a^6 + 218790*a^4 + 230945*a^2 + 88179)*b^13*c^6*d^
5 + 110*(21*a^12 + 462*a^10 + 3003*a^8 + 8580*a^6 + 12155*a^4 + 8398*a^2 +
2261)*b^11*c^7*d^4 + 264*(5*a^14 + 105*a^12 + 693*a^10 + 2145*a^8 + 3575*a^
6 + 3315*a^4 + 1615*a^2 + 323)*b^9*c^8*d^3 + 33*(15*a^16 + 280*a^14 + 1764*
a^12 + 5544*a^10 + 10010*a^8 + 10920*a^6 + 7140*a^4 + 2584*a^2 + 399)*b^7*c
^9*d^2 + 110*(a^18 + 15*a^16 + 84*a^14 + 252*a^12 + 462*a^10 + 546*a^8 + 42
0*a^6 + 204*a^4 + 57*a^2 + 7)*b^5*c^10*d + 11*(a^20 + 10*a^18 + 45*a^16 + 1
20*a^14 + 210*a^12 + 252*a^10 + 210*a^8 + 120*a^6 + 45*a^4 + 10*a^2 + 1)*b^
3*c^11)*x^2 + 2*(11*a*b^22*c*d^10 + 110*(a^3 + 7*a)*b^20*c^2*d^9 + 33*(15*a
^5 + 190*a^3 + 399*a)*b^18*c^3*d^8 + 264*(5*a^7 + 85*a^5 + 323*a^3 + 323*a)
*b^16*c^4*d^7 + 110*(21*a^9 + 420*a^7 + 2142*a^5 + 3876*a^3 + 2261*a)*b^14*
c^5*d^6 + 4*(693*a^11 + 15015*a^9 + 90090*a^7 + 218790*a^5 + 230945*a^3 + 8
8179*a)*b^12*c^6*d^5 + 110*(21*a^13 + 462*a^11 + 3003*a^9 + 8580*a^7 + 1215
5*a^5 + 8398*a^3 + 2261*a)*b^10*c^7*d^4 + 264*(5*a^15 + 105*a^13 + 693*a^11
 + 2145*a^9 + 3575*a^7 + 3315*a^5 + 1615*a^3 + 323*a)*b^8*c^8*d^3 + 33*(15*
a^17 + 280*a^15 + 1764*a^13 + 5544*a^11 + 10010*a^9 + 10920*a^7 + 7140*a^5
+ 2584*a^3 + 399*a)*b^6*c^9*d^2 + 110*(a^19 + 15*a^17 + 84*a^15 + 252*a^13
+ 462*a^11 + 546*a^9 + 420*a^7 + 204*a^5 + 57*a^3 + 7*a)*b^4*c^10*d + 11*(a
^21 + 10*a^19 + 45*a^17 + 120*a^15 + 210*a^13 + 252*a^11 + 210*a^9 + 120*a^
```

```
7 + 45*a^5 + 10*a^3 + a)*b^2*c^11)*x)*sqrt(c)*sqrt(d) + 2*(a*b^23*c*d^11 +
11*(a^3 + 21*a)*b^21*c^2*d^10 + 55*(a^5 + 38*a^3 + 133*a)*b^19*c^3*d^9 + 33
*(5*a^7 + 255*a^5 + 1615*a^3 + 2261*a)*b^17*c^4*d^8 + 330*(a^9 + 60*a^7 + 5
10*a^5 + 1292*a^3 + 969*a)*b^15*c^5*d^7 + 22*(21*a^11 + 1365*a^9 + 13650*a^
7 + 46410*a^5 + 62985*a^3 + 29393*a)*b^13*c^6*d^6 + 22*(21*a^13 + 1386*a^11
 + 15015*a^9 + 60060*a^7 + 109395*a^5 + 92378*a^3 + 29393*a)*b^11*c^7*d^5 +
 330*(a^15 + 63*a^13 + 693*a^11 + 3003*a^9 + 6435*a^7 + 7293*a^5 + 4199*a^3
 + 969*a)*b^9*c^8*d^4 + 33*(5*a^17 + 280*a^15 + 2940*a^13 + 12936*a^11 + 30
030*a^9 + 40040*a^7 + 30940*a^5 + 12920*a^3 + 2261*a)*b^7*c^9*d^3 + 55*(a^1
9 + 45*a^17 + 420*a^15 + 1764*a^13 + 4158*a^11 + 6006*a^9 + 5460*a^7 + 3060
*a^5 + 969*a^3 + 133*a)*b^5*c^10*d^2 + 11*(a^21 + 30*a^19 + 225*a^17 + 840*
a^15 + 1890*a^13 + 2772*a^11 + 2730*a^9 + 1800*a^7 + 765*a^5 + 190*a^3 + 21
*a)*b^3*c^11*d + (a^23 + 11*a^21 + 55*a^19 + 165*a^17 + 330*a^15 + 462*a^13
 + 462*a^11 + 330*a^9 + 165*a^7 + 55*a^5 + 11*a^3 + a)*b*c^12)*x)/(b^24*d^1
2 + 12*(a^2 + 23)*b^22*c*d^11 + 66*(a^4 + 42*a^2 + 161)*b^20*c^2*d^10 + 44*
(5*a^6 + 285*a^4 + 1995*a^2 + 3059)*b^18*c^3*d^9 + 99*(5*a^8 + 340*a^6 + 32
30*a^4 + 9044*a^2 + 7429)*b^16*c^4*d^8 + 264*(3*a^10 + 225*a^8 + 2550*a^6 +
 9690*a^4 + 14535*a^2 + 7429)*b^14*c^5*d^7 + 4*(231*a^12 + 18018*a^10 + 225
225*a^8 + 1021020*a^6 + 2078505*a^4 + 1939938*a^2 + 676039)*b^12*c^6*d^6 +
264*(3*a^14 + 231*a^12 + 3003*a^10 + 15015*a^8 + 36465*a^6 + 46189*a^4 + 29
393*a^2 + 7429)*b^10*c^7*d^5 + 99*(5*a^16 + 360*a^14 + 4620*a^12 + 24024*a^
10 + 64350*a^8 + 97240*a^6 + 83980*a^4 + 38760*a^2 + 7429)*b^8*c^8*d^4 + 44
*(5*a^18 + 315*a^16 + 3780*a^14 + 19404*a^12 + 54054*a^10 + 90090*a^8 + 928
20*a^6 + 58140*a^4 + 20349*a^2 + 3059)*b^6*c^9*d^3 + 66*(a^20 + 50*a^18 + 5
25*a^16 + 2520*a^14 + 6930*a^12 + 12012*a^10 + 13650*a^8 + 10200*a^6 + 4845
*a^4 + 1330*a^2 + 161)*b^4*c^10*d^2 + 12*(a^22 + 33*a^20 + 275*a^18 + 1155*
a^16 + 2970*a^14 + 5082*a^12 + 6006*a^10 + 4950*a^8 + 2805*a^6 + 1045*a^4 +
 231*a^2 + 23)*b^2*c^11*d + (a^24 + 12*a^22 + 66*a^20 + 220*a^18 + 495*a^16
 + 792*a^14 + 924*a^12 + 792*a^10 + 495*a^8 + 220*a^6 + 66*a^4 + 12*a^2 + 1
)*c^12 - 8*(3*b^23*d^11 + 11*(3*a^2 + 23)*b^21*c*d^10 + 33*(5*a^4 + 70*a^2
+ 161)*b^19*c^2*d^9 + 99*(5*a^6 + 95*a^4 + 399*a^2 + 437)*b^17*c^3*d^8 + 22
*(45*a^8 + 1020*a^6 + 5814*a^4 + 11628*a^2 + 7429)*b^15*c^4*d^7 + 6*(231*a^
10 + 5775*a^8 + 39270*a^6 + 106590*a^4 + 124355*a^2 + 52003)*b^13*c^5*d^6 +
 6*(231*a^12 + 6006*a^10 + 45045*a^8 + 145860*a^6 + 230945*a^4 + 176358*a^2
 + 52003)*b^11*c^6*d^5 + 22*(45*a^14 + 1155*a^12 + 9009*a^10 + 32175*a^8 +
60775*a^6 + 62985*a^4 + 33915*a^2 + 7429)*b^9*c^7*d^4 + 99*(5*a^16 + 120*a^
14 + 924*a^12 + 3432*a^10 + 7150*a^8 + 8840*a^6 + 6460*a^4 + 2584*a^2 + 437
)*b^7*c^8*d^3 + 33*(5*a^18 + 105*a^16 + 756*a^14 + 2772*a^12 + 6006*a^10 +
8190*a^8 + 7140*a^6 + 3876*a^4 + 1197*a^2 + 161)*b^5*c^9*d^2 + 11*(3*a^20 +
 50*a^18 + 315*a^16 + 1080*a^14 + 2310*a^12 + 3276*a^10 + 3150*a^8 + 2040*a
^6 + 855*a^4 + 210*a^2 + 23)*b^3*c^10*d + 3*(a^22 + 11*a^20 + 55*a^18 + 165
*a^16 + 330*a^14 + 462*a^12 + 462*a^10 + 330*a^8 + 165*a^6 + 55*a^4 + 11*a^
2 + 1)*b*c^11)*sqrt(c)*sqrt(d))) + 2*b*dilog(((a + I)*b*c*x + b^2*d + (I*b^
2*x + (-I*a + 1)*b)*sqrt(c)*sqrt(d))/(2*(-I*a + 1)*b*sqrt(c)*sqrt(d) + b^2*
d - (a^2 + 2*I*a - 1)*c)) - 2*b*dilog(-((a + I)*b*c*x + b^2*d - (I*b^2*x +
(-I*a + 1)*b)*sqrt(c)*sqrt(d))/(2*(-I*a + 1)*b*sqrt(c)*sqrt(d) - b^2*d + (a
^2 + 2*I*a - 1)*c)) - 2*b*dilog(((a - I)*b*c*x + b^2*d + (I*b^2*x + (-I*a -
 1)*b)*sqrt(c)*sqrt(d))/(2*(-I*a - 1)*b*sqrt(c)*sqrt(d) + b^2*d - (a^2 - 2*
I*a - 1)*c)) + 2*b*dilog(-((a - I)*b*c*x + b^2*d - (I*b^2*x + (-I*a - 1)*b)
*sqrt(c)*sqrt(d))/(2*(-I*a - 1)*b*sqrt(c)*sqrt(d) - b^2*d + (a^2 - 2*I*a -
1)*c)))*sqrt(c)*sqrt(d) - 4*c*log(b^2*x^2 + 2*a*b*x + a^2 + 1))/(b*c^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a + b x)}{c + \frac{d}{x^2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)/(c + d/x^2),x)
```

```
[Out] int(atan(a + b*x)/(c + d/x^2), x)
```

sympy [**F(-1)**]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(c+d/x**2),x)
```

```
[Out] Timed out
```

3.57 $\int \frac{\tan^{-1}(a+bx)}{c+\frac{d}{x^3}}\,dx$

Optimal. Leaf size=933

$$-\frac{(ia+ibx+1)\log(ia+ibx+1)}{2bc}+\frac{i\sqrt[3]{d}\log\left(\frac{b\left(\sqrt[3]{c}x+\sqrt[3]{d}\right)}{\sqrt[3]{c}(i-a)+b\sqrt[3]{d}}\right)\log(ia+ibx+1)}{6c^{4/3}}-\frac{\sqrt[6]{-1}\sqrt[3]{d}\log\left(-\frac{b\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{c}x\right)}{\sqrt[3]{-1}(i-a)\sqrt[3]{c}-b\sqrt[3]{d}}\right)\log(}{6c^{4/3}}$$

```
[Out] -1/2*(1+I*a+I*b*x)*ln(1+I*a+I*b*x)/b/c-1/2*(1-I*a-I*b*x)*ln(-I*(I+a+b*x))/b
/c-1/6*I*d^(1/3)*ln(1-I*a-I*b*x)*ln(-b*(d^(1/3)+c^(1/3)*x)/((I+a)*c^(1/3)-b
*d^(1/3)))/c^(4/3)+1/6*I*d^(1/3)*ln(1+I*a+I*b*x)*ln(b*(d^(1/3)+c^(1/3)*x)/(
(I-a)*c^(1/3)+b*d^(1/3)))/c^(4/3)-1/6*(-1)^(1/6)*d^(1/3)*ln(1+I*a+I*b*x)*ln
(-b*(d^(1/3)-(-1)^(1/3)*c^(1/3)*x)/((-1)^(1/3)*(I-a)*c^(1/3)-b*d^(1/3)))/c^
(4/3)+1/6*(-1)^(1/6)*d^(1/3)*ln(1-I*a-I*b*x)*ln(b*(d^(1/3)-(-1)^(1/3)*c^(1/
3)*x)/((-1)^(1/3)*(I+a)*c^(1/3)+b*d^(1/3)))/c^(4/3)-1/6*(-1)^(5/6)*d^(1/3)*
ln(1+I*a+I*b*x)*ln(b*(d^(1/3)+(-1)^(2/3)*c^(1/3)*x)/((-1)^(2/3)*(I-a)*c^(1/
3)+b*d^(1/3)))/c^(4/3)+1/6*(-1)^(5/6)*d^(1/3)*ln(1-I*a-I*b*x)*ln(b*(d^(1/3)
+(-1)^(2/3)*c^(1/3)*x)/((-1)^(1/6)*(1-I*a)*c^(1/3)+b*d^(1/3)))/c^(4/3)-1/6*
(-1)^(1/6)*d^(1/3)*polylog(2,(-1)^(1/3)*c^(1/3)*(I-a-b*x)/((-1)^(1/3)*(I-a)
*c^(1/3)-b*d^(1/3)))/c^(4/3)-1/6*(-1)^(5/6)*d^(1/3)*polylog(2,(-1)^(1/6)*c^
(1/3)*(I-a-b*x)/((-1)^(1/6)*(I-a)*c^(1/3)-I*b*d^(1/3)))/c^(4/3)+1/6*I*d^(1/
3)*polylog(2,c^(1/3)*(I-a-b*x)/((I-a)*c^(1/3)+b*d^(1/3)))/c^(4/3)-1/6*I*d^(
1/3)*polylog(2,c^(1/3)*(I+a+b*x)/((I+a)*c^(1/3)-b*d^(1/3)))/c^(4/3)+1/6*(-1
)^(5/6)*d^(1/3)*polylog(2,(-1)^(2/3)*c^(1/3)*(I+a+b*x)/((-1)^(2/3)*(I+a)*c^
(1/3)-b*d^(1/3)))/c^(4/3)+1/6*(-1)^(1/6)*d^(1/3)*polylog(2,(-1)^(1/3)*c^(1/
3)*(I+a+b*x)/((-1)^(1/3)*(I+a)*c^(1/3)+b*d^(1/3)))/c^(4/3)
```

Rubi [A] time = 1.37, antiderivative size = 933, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5051, 2409, 2389, 2295, 2394, 2393, 2391}

$$-\frac{(ia+ibx+1)\log(ia+ibx+1)}{2bc}+\frac{i\sqrt[3]{d}\log\left(\frac{b\left(\sqrt[3]{c}x+\sqrt[3]{d}\right)}{\sqrt[3]{c}(i-a)+b\sqrt[3]{d}}\right)\log(ia+ibx+1)}{6c^{4/3}}-\frac{\sqrt[6]{-1}\sqrt[3]{d}\log\left(-\frac{b\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{c}x\right)}{\sqrt[3]{-1}(i-a)\sqrt[3]{c}-b\sqrt[3]{d}}\right)\log(}{6c^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/(c + d/x^3),x]
```

```
[Out] -((1 + I*a + I*b*x)*Log[1 + I*a + I*b*x])/(2*b*c) - ((1 - I*a - I*b*x)*Log[
(-I)*(I + a + b*x)])/(2*b*c) - ((I/6)*d^(1/3)*Log[1 - I*a - I*b*x]*Log[-((b
*(d^(1/3) + c^(1/3)*x))/((I + a)*c^(1/3) - b*d^(1/3)))])/c^(4/3) + ((I/6)*d
^(1/3)*Log[1 + I*a + I*b*x]*Log[(b*(d^(1/3) + c^(1/3)*x))/((I - a)*c^(1/3)
+ b*d^(1/3))])/c^(4/3) - ((-1)^(1/6)*d^(1/3)*Log[1 + I*a + I*b*x]*Log[-((b*
(d^(1/3) - (-1)^(1/3)*c^(1/3)*x))/((-1)^(1/3)*(I - a)*c^(1/3) - b*d^(1/3)))
])/(6*c^(4/3)) + ((-1)^(1/6)*d^(1/3)*Log[1 - I*a - I*b*x]*Log[(b*(d^(1/3) -
 (-1)^(1/3)*c^(1/3)*x))/((-1)^(1/3)*(I + a)*c^(1/3) + b*d^(1/3))])/(6*c^(4/
3)) - ((-1)^(5/6)*d^(1/3)*Log[1 + I*a + I*b*x]*Log[(b*(d^(1/3) + (-1)^(2/3)
*c^(1/3)*x))/((-1)^(2/3)*(I - a)*c^(1/3) + b*d^(1/3))])/(6*c^(4/3)) + ((-1)
^(5/6)*d^(1/3)*Log[1 - I*a - I*b*x]*Log[(b*(d^(1/3) + (-1)^(2/3)*c^(1/3)*x)
)/((-1)^(1/6)*(1 - I*a)*c^(1/3) + b*d^(1/3))])/(6*c^(4/3)) - ((-1)^(1/6)*d^
(1/3)*PolyLog[2, ((-1)^(1/3)*c^(1/3)*(I - a - b*x))/((-1)^(1/3)*(I - a)*c^(
1/3) - b*d^(1/3))])/(6*c^(4/3)) - ((-1)^(5/6)*d^(1/3)*PolyLog[2, ((-1)^(1/6
)*c^(1/3)*(I - a - b*x))/((-1)^(1/6)*(I - a)*c^(1/3) - I*b*d^(1/3))])/(6*c^
(4/3)) + ((I/6)*d^(1/3)*PolyLog[2, (c^(1/3)*(I - a - b*x))/((I - a)*c^(1/3)
 + b*d^(1/3))])/c^(4/3) - ((I/6)*d^(1/3)*PolyLog[2, (c^(1/3)*(I + a + b*x))
/((I + a)*c^(1/3) - b*d^(1/3))])/c^(4/3) + ((-1)^(5/6)*d^(1/3)*PolyLog[2, (
```

```
(-1)^(2/3)*c^(1/3)*(I + a + b*x))/((-1)^(2/3)*(I + a)*c^(1/3) - b*d^(1/3))]
)/(6*c^(4/3)) + ((-1)^(1/6)*d^(1/3)*PolyLog[2, ((-1)^(1/3)*c^(1/3)*(I + a +
 b*x))/((-1)^(1/3)*(I + a)*c^(1/3) + b*d^(1/3))])/(6*c^(4/3))
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 5051

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{c+\frac{d}{x^3}}\,dx = \frac{1}{2}i\int \frac{\log(1-ia-ibx)}{c+\frac{d}{x^3}}\,dx - \frac{1}{2}i\int \frac{\log(1+ia+ibx)}{c+\frac{d}{x^3}}\,dx$$

$$= \frac{1}{2}i\int\left(\frac{\log(1-ia-ibx)}{c} - \frac{d\log(1-ia-ibx)}{c\left(d+cx^3\right)}\right)dx - \frac{1}{2}i\int\left(\frac{\log(1+ia+ibx)}{c} - \frac{d\log(1+i}{c\left(d+c\right.}\right.$$

$$= \frac{i\int\log(1-ia-ibx)\,dx}{2c} - \frac{i\int\log(1+ia+ibx)\,dx}{2c} - \frac{(id)\int\frac{\log(1-ia-ibx)}{d+cx^3}\,dx}{2c} + \frac{(id)\int\frac{\log(1+ia+}{d+cx^3}}{2c}$$

$$= -\frac{\mathrm{Subst}(\int\log(x)\,dx,x,1-ia-ibx)}{2bc} - \frac{\mathrm{Subst}(\int\log(x)\,dx,x,1+ia+ibx)}{2bc} - \frac{(id)\int\left(-\frac{\log(}{3d^{2/3}(}\right.}{}$$

$$= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} + \frac{\left(i\sqrt[3]{d}\right)\int\frac{\log(1-ia-ibx}{-\sqrt[3]{d}-\sqrt[3]{c}\,x}}{6c}$$

$$= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} - \frac{i\sqrt[3]{d}\,\log(1-ia-ib}{6}$$

$$= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} - \frac{i\sqrt[3]{d}\,\log(1-ia-ib}{6}$$

$$= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} - \frac{i\sqrt[3]{d}\,\log(1-ia-ib}{6}$$

Mathematica [C] time = 7.38, size = 933, normalized size = 1.00

$$\frac{6\left((a+bx)\tan^{-1}(a+bx)+\log\left(\frac{1}{\sqrt{(a+bx)^2+1}}\right)\right) - b^3 d\,\mathrm{RootSum}\left[c\#1^3a^3+3c\#1^2a^3+ca^3+3c\#1a^3+3ic\#1^3a^2+3ic\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a + b*x]/(c + d/x^3),x]
```

```
[Out] (6*((a + b*x)*ArcTan[a + b*x] + Log[1/Sqrt[1 + (a + b*x)^2]]) - b^3*d*RootS
um[I*c - 3*a*c - (3*I)*a^2*c + a^3*c - b^3*d - (3*I)*c*#1 + 3*a*c*#1 - (3*I
)*a^2*c*#1 + 3*a^3*c*#1 - 3*b^3*d*#1 + (3*I)*c*#1^2 + 3*a*c*#1^2 + (3*I)*a^
2*c*#1^2 + 3*a^3*c*#1^2 - 3*b^3*d*#1^2 - I*c*#1^3 - 3*a*c*#1^3 + (3*I)*a^2*
c*#1^3 + a^3*c*#1^3 - b^3*d*#1^3 & , (-(Pi*ArcTan[a + b*x]) - 2*ArcTan[a +
b*x]^2 + (2*I)*ArcTan[a + b*x]*ArcTanh[(-1 + #1)/(1 + #1)] + I*Pi*Log[1 + E
^((-2*I)*ArcTan[a + b*x])] + (2*I)*ArcTan[a + b*x]*Log[1 - E^((2*I)*ArcTan[
a + b*x] - 2*ArcTanh[(-1 + #1)/(1 + #1)])] - 2*ArcTanh[(-1 + #1)/(1 + #1)]*
Log[1 - E^((2*I)*ArcTan[a + b*x] - 2*ArcTanh[(-1 + #1)/(1 + #1)])] - I*Pi*L
og[1/Sqrt[1 + (a + b*x)^2]] + 2*ArcTanh[(-1 + #1)/(1 + #1)]*Log[Sin[ArcTan[
a + b*x] + I*ArcTanh[(-1 + #1)/(1 + #1)]]] + PolyLog[2, E^((2*I)*ArcTan[a +
 b*x] - 2*ArcTanh[(-1 + #1)/(1 + #1)])] - 2*ArcTan[a + b*x]^2*#1 + Pi*ArcTa
n[a + b*x]*#1^2 - (2*I)*ArcTan[a + b*x]*ArcTanh[(-1 + #1)/(1 + #1)]*#1^2 -
I*Pi*Log[1 + E^((-2*I)*ArcTan[a + b*x])]*#1^2 - (2*I)*ArcTan[a + b*x]*Log[1
 - E^((2*I)*ArcTan[a + b*x] - 2*ArcTanh[(-1 + #1)/(1 + #1)])]*#1^2 + 2*ArcT
anh[(-1 + #1)/(1 + #1)]*Log[1 - E^((2*I)*ArcTan[a + b*x] - 2*ArcTanh[(-1 +
```

```
#1)/(1 + #1)])]*#1^2 + I*Pi*Log[1/Sqrt[1 + (a + b*x)^2]]*#1^2 - 2*ArcTanh[(
-1 + #1)/(1 + #1)]*Log[Sin[ArcTan[a + b*x] + I*ArcTanh[(-1 + #1)/(1 + #1)]]
]*#1^2 - PolyLog[2, E^((2*I)*ArcTan[a + b*x] - 2*ArcTanh[(-1 + #1)/(1 + #1)
])]*#1^2 + 2*E^ArcTanh[(1 - #1)/(1 + #1)]*ArcTan[a + b*x]^2*Sqrt[#1/(1 + #1
)^2] + 4*E^ArcTanh[(1 - #1)/(1 + #1)]*ArcTan[a + b*x]^2*#1*Sqrt[#1/(1 + #1)
^2] + 2*E^ArcTanh[(1 - #1)/(1 + #1)]*ArcTan[a + b*x]^2*#1^2*Sqrt[#1/(1 + #1
)^2])/(-(a*c) - (2*I)*a^2*c + a^3*c - b^3*d + 2*a*c*#1 + 2*a^3*c*#1 - 2*b^3
*d*#1 - a*c*#1^2 + (2*I)*a^2*c*#1^2 + a^3*c*#1^2 - b^3*d*#1^2) & ])/(6*b*c)
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3 \arctan(bx+a)}{cx^3+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x^3),x, algorithm="fricas")
```

```
[Out] integral(x^3*arctan(b*x + a)/(c*x^3 + d), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x^3),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 1.24, size = 682, normalized size = 0.73

$$\frac{\arctan(bx+a)\,x}{c}+\frac{\arctan(bx+a)\,a}{bc}-\frac{\ln\left(1+(bx+a)^2\right)}{2bc}-\frac{2b^2d\left(\sum_{\_R1=\text{RootOf}\left(\left(a^3c+3ia^2c-d\,b^3-3ac-ic\right)\_Z^6+\left(3a^3c+3ia^2c-3d\right.\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(c+d/x^3),x)
```

```
[Out] arctan(b*x+a)/c*x+1/b*arctan(b*x+a)/c*a-1/2/b/c*ln(1+(b*x+a)^2)-2/3*b^2/c*d
*sum(1/(a^3*c*_R1^4+3*I*_R1^4*a^2*c-b^3*d*_R1^4-3*a*c*_R1^4-I*_R1^4*c+2*a^3
*c*_R1^2+2*I*_R1^2*a^2*c-2*b^3*d*_R1^2+2*_R1^2*a*c+2*I*_R1^2*c+a^3*c-I*a^2*
c-d*b^3+a*c-I*c)*(I*arctan(b*x+a)*ln((_R1-(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)
)/_R1)+dilog((_R1-(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))/_R1)),_R1=RootOf((3*I*
a^2*c+a^3*c-d*b^3-I*c-3*a*c)*_Z^6+(3*I*a^2*c+3*a^3*c-3*d*b^3+3*I*c+3*a*c)*_
Z^4+(-3*I*a^2*c+3*a^3*c-3*d*b^3-3*I*c+3*a*c)*_Z^2-3*I*a^2*c+a^3*c-d*b^3+I*c
-3*a*c))-2/3*b^2/c*d*sum(_R1^2/(a^3*c*_R1^4+3*I*_R1^4*a^2*c-b^3*d*_R1^4-3*a
*c*_R1^4-I*_R1^4*c+2*a^3*c*_R1^2+2*I*_R1^2*a^2*c-2*b^3*d*_R1^2+2*_R1^2*a*c+
2*I*_R1^2*c+a^3*c-I*a^2*c-d*b^3+a*c-I*c)*(I*arctan(b*x+a)*ln((_R1-(1+I*(b*x
+a))/(1+(b*x+a)^2)^(1/2))/_R1)+dilog((_R1-(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)
)/_R1)),_R1=RootOf((3*I*a^2*c+a^3*c-d*b^3-I*c-3*a*c)*_Z^6+(3*I*a^2*c+3*a^3*
c-3*d*b^3+3*I*c+3*a*c)*_Z^4+(-3*I*a^2*c+3*a^3*c-3*d*b^3-3*I*c+3*a*c)*_Z^2-3
*I*a^2*c+a^3*c-d*b^3+I*c-3*a*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan(bx+a)}{c+\frac{d}{x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x^3),x, algorithm="maxima")
```

```
[Out] integrate(arctan(b*x + a)/(c + d/x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a+b\,x)}{c+\frac{d}{x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)/(c + d/x^3),x)
```

```
[Out] int(atan(a + b*x)/(c + d/x^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(c+d/x**3),x)
```

```
[Out] Timed out
```

3.58 $\int \frac{\tan^{-1}(a+bx)}{c+d\sqrt{x}}\,dx$

Optimal. Leaf size=673

$$\frac{ic\mathrm{Li}_2\left(\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}c-\sqrt{-a-i}d}\right)}{d^2}+\frac{ic\mathrm{Li}_2\left(\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-a-i}d}\right)}{d^2}-\frac{ic\mathrm{Li}_2\left(\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}c-\sqrt{i-a}d}\right)}{d^2}-\frac{ic\mathrm{Li}_2\left(\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}c+\sqrt{i-a}d}\right)}{d^2}+\frac{ic\log\left(c+d\sqrt{x}\right)\log\left(\frac{d(-\sqrt{b}}{\sqrt{b}c}\right.}{d^2}$$

```
[Out] -I*c*ln(1-I*a-I*b*x)*ln(c+d*x^(1/2))/d^2+I*c*ln(1+I*a+I*b*x)*ln(c+d*x^(1/2)
)/d^2+I*c*ln(c+d*x^(1/2))*ln(d*((-I-a)^(1/2)-b^(1/2)*x^(1/2))/(d*(-I-a)^(1/
2)+c*b^(1/2)))/d^2-I*c*ln(c+d*x^(1/2))*ln(d*((I-a)^(1/2)-b^(1/2)*x^(1/2))/(
d*(I-a)^(1/2)+c*b^(1/2)))/d^2+I*c*ln(c+d*x^(1/2))*ln(-d*((-I-a)^(1/2)+b^(1/
2)*x^(1/2))/(-d*(-I-a)^(1/2)+c*b^(1/2)))/d^2-I*c*ln(c+d*x^(1/2))*ln(-d*((I-
a)^(1/2)+b^(1/2)*x^(1/2))/(-d*(I-a)^(1/2)+c*b^(1/2)))/d^2+I*c*polylog(2,b^(
1/2)*(c+d*x^(1/2))/(-d*(-I-a)^(1/2)+c*b^(1/2)))/d^2+I*c*polylog(2,b^(1/2)*(
c+d*x^(1/2))/(d*(-I-a)^(1/2)+c*b^(1/2)))/d^2-I*c*polylog(2,b^(1/2)*(c+d*x^(
1/2))/(-d*(I-a)^(1/2)+c*b^(1/2)))/d^2-I*c*polylog(2,b^(1/2)*(c+d*x^(1/2))/(
d*(I-a)^(1/2)+c*b^(1/2)))/d^2-2*I*arctanh(b^(1/2)*x^(1/2)/(I-a)^(1/2))*(I-a
)^(1/2)/d/b^(1/2)+2*I*arctan(b^(1/2)*x^(1/2)/(I+a)^(1/2))*(I+a)^(1/2)/d/b^(
1/2)+I*ln(1-I*a-I*b*x)*x^(1/2)/d-I*ln(1+I*a+I*b*x)*x^(1/2)/d
```

Rubi [A] time = 0.90, antiderivative size = 673, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {5051, 2408, 2466, 2448, 321, 205, 2462, 260, 2416, 2394, 2393, 2391, 208}

$$\frac{ic\text{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}c-\sqrt{-a-i}d}\right)}{d^2}+\frac{ic\text{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-a-i}d}\right)}{d^2}-\frac{ic\text{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}c-\sqrt{-a+i}d}\right)}{d^2}-\frac{ic\text{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\right.}{\sqrt{b}c+\sqrt{-}}\right.}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/(c + d*Sqrt[x]),x]
```

```
[Out] ((2*I)*Sqrt[I + a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]])/(Sqrt[b]*d) - ((2
*I)*Sqrt[I - a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I - a]])/(Sqrt[b]*d) + (I*c*
Log[(d*(Sqrt[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-I - a]*d)]*Log[
c + d*Sqrt[x]])/d^2 - (I*c*Log[(d*(Sqrt[I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]
*c + Sqrt[I - a]*d)]*Log[c + d*Sqrt[x]])/d^2 + (I*c*Log[-((d*(Sqrt[-I - a]
+ Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c - Sqrt[-I - a]*d))]*Log[c + d*Sqrt[x]])/d^2
- (I*c*Log[-((d*(Sqrt[I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c - Sqrt[I - a]*d
))]*Log[c + d*Sqrt[x]])/d^2 + (I*Sqrt[x]*Log[1 - I*a - I*b*x])/d - (I*c*Log
[c + d*Sqrt[x]]*Log[1 - I*a - I*b*x])/d^2 - (I*Sqrt[x]*Log[1 + I*a + I*b*x]
)/d + (I*c*Log[c + d*Sqrt[x]]*Log[1 + I*a + I*b*x])/d^2 + (I*c*PolyLog[2, (
Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[-I - a]*d)])/d^2 + (I*c*PolyLog[
2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-I - a]*d)])/d^2 - (I*c*Poly
Log[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[I - a]*d)])/d^2 - (I*c*P
olyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[I - a]*d)])/d^2
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2408

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_)^(r_))^(q_.), x_Symbol] :> With[{k = Denominator[r]}, Dist[k, Subst[
Int[x^(k - 1)*(f + g*x^(k*r))^q*(a + b*Log[c*(d + e*x^k)^n])^p, x], x, x^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && FractionQ[r] && IG
tQ[p, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 5051

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{c+d\sqrt{x}}\,dx = \frac{1}{2}i\int \frac{\log(1-ia-ibx)}{c+d\sqrt{x}}\,dx - \frac{1}{2}i\int \frac{\log(1+ia+ibx)}{c+d\sqrt{x}}\,dx$$

$$= i\,\text{Subst}\left(\int \frac{x\log\left(1-ia-ibx^2\right)}{c+dx}\,dx, x, \sqrt{x}\right) - i\,\text{Subst}\left(\int \frac{x\log\left(1+ia+ibx^2\right)}{c+dx}\,dx, x, \sqrt{x}\right.$$

$$= i\,\text{Subst}\left(\int \left(\frac{\log\left(1-ia-ibx^2\right)}{d} - \frac{c\log\left(1-ia-ibx^2\right)}{d(c+dx)}\right)dx, x, \sqrt{x}\right) - i\,\text{Subst}\left(\int \left(\frac{\log\left(\right.}{}\right.\right.$$

$$= \frac{i\,\text{Subst}\left(\int \log\left(1-ia-ibx^2\right)dx, x, \sqrt{x}\right)}{d} - \frac{i\,\text{Subst}\left(\int \log\left(1+ia+ibx^2\right)dx, x, \sqrt{x}\right)}{d} -$$

$$= \frac{i\sqrt{x}\log(1-ia-ibx)}{d} - \frac{ic\log\left(c+d\sqrt{x}\right)\log(1-ia-ibx)}{d^2} - \frac{i\sqrt{x}\log(1+ia+ibx)}{d} + \frac{i}{}$$

$$= \frac{i\sqrt{x}\log(1-ia-ibx)}{d} - \frac{ic\log\left(c+d\sqrt{x}\right)\log(1-ia-ibx)}{d^2} - \frac{i\sqrt{x}\log(1+ia+ibx)}{d} + \frac{i}{}$$

$$= \frac{2i\sqrt{i+a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}\,d} - \frac{2i\sqrt{i-a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}\,d} + \frac{i\sqrt{x}\log(1-ia-ibx)}{d} - \frac{ic\log\left(c\right.}{}$$

$$= \frac{2i\sqrt{i+a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}\,d} - \frac{2i\sqrt{i-a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}\,d} + \frac{ic\log\left(\frac{d\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-i-a}\,d}\right)\log\left(c+\right.}{d^2}$$

$$= \frac{2i\sqrt{i+a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}\,d} - \frac{2i\sqrt{i-a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}\,d} + \frac{ic\log\left(\frac{d\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-i-a}\,d}\right)\log\left(c+\right.}{d^2}$$

$$= \frac{2i\sqrt{i+a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}\,d} - \frac{2i\sqrt{i-a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}\,d} + \frac{ic\log\left(\frac{d\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-i-a}\,d}\right)\log\left(c+\right.}{d^2}$$

Mathematica [A] time = 0.60, size = 604, normalized size = 0.90

$$i\left(c\text{Li}_2\left(\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}c-\sqrt{-a-i}\,d}\right) + c\text{Li}_2\left(\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-a-i}\,d}\right) - c\text{Li}_2\left(\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}c-\sqrt{i-a}\,d}\right) - c\text{Li}_2\left(\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}c+\sqrt{i-a}\,d}\right) + c\log\left(c+d\sqrt{x}\right)\log\left(\frac{d(-\sqrt{}}{\sqrt{}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x]/(c + d*Sqrt[x]),x]
```

```
[Out] (I*((2*Sqrt[I + a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]])/Sqrt[b] - (2*Sq
rt[I - a]*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I - a]])/Sqrt[b] + c*Log[(d*(Sqr
t[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-I - a]*d)]*Log[c + d*Sqrt[
x]] - c*Log[(d*(Sqrt[I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[I - a]*d)
]*Log[c + d*Sqrt[x]] + c*Log[(d*(Sqrt[-I - a] + Sqrt[b]*Sqrt[x]))/(-(Sqrt[b
]*c) + Sqrt[-I - a]*d)]*Log[c + d*Sqrt[x]] - c*Log[(d*(Sqrt[I - a] + Sqrt[b
]*Sqrt[x]))/(-(Sqrt[b]*c) + Sqrt[I - a]*d)]*Log[c + d*Sqrt[x]] - d*Sqrt[x]*
Log[1 + I*a + I*b*x] + c*Log[c + d*Sqrt[x]]*Log[1 + I*a + I*b*x] + d*Sqrt[x
]*Log[(-I)*(I + a + b*x)] - c*Log[c + d*Sqrt[x]]*Log[(-I)*(I + a + b*x)] +
c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[-I - a]*d)] + c*Po
lyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-I - a]*d)] - c*PolyLo
g[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[I - a]*d)] - c*PolyLog[2,
(Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[I - a]*d)]))/d^2
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d\sqrt{x}\arctan(bx+a)-c\arctan(bx+a)}{d^2x-c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d*x^(1/2)),x, algorithm="fricas")
```

```
[Out] integral((d*sqrt(x)*arctan(b*x + a) - c*arctan(b*x + a))/(d^2*x - c^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d*x^(1/2)),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.33, size = 344, normalized size = 0.51

$$\frac{2\arctan(bx+a)\sqrt{x}}{d}-\frac{2\arctan(bx+a)c\ln\left(c+d\sqrt{x}\right)}{d^2}+c\left(\sum_{\_R1=\mathrm{RootOf}\left(b^2\_Z^4-4cb^2\_Z^3+\left(2abd^2+6b^2c^2\right)\_Z^2+\left(-4abcd^2-4b^2c^3\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(c+d*x^(1/2)),x)
```

```
[Out] 2*arctan(b*x+a)/d*x^(1/2)-2*arctan(b*x+a)*c/d^2*ln(c+d*x^(1/2))+c*sum(1/(_R
1^2*b-2*_R1*b*c+a*d^2+b*c^2)*(ln(c+d*x^(1/2))*ln((-d*x^(1/2)+_R1-c)/_R1)+di
log((-d*x^(1/2)+_R1-c)/_R1)),_R1=RootOf(b^2*_Z^4-4*c*b^2*_Z^3+(2*a*b*d^2+6*
b^2*c^2)*_Z^2+(-4*a*b*c*d^2-4*b^2*c^3)*_Z+a^2*d^4+2*a*b*c^2*d^2+b^2*c^4+d^4
))-sum((_R^2-2*_R*c+c^2)/(_R^3*b-3*_R^2*b*c+_R*a*d^2+3*_R*b*c^2-a*c*d^2-b*c
^3)*ln(d*x^(1/2)-_R+c),_R=RootOf(b^2*_Z^4-4*c*b^2*_Z^3+(2*a*b*d^2+6*b^2*c^2
)*_Z^2+(-4*a*b*c*d^2-4*b^2*c^3)*_Z+a^2*d^4+2*a*b*c^2*d^2+b^2*c^4+d^4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\arctan(bx+a)}{d\sqrt{x}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d*x^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(arctan(b*x + a)/(d*sqrt(x) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}\left(a+b\,x\right)}{c+d\,\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)/(c + d*x^(1/2)),x)
```

```
[Out] int(atan(a + b*x)/(c + d*x^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(c+d*x**(1/2)),x)
```

```
[Out] Timed out
```

3.59 $\int \frac{\tan^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx$

Optimal. Leaf size=770

$$-\frac{id^2\text{Li}_2\left(-\frac{\sqrt{b}\left(\sqrt{x}c+d\right)}{\sqrt{-a-i}c-\sqrt{b}d}\right)}{c^3}+\frac{id^2\text{Li}_2\left(-\frac{\sqrt{b}\left(\sqrt{x}c+d\right)}{\sqrt{i-a}c-\sqrt{b}d}\right)}{c^3}-\frac{id^2\text{Li}_2\left(\frac{\sqrt{b}\left(\sqrt{x}c+d\right)}{\sqrt{-a-i}c+\sqrt{b}d}\right)}{c^3}+\frac{id^2\text{Li}_2\left(\frac{\sqrt{b}\left(\sqrt{x}c+d\right)}{\sqrt{i-a}c+\sqrt{b}d}\right)}{c^3}-\frac{id^2\log\left(c\sqrt{x}+d\right)\log}{c^3}$$

```
[Out] -1/2*(1+I*a+I*b*x)*ln(1+I*a+I*b*x)/b/c-1/2*(1-I*a-I*b*x)*ln(-I*(I+a+b*x))/b
/c-2*I*d*arctan(b^(1/2)*x^(1/2)/(I+a)^(1/2))*(I+a)^(1/2)/c^2/b^(1/2)-I*d*ln
(1-I*a-I*b*x)*x^(1/2)/c^2+I*d*ln(1+I*a+I*b*x)*x^(1/2)/c^2-I*d^2*ln(d+c*x^(1
/2))*ln(c*((-I-a)^(1/2)+b^(1/2)*x^(1/2))/(c*(-I-a)^(1/2)-d*b^(1/2)))/c^3+I*
d^2*polylog(2,-b^(1/2)*(d+c*x^(1/2))/(c*(I-a)^(1/2)-d*b^(1/2)))/c^3+I*d^2*l
n(d+c*x^(1/2))*ln(c*((I-a)^(1/2)+b^(1/2)*x^(1/2))/(c*(I-a)^(1/2)-d*b^(1/2))
)/c^3-I*d^2*ln(d+c*x^(1/2))*ln(c*((-I-a)^(1/2)-b^(1/2)*x^(1/2))/(c*(-I-a)^(
1/2)+d*b^(1/2)))/c^3+I*d^2*ln(d+c*x^(1/2))*ln(c*((I-a)^(1/2)-b^(1/2)*x^(1/2
))/(c*(I-a)^(1/2)+d*b^(1/2)))/c^3+2*I*d*arctanh(b^(1/2)*x^(1/2)/(I-a)^(1/2)
)*(I-a)^(1/2)/c^2/b^(1/2)+I*d^2*polylog(2,b^(1/2)*(d+c*x^(1/2))/(c*(I-a)^(1
/2)+d*b^(1/2)))/c^3+I*d^2*ln(1-I*a-I*b*x)*ln(d+c*x^(1/2))/c^3-I*d^2*ln(1+I*
a+I*b*x)*ln(d+c*x^(1/2))/c^3-I*d^2*polylog(2,-b^(1/2)*(d+c*x^(1/2))/(c*(-I-
a)^(1/2)-d*b^(1/2)))/c^3-I*d^2*polylog(2,b^(1/2)*(d+c*x^(1/2))/(c*(-I-a)^(1
/2)+d*b^(1/2)))/c^3
```

Rubi [A] time = 0.98, antiderivative size = 770, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 16, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {5051, 2408, 2476, 2448, 321, 205, 2454, 2389, 2295, 2462, 260, 2416, 2394, 2393, 2391, 208}

$$-\frac{id^2\text{PolyLog}\left(2,-\frac{\sqrt{b}\left(c\sqrt{x}+d\right)}{-\sqrt{b}d+\sqrt{-a-i}c}\right)}{c^3}+\frac{id^2\text{PolyLog}\left(2,-\frac{\sqrt{b}\left(c\sqrt{x}+d\right)}{-\sqrt{b}d+\sqrt{-a+i}c}\right)}{c^3}-\frac{id^2\text{PolyLog}\left(2,\frac{\sqrt{b}\left(c\sqrt{x}+d\right)}{\sqrt{b}d+\sqrt{-a-i}c}\right)}{c^3}+\frac{id^2\text{PolyLog}\left(2\right.}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/(c + d/Sqrt[x]),x]
```

```
[Out] ((-2*I)*Sqrt[I + a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]])/(Sqrt[b]*c^2)
+ ((2*I)*Sqrt[I - a]*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I - a]])/(Sqrt[b]*c^2
) - (I*d^2*Log[(c*(Sqrt[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c + Sqrt[
b]*d)]*Log[d + c*Sqrt[x]])/c^3 + (I*d^2*Log[(c*(Sqrt[I - a] - Sqrt[b]*Sqrt[
x]))/(Sqrt[I - a]*c + Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 - (I*d^2*Log[(c*(
Sqrt[-I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c - Sqrt[b]*d)]*Log[d + c*Sq
rt[x]])/c^3 + (I*d^2*Log[(c*(Sqrt[I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[I - a]*c
 - Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 - (I*d*Sqrt[x]*Log[1 - I*a - I*b*x])
/c^2 + (I*d^2*Log[d + c*Sqrt[x]]*Log[1 - I*a - I*b*x])/c^3 + (I*d*Sqrt[x]*L
og[1 + I*a + I*b*x])/c^2 - ((1 + I*a + I*b*x)*Log[1 + I*a + I*b*x])/(2*b*c)
 - (I*d^2*Log[d + c*Sqrt[x]]*Log[1 + I*a + I*b*x])/c^3 - ((1 - I*a - I*b*x)
*Log[(-I)*(I + a + b*x)])/(2*b*c) - (I*d^2*PolyLog[2, -((Sqrt[b]*(d + c*Sqr
t[x]))/(Sqrt[-I - a]*c - Sqrt[b]*d))])/c^3 + (I*d^2*PolyLog[2, -((Sqrt[b]*(
d + c*Sqrt[x]))/(Sqrt[I - a]*c - Sqrt[b]*d))])/c^3 - (I*d^2*PolyLog[2, (Sqr
t[b]*(d + c*Sqrt[x]))/(Sqrt[-I - a]*c + Sqrt[b]*d)])/c^3 + (I*d^2*PolyLog[2
, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[I - a]*c + Sqrt[b]*d)])/c^3
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2408

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_)^(r_))^(q_.), x_Symbol] :> With[{k = Denominator[r]}, Dist[k, Subst[
Int[x^(k - 1)*(f + g*x^(k*r))^q*(a + b*Log[c*(d + e*x^k)^n])^p, x], x, x^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && FractionQ[r] && IG
tQ[p, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
```

```
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 5051

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \frac{1}{2}i\int \frac{\log(1-ia-ibx)}{c+\frac{d}{\sqrt{x}}}\,dx - \frac{1}{2}i\int \frac{\log(1+ia+ibx)}{c+\frac{d}{\sqrt{x}}}\,dx$$

$$= i\,\text{Subst}\left(\int \frac{x\log\left(1-ia-ibx^2\right)}{c+\frac{d}{x}}\,dx, x, \sqrt{x}\right) - i\,\text{Subst}\left(\int \frac{x\log\left(1+ia+ibx^2\right)}{c+\frac{d}{x}}\,dx, x, \sqrt{x}\right.$$

$$= i\,\text{Subst}\left(\int \left(-\frac{d\log\left(1-ia-ibx^2\right)}{c^2} + \frac{x\log\left(1-ia-ibx^2\right)}{c} + \frac{d^2\log\left(1-ia-ibx^2\right)}{c^2(d+cx)}\right)dx,\right.$$

$$= \frac{i\,\text{Subst}\left(\int x\log\left(1-ia-ibx^2\right)dx, x, \sqrt{x}\right)}{c} - \frac{i\,\text{Subst}\left(\int x\log\left(1+ia+ibx^2\right)dx, x, \sqrt{x}\right)}{c}$$

$$= -\frac{id\sqrt{x}\log(1-ia-ibx)}{c^2} + \frac{id^2\log\left(d+c\sqrt{x}\right)\log(1-ia-ibx)}{c^3} + \frac{id\sqrt{x}\log(1+ia+ibx}{c^2}$$

$$= -\frac{id\sqrt{x}\log(1-ia-ibx)}{c^2} + \frac{id^2\log\left(d+c\sqrt{x}\right)\log(1-ia-ibx)}{c^3} + \frac{id\sqrt{x}\log(1+ia+ibx}{c^2}$$

$$= -\frac{2i\sqrt{i+a}\,d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}\,c^2} + \frac{2i\sqrt{i-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}\,c^2} - \frac{id\sqrt{x}\log(1-ia-ibx)}{c^2} + \frac{id^2}{}$$

$$= -\frac{2i\sqrt{i+a}\,d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}\,c^2} + \frac{2i\sqrt{i-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}\,c^2} - \frac{id^2\log\left(\frac{c\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}\,c+\sqrt{b}\,d}\right)\log}{c^3}$$

$$= -\frac{2i\sqrt{i+a}\,d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}\,c^2} + \frac{2i\sqrt{i-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}\,c^2} - \frac{id^2\log\left(\frac{c\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}\,c+\sqrt{b}\,d}\right)\log}{c^3}$$

$$= -\frac{2i\sqrt{i+a}\,d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}\,c^2} + \frac{2i\sqrt{i-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}\,c^2} - \frac{id^2\log\left(\frac{c\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}\,c+\sqrt{b}\,d}\right)\log}{c^3}$$

Mathematica [A] time = 0.83, size = 666, normalized size = 0.86

$$\frac{i\left(-\frac{c^2(a+bx-i)\log(ia+ibx+1)}{b} + \frac{c^2(a+bx+i)\log(-i(a+bx+i))}{b} - 2d^2\left(\text{Li}_2\left(\frac{\sqrt{b}\left(\sqrt{x}c+d\right)}{\sqrt{b}d-\sqrt{-a-i}c}\right) + \text{Li}_2\left(\frac{\sqrt{b}\left(\sqrt{x}c+d\right)}{\sqrt{-a-i}c+\sqrt{b}d}\right) + \log\left(c\sqrt{x}+d\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x]/(c + d/Sqrt[x]),x]
```

```
[Out] ((I/2)*(4*c*d*(Sqrt[x] - (Sqrt[I + a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]]
)/Sqrt[b]) - 4*c*d*(Sqrt[x] - (Sqrt[I - a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I
 - a]])/Sqrt[b]) + 2*c*d*Sqrt[x]*Log[1 + I*a + I*b*x] - (c^2*(-I + a + b*x)
*Log[1 + I*a + I*b*x])/b - 2*d^2*Log[d + c*Sqrt[x]]*Log[1 + I*a + I*b*x] -
2*c*d*Sqrt[x]*Log[(-I)*(I + a + b*x)] + (c^2*(I + a + b*x)*Log[(-I)*(I + a
+ b*x)])/b + 2*d^2*Log[d + c*Sqrt[x]]*Log[(-I)*(I + a + b*x)] - 2*d^2*((Log
[(c*(Sqrt[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c + Sqrt[b]*d)] + Log[(
c*(Sqrt[-I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c - Sqrt[b]*d)])*Log[d +
c*Sqrt[x]] + PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(-(Sqrt[-I - a]*c) + Sqrt
[b]*d)] + PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[-I - a]*c + Sqrt[b]*d)
```

```
]) + 2*d^2*((Log[(c*(Sqrt[I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[I - a]*c + Sqrt[
b]*d)] + Log[(c*(Sqrt[I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[I - a]*c - Sqrt[b]*d
)])*Log[d + c*Sqrt[x]] + PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(-(Sqrt[I - a
]*c) + Sqrt[b]*d)] + PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[I - a]*c +
Sqrt[b]*d)])))/c^3
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{cx\arctan\left(bx+a\right)-d\sqrt{x}\arctan\left(bx+a\right)}{c^2x-d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x^(1/2)),x, algorithm="fricas")
```

```
[Out] integral((c*x*arctan(b*x + a) - d*sqrt(x)*arctan(b*x + a))/(c^2*x - d^2), x
)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x^(1/2)),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.31, size = 377, normalized size = 0.49

$$\frac{\arctan\left(bx+a\right)x}{c}-\frac{2\arctan\left(bx+a\right)d\sqrt{x}}{c^2}+\frac{2\arctan\left(bx+a\right)d^2\ln\left(d+c\sqrt{x}\right)}{c^3}-\frac{d^2\left(\sum_{\_R1=\mathrm{RootOf}\left(b^2\_Z^4-4b^2d\_Z^3+\left(2c^2ab+\right.\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(c+d/x^(1/2)),x)
```

```
[Out] arctan(b*x+a)/c*x-2*arctan(b*x+a)/c^2*d*x^(1/2)+2*arctan(b*x+a)/c^3*d^2*ln(
d+c*x^(1/2))-1/c*d^2*sum(1/(_R1^2*b-2*_R1*b*d+a*c^2+b*d^2)*(ln(d+c*x^(1/2))
*ln((-c*x^(1/2)+_R1-d)/_R1)+dilog((-c*x^(1/2)+_R1-d)/_R1)),_R1=RootOf(b^2*_
Z^4-4*b^2*d*_Z^3+(2*a*b*c^2+6*b^2*d^2)*_Z^2+(-4*a*b*c^2*d-4*b^2*d^3)*_Z+a^2
*c^4+2*a*b*c^2*d^2+b^2*d^4+c^4))-1/2/c*sum((_R^3-5*_R^2*d+7*_R*d^2-3*d^3)/(
_R^3*b-3*_R^2*b*d+_R*a*c^2+3*_R*b*d^2-a*c^2*d-b*d^3)*ln(c*x^(1/2)-_R+d),_R=
RootOf(b^2*_Z^4-4*b^2*d*_Z^3+(2*a*b*c^2+6*b^2*d^2)*_Z^2+(-4*a*b*c^2*d-4*b^2
*d^3)*_Z+a^2*c^4+2*a*b*c^2*d^2+b^2*d^4+c^4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\arctan\left(bx+a\right)}{c+\frac{d}{\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(arctan(b*x + a)/(c + d/sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\mathrm{atan}\left(a+b\,x\right)}{c+\frac{d}{\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)/(c + d/x^(1/2)),x)
```

```
[Out] int(atan(a + b*x)/(c + d/x^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(c+d/x**(1/2)),x)
```

```
[Out] Timed out
```

3.60 $\int \frac{\tan^{-1}(a+bx)}{1+x^2}\,dx$

Optimal. Leaf size=274

$$-\frac{1}{4}\text{Li}_2\left(-\frac{-a-bx+i}{a-i(1-b)}\right)+\frac{1}{4}\text{Li}_2\left(-\frac{-a-bx+i}{a-i(b+1)}\right)-\frac{1}{4}\text{Li}_2\left(\frac{a+bx+i}{a-ib+i}\right)+\frac{1}{4}\text{Li}_2\left(\frac{a+bx+i}{a+i(b+1)}\right)+\frac{1}{4}\log\left(\frac{b(-x+i)}{a+i(b+1)}\right)\log(-$$

```
[Out] 1/4*ln(b*(I-x)/(a+I*(1+b)))*ln(1-I*a-I*b*x)-1/4*ln(-b*(I+x)/(a+I*(1-b)))*ln
(1-I*a-I*b*x)-1/4*ln(b*(I-x)/(a-I*(1-b)))*ln(1+I*a+I*b*x)+1/4*ln(-b*(I+x)/(
a-I*(1+b)))*ln(1+I*a+I*b*x)-1/4*polylog(2,(-I+a+b*x)/(a-I*(1-b)))+1/4*polyl
og(2,(-I+a+b*x)/(a-I*(1+b)))-1/4*polylog(2,(I+a+b*x)/(I+a-I*b))+1/4*polylog
(2,(I+a+b*x)/(a+I*(1+b)))
```

Rubi [A] time = 0.27, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5051, 2409, 2394, 2393, 2391}

$$-\frac{1}{4}\text{PolyLog}\left(2,-\frac{-a-bx+i}{a-i(1-b)}\right)+\frac{1}{4}\text{PolyLog}\left(2,-\frac{-a-bx+i}{a-i(b+1)}\right)-\frac{1}{4}\text{PolyLog}\left(2,\frac{a+bx+i}{a-ib+i}\right)+\frac{1}{4}\text{PolyLog}\left(2,\frac{a+bx}{a+i(b}\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/(1 + x^2),x]
```

```
[Out] (Log[(b*(I - x))/(a + I*(1 + b))]*Log[1 - I*a - I*b*x])/4 - (Log[-((b*(I +
x))/(a + I*(1 - b)))]*Log[1 - I*a - I*b*x])/4 - (Log[(b*(I - x))/(a - I*(1
- b))]*Log[1 + I*a + I*b*x])/4 + (Log[-((b*(I + x))/(a - I*(1 + b)))]*Log[1
 + I*a + I*b*x])/4 - PolyLog[2, -((I - a - b*x)/(a - I*(1 - b)))]/4 + PolyL
og[2, -((I - a - b*x)/(a - I*(1 + b)))]/4 - PolyLog[2, (I + a + b*x)/(I + a
 - I*b)]/4 + PolyLog[2, (I + a + b*x)/(a + I*(1 + b))]/4
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 5051

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(a+bx)}{1+x^2}\,dx &= \frac{1}{2}i\int \frac{\log(1-ia-ibx)}{1+x^2}\,dx-\frac{1}{2}i\int \frac{\log(1+ia+ibx)}{1+x^2}\,dx\\ &= \frac{1}{2}i\int \left(\frac{i\log(1-ia-ibx)}{2(i-x)}+\frac{i\log(1-ia-ibx)}{2(i+x)}\right)dx-\frac{1}{2}i\int \left(\frac{i\log(1+ia+ibx)}{2(i-x)}+\frac{i\log(\right.\\ &= -\left(\frac{1}{4}\int \frac{\log(1-ia-ibx)}{i-x}\,dx\right)-\frac{1}{4}\int \frac{\log(1-ia-ibx)}{i+x}\,dx+\frac{1}{4}\int \frac{\log(1+ia+ibx)}{i-x}\,dx+\\ &= \frac{1}{4}\log\left(\frac{b(i-x)}{a+i(1+b)}\right)\log(1-ia-ibx)-\frac{1}{4}\log\left(-\frac{b(i+x)}{a+i(1-b)}\right)\log(1-ia-ibx)-\frac{1}{4}\log\\ &= \frac{1}{4}\log\left(\frac{b(i-x)}{a+i(1+b)}\right)\log(1-ia-ibx)-\frac{1}{4}\log\left(-\frac{b(i+x)}{a+i(1-b)}\right)\log(1-ia-ibx)-\frac{1}{4}\log\\ &= \frac{1}{4}\log\left(\frac{b(i-x)}{a+i(1+b)}\right)\log(1-ia-ibx)-\frac{1}{4}\log\left(-\frac{b(i+x)}{a+i(1-b)}\right)\log(1-ia-ibx)-\frac{1}{4}\log\end{aligned}$$

Mathematica [A] time = 0.06, size = 283, normalized size = 1.03

$$-\frac{1}{4}\text{Li}_2\left(\frac{-ia-ibx+1}{-ia-b+1}\right)+\frac{1}{4}\text{Li}_2\left(\frac{-ia-ibx+1}{-ia+b+1}\right)-\frac{1}{4}\text{Li}_2\left(\frac{ia+ibx+1}{ia-b+1}\right)+\frac{1}{4}\text{Li}_2\left(\frac{ia+ibx+1}{ia+b+1}\right)+\frac{1}{4}\log\left(\frac{b(-x+i)}{a+i(b+1)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x]/(1 + x^2),x]
```

```
[Out] (Log[(b*(I - x))/(a + I*(1 + b))]*Log[1 - I*a - I*b*x])/4 - (Log[-((b*(I +
x))/(a + I*(1 - b)))]*Log[1 - I*a - I*b*x])/4 - (Log[(b*(I - x))/(a - I*(1
- b))]*Log[1 + I*a + I*b*x])/4 + (Log[-((b*(I + x))/(a - I*(1 + b)))]*Log[1
 + I*a + I*b*x])/4 - PolyLog[2, (1 - I*a - I*b*x)/(1 - I*a - b)]/4 + PolyLo
g[2, (1 - I*a - I*b*x)/(1 - I*a + b)]/4 - PolyLog[2, (1 + I*a + I*b*x)/(1 +
 I*a - b)]/4 + PolyLog[2, (1 + I*a + I*b*x)/(1 + I*a + b)]/4
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\arctan(bx+a)}{x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(x^2+1),x, algorithm="fricas")
```

```
[Out] integral(arctan(b*x + a)/(x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(x^2+1),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 1.06, size = 833, normalized size = 3.04

$$\arctan(x)\arctan(bx+a)+\frac{i\arctan\left(\frac{bx+a}{b}-\frac{a}{b}\right)\ln\left(1-\frac{(-ib+a-i)\left(1+i\left(\frac{bx+a}{b}-\frac{a}{b}\right)\right)^2}{\left(\left(\frac{bx+a}{b}-\frac{a}{b}\right)^2+1\right)(-ib-a+i)}\right)}{2}+\frac{\arctan\left(\frac{bx+a}{b}-\frac{a}{b}\right)^2}{2}+\frac{\operatorname{polylog}\left(2,\frac{(-\cdots}{((\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(x^2+1),x)
```

```
[Out] arctan(x)*arctan(b*x+a)+1/2*I*arctan((b*x+a)/b-a/b)*ln(1-(-I*b+a-I)*(1+I*((
b*x+a)/b-a/b))^2/(((b*x+a)/b-a/b)^2+1)/(-I*b+I-a))+1/2*arctan((b*x+a)/b-a/b
)^2+1/4*polylog(2,(-I*b+a-I)*(1+I*((b*x+a)/b-a/b))^2/(((b*x+a)/b-a/b)^2+1)/
(-I*b+I-a))+1/2*b/(I*b+I+a)*ln(1-(I+a-I*b)*(1+I*((b*x+a)/b-a/b))^2/(((b*x+a
)/b-a/b)^2+1)/(-I*b-I-a))*arctan((b*x+a)/b-a/b)+1/2/(I*b+I+a)*ln(1-(I+a-I*b
)*(1+I*((b*x+a)/b-a/b))^2/(((b*x+a)/b-a/b)^2+1)/(-I*b-I-a))*arctan((b*x+a)/
b-a/b)-1/2*I/(I*b+I+a)*ln(1-(I+a-I*b)*(1+I*((b*x+a)/b-a/b))^2/(((b*x+a)/b-a
/b)^2+1)/(-I*b-I-a))*arctan((b*x+a)/b-a/b)*a-1/2*I*b/(I*b+I+a)*arctan((b*x+
a)/b-a/b)^2-1/4*I*b/(I*b+I+a)*polylog(2,(I+a-I*b)*(1+I*((b*x+a)/b-a/b))^2/(
((b*x+a)/b-a/b)^2+1)/(-I*b-I-a))-1/2*I/(I*b+I+a)*arctan((b*x+a)/b-a/b)^2-1/
2/(I*b+I+a)*arctan((b*x+a)/b-a/b)^2*a-1/4*I/(I*b+I+a)*polylog(2,(I+a-I*b)*(
1+I*((b*x+a)/b-a/b))^2/(((b*x+a)/b-a/b)^2+1)/(-I*b-I-a))-1/4/(I*b+I+a)*poly
log(2,(I+a-I*b)*(1+I*((b*x+a)/b-a/b))^2/(((b*x+a)/b-a/b)^2+1)/(-I*b-I-a))*a
```

maxima [A] time = 0.55, size = 328, normalized size = 1.20

$$\frac{1}{8}b\left(\frac{8\arctan(x)\arctan\left(\frac{b^2x+ab}{b}\right)}{b}-\frac{4\arctan(x)\arctan\left(\frac{ab+(b^2+b)x}{a^2+b^2+2b+1},\frac{abx+a^2+b+1}{a^2+b^2+2b+1}\right)-4\arctan(x)\arctan\left(\frac{ab+(b^2-b)x}{a^2+b^2-2b+1}\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/8*b*(8*arctan(x)*arctan((b^2*x + a*b)/b)/b - (4*arctan(x)*arctan2((a*b +
(b^2 + b)*x)/(a^2 + b^2 + 2*b + 1), (a*b*x + a^2 + b + 1)/(a^2 + b^2 + 2*b
+ 1)) - 4*arctan(x)*arctan2((a*b + (b^2 - b)*x)/(a^2 + b^2 - 2*b + 1), (a*b
*x + a^2 - b + 1)/(a^2 + b^2 - 2*b + 1)) + log(x^2 + 1)*log((b^2*x^2 + 2*a*
b*x + a^2 + 1)/(a^2 + b^2 + 2*b + 1)) - log(x^2 + 1)*log((b^2*x^2 + 2*a*b*x
 + a^2 + 1)/(a^2 + b^2 - 2*b + 1)) + 2*dilog(-(I*b*x - b)/(I*a + b + 1)) -
2*dilog(-(I*b*x - b)/(I*a + b - 1)) + 2*dilog((I*b*x + b)/(-I*a + b + 1)) -
 2*dilog((I*b*x + b)/(-I*a + b - 1)))/b) + arctan(b*x + a)*arctan(x) - arct
an(x)*arctan((b^2*x + a*b)/b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a+b\,x)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)/(x^2 + 1),x)
```

```
[Out] int(atan(a + b*x)/(x^2 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}(a+bx)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(x**2+1),x)
```

```
[Out] Integral(atan(a + b*x)/(x**2 + 1), x)
```

3.61 $\int \frac{\tan^{-1}(d+ex)}{a+bx^2} \, dx$

Optimal. Leaf size=543

$$-\frac{i\text{Li}_2\left(\frac{\sqrt{b}(-d-ex+i)}{\sqrt{b}(i-d)-\sqrt{-a}e}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{i\text{Li}_2\left(\frac{\sqrt{b}(-d-ex+i)}{\sqrt{b}(i-d)+\sqrt{-a}e}\right)}{4\sqrt{-a}\sqrt{b}}-\frac{i\text{Li}_2\left(\frac{\sqrt{b}(d+ex+i)}{\sqrt{b}(d+i)-\sqrt{-a}e}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{i\text{Li}_2\left(\frac{\sqrt{b}(d+ex+i)}{\sqrt{b}(d+i)+\sqrt{-a}e}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{i\log(-id-iex+1)\log\left(\frac{e}{\sqrt{}}\right.}{4\sqrt{-a}\sqrt{b}}$$

```
[Out] -1/4*I*ln(1+I*d+I*e*x)*ln(-e*((-a)^(1/2)-x*b^(1/2))/(-e*(-a)^(1/2)+(I-d)*b^
(1/2)))/(-a)^(1/2)/b^(1/2)+1/4*I*ln(1-I*d-I*e*x)*ln(e*((-a)^(1/2)-x*b^(1/2)
)/(e*(-a)^(1/2)+(I+d)*b^(1/2)))/(-a)^(1/2)/b^(1/2)+1/4*I*ln(1+I*d+I*e*x)*ln
(e*((-a)^(1/2)+x*b^(1/2))/(e*(-a)^(1/2)+(I-d)*b^(1/2)))/(-a)^(1/2)/b^(1/2)-
1/4*I*ln(1-I*d-I*e*x)*ln(-e*((-a)^(1/2)+x*b^(1/2))/(-e*(-a)^(1/2)+(I+d)*b^(
1/2)))/(-a)^(1/2)/b^(1/2)-1/4*I*polylog(2,(I-d-e*x)*b^(1/2)/(-e*(-a)^(1/2)+
(I-d)*b^(1/2)))/(-a)^(1/2)/b^(1/2)+1/4*I*polylog(2,(I-d-e*x)*b^(1/2)/(e*(-a
)^(1/2)+(I-d)*b^(1/2)))/(-a)^(1/2)/b^(1/2)-1/4*I*polylog(2,(I+d+e*x)*b^(1/2
)/(-e*(-a)^(1/2)+(I+d)*b^(1/2)))/(-a)^(1/2)/b^(1/2)+1/4*I*polylog(2,(I+d+e*
x)*b^(1/2)/(e*(-a)^(1/2)+(I+d)*b^(1/2)))/(-a)^(1/2)/b^(1/2)
```

Rubi [A] time = 0.60, antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5051, 2409, 2394, 2393, 2391}

$$-\frac{i\text{PolyLog}\left(2,\frac{\sqrt{b}(-d-ex+i)}{-\sqrt{-a}e+\sqrt{b}(-d+i)}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{i\text{PolyLog}\left(2,\frac{\sqrt{b}(-d-ex+i)}{\sqrt{-a}e+\sqrt{b}(-d+i)}\right)}{4\sqrt{-a}\sqrt{b}}-\frac{i\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex+i)}{-\sqrt{-a}e+\sqrt{b}(d+i)}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{i\text{PolyLog}\left(2,\frac{\sqrt{b}(}{\sqrt{-a}e}\right.}{4\sqrt{-a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[d + e*x]/(a + b*x^2),x]
```

```
[Out] ((I/4)*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*(I + d) + Sqrt[-a]*e)]*Log[1
 - I*d - I*e*x])/(Sqrt[-a]*Sqrt[b]) - ((I/4)*Log[-((e*(Sqrt[-a] + Sqrt[b]*x
))/(Sqrt[b]*(I + d) - Sqrt[-a]*e))]*Log[1 - I*d - I*e*x])/(Sqrt[-a]*Sqrt[b]
) - ((I/4)*Log[-((e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*(I - d) - Sqrt[-a]*e))
]*Log[1 + I*d + I*e*x])/(Sqrt[-a]*Sqrt[b]) + ((I/4)*Log[(e*(Sqrt[-a] + Sqrt
[b]*x))/(Sqrt[b]*(I - d) + Sqrt[-a]*e)]*Log[1 + I*d + I*e*x])/(Sqrt[-a]*Sqr
t[b]) - ((I/4)*PolyLog[2, (Sqrt[b]*(I - d - e*x))/(Sqrt[b]*(I - d) - Sqrt[-
a]*e)])/(Sqrt[-a]*Sqrt[b]) + ((I/4)*PolyLog[2, (Sqrt[b]*(I - d - e*x))/(Sqr
t[b]*(I - d) + Sqrt[-a]*e)])/(Sqrt[-a]*Sqrt[b]) - ((I/4)*PolyLog[2, (Sqrt[b
]*(I + d + e*x))/(Sqrt[b]*(I + d) - Sqrt[-a]*e)])/(Sqrt[-a]*Sqrt[b]) + ((I/
4)*PolyLog[2, (Sqrt[b]*(I + d + e*x))/(Sqrt[b]*(I + d) + Sqrt[-a]*e)])/(Sqr
t[-a]*Sqrt[b])
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
```

```
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 5051

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(d+ex)}{a+bx^2}\,dx &= \frac{1}{2}i\int \frac{\log(1-id-iex)}{a+bx^2}\,dx - \frac{1}{2}i\int \frac{\log(1+id+iex)}{a+bx^2}\,dx \\
&= \frac{1}{2}i\int \left(\frac{\sqrt{-a}\log(1-id-iex)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{-a}\log(1-id-iex)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx - \frac{1}{2}i\int \left(\frac{\sqrt{-a}\log(1+id+i}{2a\left(\sqrt{-a}-\sqrt{b}x\right.}\right. \\
&= -\frac{i\int \frac{\log(1-id-iex)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4\sqrt{-a}} - \frac{i\int \frac{\log(1-id-iex)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4\sqrt{-a}} + \frac{i\int \frac{\log(1+id+iex)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4\sqrt{-a}} + \frac{i\int \frac{\log(1+id+iex)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4\sqrt{-a}} \\
&= \frac{i\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}(i+d)+\sqrt{-a}e}\right)\log(1-id-iex)}{4\sqrt{-a}\sqrt{b}} - \frac{i\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}(i+d)-\sqrt{-a}e}\right)\log(1-id-iex)}{4\sqrt{-a}\sqrt{b}} - \frac{i\log\left(\right.}{} \\
&= \frac{i\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}(i+d)+\sqrt{-a}e}\right)\log(1-id-iex)}{4\sqrt{-a}\sqrt{b}} - \frac{i\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}(i+d)-\sqrt{-a}e}\right)\log(1-id-iex)}{4\sqrt{-a}\sqrt{b}} - \frac{i\log\left(\right.}{} \\
&= \frac{i\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}(i+d)+\sqrt{-a}e}\right)\log(1-id-iex)}{4\sqrt{-a}\sqrt{b}} - \frac{i\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}(i+d)-\sqrt{-a}e}\right)\log(1-id-iex)}{4\sqrt{-a}\sqrt{b}} - \frac{i\log\left(\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.40, size = 409, normalized size = 0.75

$$\frac{i\left(\text{Li}_2\left(\frac{\sqrt{b}(d+ex-i)}{\sqrt{b}(d-i)-\sqrt{-a}e}\right) - \text{Li}_2\left(\frac{\sqrt{b}(d+ex-i)}{\sqrt{b}(d-i)+\sqrt{-a}e}\right) - \text{Li}_2\left(\frac{\sqrt{b}(d+ex+i)}{\sqrt{b}(d+i)-\sqrt{-a}e}\right) + \text{Li}_2\left(\frac{\sqrt{b}(d+ex+i)}{\sqrt{b}(d+i)+\sqrt{-a}e}\right) + \log(id+iex+1)\left(-\log\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[d + e*x]/(a + b*x^2),x]
```

```
[Out] ((I/4)*(-(Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*(-I + d) + Sqrt[-a]*e)]*L
og[1 + I*d + I*e*x]) + Log[(e*(Sqrt[-a] + Sqrt[b]*x))/(-(Sqrt[b]*(-I + d))
+ Sqrt[-a]*e)]*Log[1 + I*d + I*e*x] + Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[
b]*(I + d) + Sqrt[-a]*e)]*Log[(-I)*(I + d + e*x)] - Log[(e*(Sqrt[-a] + Sqrt
[b]*x))/(-(Sqrt[b]*(I + d)) + Sqrt[-a]*e)]*Log[(-I)*(I + d + e*x)] + PolyLo
g[2, (Sqrt[b]*(-I + d + e*x))/(Sqrt[b]*(-I + d) - Sqrt[-a]*e)] - PolyLog[2,
 (Sqrt[b]*(-I + d + e*x))/(Sqrt[b]*(-I + d) + Sqrt[-a]*e)] - PolyLog[2, (Sq
```

```
rt[b]*(I + d + e*x))/(Sqrt[b]*(I + d) - Sqrt[-a]*e)] + PolyLog[2, (Sqrt[b]*
(I + d + e*x))/(Sqrt[b]*(I + d) + Sqrt[-a]*e)]))/(Sqrt[-a]*Sqrt[b])
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\arctan(ex+d)}{bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x+d)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral(arctan(e*x + d)/(b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x+d)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 1.24, size = 2192, normalized size = 4.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(e*x+d)/(b*x^2+a),x)
```

```
[Out] 1/2*I/e*(a*b*e^2)^(1/2)/a/(a*e^2+b*d^2+2*(a*b*e^2)^(1/2)+b)*ln(1-(2*I*b*d+a
*e^2+b*d^2-b)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2-b*d^2-2*(a*b*e^2)^(1/2)
-b))*arctan(e*x+d)+1/2*I/e*(a*b*e^2)^(1/2)/a/(a*e^2+b*d^2+2*(a*b*e^2)^(1/2)
+b)*ln(1-(2*I*b*d+a*e^2+b*d^2-b)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2-b*d^
2-2*(a*b*e^2)^(1/2)-b))*arctan(e*x+d)*d^2-1/2*I*e/b*(a*b*e^2)^(1/2)/(a*e^2+
b*d^2-2*(a*b*e^2)^(1/2)+b)*ln(1-(2*I*b*d+a*e^2+b*d^2-b)*(1+I*(e*x+d))^2/((e
*x+d)^2+1)/(-a*e^2-b*d^2+2*(a*b*e^2)^(1/2)-b))*arctan(e*x+d)+I*e/(a*e^2+b*d
^2+2*(a*b*e^2)^(1/2)+b)*ln(1-(2*I*b*d+a*e^2+b*d^2-b)*(1+I*(e*x+d))^2/((e*x+
d)^2+1)/(-a*e^2-b*d^2-2*(a*b*e^2)^(1/2)-b))*arctan(e*x+d)-1/2*e/b*(a*b*e^2)
^(1/2)/(a*e^2+b*d^2-2*(a*b*e^2)^(1/2)+b)*arctan(e*x+d)^2+e/(a*e^2+b*d^2-2*(
a*b*e^2)^(1/2)+b)*arctan(e*x+d)^2-1/2/e*(a*b*e^2)^(1/2)/a/(a*e^2+b*d^2-2*(a
*b*e^2)^(1/2)+b)*arctan(e*x+d)^2-1/2/e*(a*b*e^2)^(1/2)/a/(a*e^2+b*d^2-2*(a*
b*e^2)^(1/2)+b)*arctan(e*x+d)^2*d^2-1/4*e/b*(a*b*e^2)^(1/2)/(a*e^2+b*d^2-2*
(a*b*e^2)^(1/2)+b)*polylog(2,(2*I*b*d+a*e^2+b*d^2-b)*(1+I*(e*x+d))^2/((e*x+
d)^2+1)/(-a*e^2-b*d^2+2*(a*b*e^2)^(1/2)-b))+1/2*e/(a*e^2+b*d^2-2*(a*b*e^2)^
(1/2)+b)*polylog(2,(2*I*b*d+a*e^2+b*d^2-b)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-
a*e^2-b*d^2+2*(a*b*e^2)^(1/2)-b))-1/4/e*(a*b*e^2)^(1/2)/a/(a*e^2+b*d^2-2*(a
*b*e^2)^(1/2)+b)*polylog(2,(2*I*b*d+a*e^2+b*d^2-b)*(1+I*(e*x+d))^2/((e*x+d)
^2+1)/(-a*e^2-b*d^2+2*(a*b*e^2)^(1/2)-b))-1/4/e*(a*b*e^2)^(1/2)/a/(a*e^2+b*
d^2-2*(a*b*e^2)^(1/2)+b)*polylog(2,(2*I*b*d+a*e^2+b*d^2-b)*(1+I*(e*x+d))^2/
((e*x+d)^2+1)/(-a*e^2-b*d^2+2*(a*b*e^2)^(1/2)-b))*d^2+I*e/(a*e^2+b*d^2-2*(a
*b*e^2)^(1/2)+b)*ln(1-(2*I*b*d+a*e^2+b*d^2-b)*(1+I*(e*x+d))^2/((e*x+d)^2+1)
/(-a*e^2-b*d^2+2*(a*b*e^2)^(1/2)-b))*arctan(e*x+d)-1/2*I/e*(a*b*e^2)^(1/2)/
a/(a*e^2+b*d^2-2*(a*b*e^2)^(1/2)+b)*ln(1-(2*I*b*d+a*e^2+b*d^2-b)*(1+I*(e*x+
d))^2/((e*x+d)^2+1)/(-a*e^2-b*d^2+2*(a*b*e^2)^(1/2)-b))*arctan(e*x+d)*d^2-1
/2*I/e*(a*b*e^2)^(1/2)/a/(a*e^2+b*d^2-2*(a*b*e^2)^(1/2)+b)*ln(1-(2*I*b*d+a*
e^2+b*d^2-b)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2-b*d^2+2*(a*b*e^2)^(1/2)-
b))*arctan(e*x+d)+1/2*I*e/b*(a*b*e^2)^(1/2)/(a*e^2+b*d^2+2*(a*b*e^2)^(1/2)+
b)*ln(1-(2*I*b*d+a*e^2+b*d^2-b)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2-b*d^2
-2*(a*b*e^2)^(1/2)-b))*arctan(e*x+d)+1/2*e/b*(a*b*e^2)^(1/2)/(a*e^2+b*d^2+2
```

```
*(a*b*e^2)^(1/2)+b)*arctan(e*x+d)^2+e/(a*e^2+b*d^2+2*(a*b*e^2)^(1/2)+b)*arc
tan(e*x+d)^2+1/2/e*(a*b*e^2)^(1/2)/a/(a*e^2+b*d^2+2*(a*b*e^2)^(1/2)+b)*arct
an(e*x+d)^2+1/2/e*(a*b*e^2)^(1/2)/a/(a*e^2+b*d^2+2*(a*b*e^2)^(1/2)+b)*arcta
n(e*x+d)^2*d^2+1/4*e/b*(a*b*e^2)^(1/2)/(a*e^2+b*d^2+2*(a*b*e^2)^(1/2)+b)*po
lylog(2,(2*I*b*d+a*e^2+b*d^2-b)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2-b*d^2
-2*(a*b*e^2)^(1/2)-b))+1/2*e/(a*e^2+b*d^2+2*(a*b*e^2)^(1/2)+b)*polylog(2,(2
*I*b*d+a*e^2+b*d^2-b)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2-b*d^2-2*(a*b*e^
2)^(1/2)-b))+1/4/e*(a*b*e^2)^(1/2)/a/(a*e^2+b*d^2+2*(a*b*e^2)^(1/2)+b)*poly
log(2,(2*I*b*d+a*e^2+b*d^2-b)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2-b*d^2-2
*(a*b*e^2)^(1/2)-b))+1/4/e*(a*b*e^2)^(1/2)/a/(a*e^2+b*d^2+2*(a*b*e^2)^(1/2)
+b)*polylog(2,(2*I*b*d+a*e^2+b*d^2-b)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2
-b*d^2-2*(a*b*e^2)^(1/2)-b))*d^2
```

maxima [B] time = 2.84, size = 14300, normalized size = 26.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x+d)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/8*e*(8*arctan(b*x/sqrt(a*b))*arctan((e^2*x + d*e)/e)/e - (4*arctan(sqrt(b
)*x/sqrt(a))*arctan2((2*a*b*d*e^2 + (a*d*e^3 + (b*d^3 + b*d)*e + (a*e^4 + (
b*d^2 + 3*b)*e^2)*x)*sqrt(a)*sqrt(b) + (3*a*b*e^3 + (b^2*d^2 + b^2)*e)*x)/(
b^2*d^4 + a^2*e^4 + 2*b^2*d^2 + 2*(a*b*d^2 + 3*a*b)*e^2 + 4*(a*e^3 + (b*d^2
 + b)*e)*sqrt(a)*sqrt(b) + b^2), (b^2*d^4 + 2*b^2*d^2 + (a*b*d^2 + 3*a*b)*e
^2 + (2*b*d*e^2*x + a*e^3 + 3*(b*d^2 + b)*e)*sqrt(a)*sqrt(b) + b^2 + (a*b*d
*e^3 + (b^2*d^3 + b^2*d)*e)*x)/(b^2*d^4 + a^2*e^4 + 2*b^2*d^2 + 2*(a*b*d^2
+ 3*a*b)*e^2 + 4*(a*e^3 + (b*d^2 + b)*e)*sqrt(a)*sqrt(b) + b^2)) + 4*arctan
(sqrt(b)*x/sqrt(a))*arctan2((2*a*b*d*e^2 - (a*d*e^3 + (b*d^3 + b*d)*e + (a*
e^4 + (b*d^2 + 3*b)*e^2)*x)*sqrt(a)*sqrt(b) + (3*a*b*e^3 + (b^2*d^2 + b^2)*
e)*x)/(b^2*d^4 + a^2*e^4 + 2*b^2*d^2 + 2*(a*b*d^2 + 3*a*b)*e^2 - 4*(a*e^3 +
 (b*d^2 + b)*e)*sqrt(a)*sqrt(b) + b^2), (b^2*d^4 + 2*b^2*d^2 + (a*b*d^2 + 3
*a*b)*e^2 - (2*b*d*e^2*x + a*e^3 + 3*(b*d^2 + b)*e)*sqrt(a)*sqrt(b) + b^2 +
 (a*b*d*e^3 + (b^2*d^3 + b^2*d)*e)*x)/(b^2*d^4 + a^2*e^4 + 2*b^2*d^2 + 2*(a
*b*d^2 + 3*a*b)*e^2 - 4*(a*e^3 + (b*d^2 + b)*e)*sqrt(a)*sqrt(b) + b^2)) + l
og(b*x^2 + a)*log((b^12*d^24 + 12*b^12*d^22 + 66*b^12*d^20 + 220*b^12*d^18
+ 495*b^12*d^16 + 792*b^12*d^14 + 924*b^12*d^12 + (a^11*b*d^2 + a^11*b)*e^2
2 + 792*b^12*d^10 + 11*(a^10*b^2*d^4 + 22*a^10*b^2*d^2 + 21*a^10*b^2)*e^20
+ 495*b^12*d^8 + 55*(a^9*b^3*d^6 + 39*a^9*b^3*d^4 + 171*a^9*b^3*d^2 + 133*a
^9*b^3)*e^18 + 220*b^12*d^6 + 33*(5*a^8*b^4*d^8 + 260*a^8*b^4*d^6 + 1870*a^
8*b^4*d^4 + 3876*a^8*b^4*d^2 + 2261*a^8*b^4)*e^16 + 66*b^12*d^4 + 330*(a^7*
b^5*d^10 + 61*a^7*b^5*d^8 + 570*a^7*b^5*d^6 + 1802*a^7*b^5*d^4 + 2261*a^7*b
^5*d^2 + 969*a^7*b^5)*e^14 + 12*b^12*d^2 + 22*(21*a^6*b^6*d^12 + 1386*a^6*b
^6*d^10 + 15015*a^6*b^6*d^8 + 60060*a^6*b^6*d^6 + 109395*a^6*b^6*d^4 + 9237
8*a^6*b^6*d^2 + 29393*a^6*b^6)*e^12 + b^12 + 22*(21*a^5*b^7*d^14 + 1407*a^5
*b^7*d^12 + 16401*a^5*b^7*d^10 + 75075*a^5*b^7*d^8 + 169455*a^5*b^7*d^6 + 2
01773*a^5*b^7*d^4 + 121771*a^5*b^7*d^2 + 29393*a^5*b^7)*e^10 + 330*(a^4*b^8
*d^16 + 64*a^4*b^8*d^14 + 756*a^4*b^8*d^12 + 3696*a^4*b^8*d^10 + 9438*a^4*b
^8*d^8 + 13728*a^4*b^8*d^6 + 11492*a^4*b^8*d^4 + 5168*a^4*b^8*d^2 + 969*a^4
*b^8)*e^8 + 33*(5*a^3*b^9*d^18 + 285*a^3*b^9*d^16 + 3220*a^3*b^9*d^14 + 158
76*a^3*b^9*d^12 + 42966*a^3*b^9*d^10 + 70070*a^3*b^9*d^8 + 70980*a^3*b^9*d^
6 + 43860*a^3*b^9*d^4 + 15181*a^3*b^9*d^2 + 2261*a^3*b^9)*e^6 + 55*(a^2*b^1
0*d^20 + 46*a^2*b^10*d^18 + 465*a^2*b^10*d^16 + 2184*a^2*b^10*d^14 + 5922*a
^2*b^10*d^12 + 10164*a^2*b^10*d^10 + 11466*a^2*b^10*d^8 + 8520*a^2*b^10*d^6
 + 4029*a^2*b^10*d^4 + 1102*a^2*b^10*d^2 + 133*a^2*b^10)*e^4 + 11*(a*b^11*d
^22 + 31*a*b^11*d^20 + 255*a*b^11*d^18 + 1065*a*b^11*d^16 + 2730*a*b^11*d^1
4 + 4662*a*b^11*d^12 + 5502*a*b^11*d^10 + 4530*a*b^11*d^8 + 2565*a*b^11*d^6
 + 955*a*b^11*d^4 + 211*a*b^11*d^2 + 21*a*b^11)*e^2 + (a^11*b*e^24 + 11*(a^
10*b^2*d^2 + 21*a^10*b^2)*e^22 + 55*(a^9*b^3*d^4 + 38*a^9*b^3*d^2 + 133*a^9
*b^3)*e^20 + 33*(5*a^8*b^4*d^6 + 255*a^8*b^4*d^4 + 1615*a^8*b^4*d^2 + 2261*
```

```
a^8*b^4)*e^18 + 330*(a^7*b^5*d^8 + 60*a^7*b^5*d^6 + 510*a^7*b^5*d^4 + 1292*
a^7*b^5*d^2 + 969*a^7*b^5)*e^16 + 22*(21*a^6*b^6*d^10 + 1365*a^6*b^6*d^8 +
13650*a^6*b^6*d^6 + 46410*a^6*b^6*d^4 + 62985*a^6*b^6*d^2 + 29393*a^6*b^6)*
e^14 + 22*(21*a^5*b^7*d^12 + 1386*a^5*b^7*d^10 + 15015*a^5*b^7*d^8 + 60060*
a^5*b^7*d^6 + 109395*a^5*b^7*d^4 + 92378*a^5*b^7*d^2 + 29393*a^5*b^7)*e^12
+ 330*(a^4*b^8*d^14 + 63*a^4*b^8*d^12 + 693*a^4*b^8*d^10 + 3003*a^4*b^8*d^8
 + 6435*a^4*b^8*d^6 + 7293*a^4*b^8*d^4 + 4199*a^4*b^8*d^2 + 969*a^4*b^8)*e^
10 + 33*(5*a^3*b^9*d^16 + 280*a^3*b^9*d^14 + 2940*a^3*b^9*d^12 + 12936*a^3*
b^9*d^10 + 30030*a^3*b^9*d^8 + 40040*a^3*b^9*d^6 + 30940*a^3*b^9*d^4 + 1292
0*a^3*b^9*d^2 + 2261*a^3*b^9)*e^8 + 55*(a^2*b^10*d^18 + 45*a^2*b^10*d^16 +
420*a^2*b^10*d^14 + 1764*a^2*b^10*d^12 + 4158*a^2*b^10*d^10 + 6006*a^2*b^10
*d^8 + 5460*a^2*b^10*d^6 + 3060*a^2*b^10*d^4 + 969*a^2*b^10*d^2 + 133*a^2*b
^10)*e^6 + 11*(a*b^11*d^20 + 30*a*b^11*d^18 + 225*a*b^11*d^16 + 840*a*b^11*
d^14 + 1890*a*b^11*d^12 + 2772*a*b^11*d^10 + 2730*a*b^11*d^8 + 1800*a*b^11*
d^6 + 765*a*b^11*d^4 + 190*a*b^11*d^2 + 21*a*b^11)*e^4 + (b^12*d^22 + 11*b^
12*d^20 + 55*b^12*d^18 + 165*b^12*d^16 + 330*b^12*d^14 + 462*b^12*d^12 + 46
2*b^12*d^10 + 330*b^12*d^8 + 165*b^12*d^6 + 55*b^12*d^4 + 11*b^12*d^2 + b^1
2)*e^2)*x^2 + 2*(11*(a^10*b*d^2 + a^10*b)*e^21 + 110*(a^9*b^2*d^4 + 8*a^9*b
^2*d^2 + 7*a^9*b^2)*e^19 + 33*(15*a^8*b^3*d^6 + 205*a^8*b^3*d^4 + 589*a^8*b
^3*d^2 + 399*a^8*b^3)*e^17 + 264*(5*a^7*b^4*d^8 + 90*a^7*b^4*d^6 + 408*a^7*
b^4*d^4 + 646*a^7*b^4*d^2 + 323*a^7*b^4)*e^15 + 110*(21*a^6*b^5*d^10 + 441*
a^6*b^5*d^8 + 2562*a^6*b^5*d^6 + 6018*a^6*b^5*d^4 + 6137*a^6*b^5*d^2 + 2261
*a^6*b^5)*e^13 + 4*(693*a^5*b^6*d^12 + 15708*a^5*b^6*d^10 + 105105*a^5*b^6*
d^8 + 308880*a^5*b^6*d^6 + 449735*a^5*b^6*d^4 + 319124*a^5*b^6*d^2 + 88179*
a^5*b^6)*e^11 + 110*(21*a^4*b^7*d^14 + 483*a^4*b^7*d^12 + 3465*a^4*b^7*d^10
 + 11583*a^4*b^7*d^8 + 20735*a^4*b^7*d^6 + 20553*a^4*b^7*d^4 + 10659*a^4*b^
7*d^2 + 2261*a^4*b^7)*e^9 + 264*(5*a^3*b^8*d^16 + 110*a^3*b^8*d^14 + 798*a^
3*b^8*d^12 + 2838*a^3*b^8*d^10 + 5720*a^3*b^8*d^8 + 6890*a^3*b^8*d^6 + 4930
*a^3*b^8*d^4 + 1938*a^3*b^8*d^2 + 323*a^3*b^8)*e^7 + 33*(15*a^2*b^9*d^18 +
295*a^2*b^9*d^16 + 2044*a^2*b^9*d^14 + 7308*a^2*b^9*d^12 + 15554*a^2*b^9*d^
10 + 20930*a^2*b^9*d^8 + 18060*a^2*b^9*d^6 + 9724*a^2*b^9*d^4 + 2983*a^2*b^
9*d^2 + 399*a^2*b^9)*e^5 + 110*(a*b^10*d^20 + 16*a*b^10*d^18 + 99*a*b^10*d^
16 + 336*a*b^10*d^14 + 714*a*b^10*d^12 + 1008*a*b^10*d^10 + 966*a*b^10*d^8
+ 624*a*b^10*d^6 + 261*a*b^10*d^4 + 64*a*b^10*d^2 + 7*a*b^10)*e^3 + (11*a^1
0*b*e^23 + 110*(a^9*b^2*d^2 + 7*a^9*b^2)*e^21 + 33*(15*a^8*b^3*d^4 + 190*a^
8*b^3*d^2 + 399*a^8*b^3)*e^19 + 264*(5*a^7*b^4*d^6 + 85*a^7*b^4*d^4 + 323*a
^7*b^4*d^2 + 323*a^7*b^4)*e^17 + 110*(21*a^6*b^5*d^8 + 420*a^6*b^5*d^6 + 21
42*a^6*b^5*d^4 + 3876*a^6*b^5*d^2 + 2261*a^6*b^5)*e^15 + 4*(693*a^5*b^6*d^1
0 + 15015*a^5*b^6*d^8 + 90090*a^5*b^6*d^6 + 218790*a^5*b^6*d^4 + 230945*a^5
*b^6*d^2 + 88179*a^5*b^6)*e^13 + 110*(21*a^4*b^7*d^12 + 462*a^4*b^7*d^10 +
3003*a^4*b^7*d^8 + 8580*a^4*b^7*d^6 + 12155*a^4*b^7*d^4 + 8398*a^4*b^7*d^2
+ 2261*a^4*b^7)*e^11 + 264*(5*a^3*b^8*d^14 + 105*a^3*b^8*d^12 + 693*a^3*b^8
*d^10 + 2145*a^3*b^8*d^8 + 3575*a^3*b^8*d^6 + 3315*a^3*b^8*d^4 + 1615*a^3*b
^8*d^2 + 323*a^3*b^8)*e^9 + 33*(15*a^2*b^9*d^16 + 280*a^2*b^9*d^14 + 1764*a
^2*b^9*d^12 + 5544*a^2*b^9*d^10 + 10010*a^2*b^9*d^8 + 10920*a^2*b^9*d^6 + 7
140*a^2*b^9*d^4 + 2584*a^2*b^9*d^2 + 399*a^2*b^9)*e^7 + 110*(a*b^10*d^18 +
15*a*b^10*d^16 + 84*a*b^10*d^14 + 252*a*b^10*d^12 + 462*a*b^10*d^10 + 546*a
*b^10*d^8 + 420*a*b^10*d^6 + 204*a*b^10*d^4 + 57*a*b^10*d^2 + 7*a*b^10)*e^5
 + 11*(b^11*d^20 + 10*b^11*d^18 + 45*b^11*d^16 + 120*b^11*d^14 + 210*b^11*d
^12 + 252*b^11*d^10 + 210*b^11*d^8 + 120*b^11*d^6 + 45*b^11*d^4 + 10*b^11*d
^2 + b^11)*e^3)*x^2 + 11*(b^11*d^22 + 11*b^11*d^20 + 55*b^11*d^18 + 165*b^1
1*d^16 + 330*b^11*d^14 + 462*b^11*d^12 + 462*b^11*d^10 + 330*b^11*d^8 + 165
*b^11*d^6 + 55*b^11*d^4 + 11*b^11*d^2 + b^11)*e + 2*(11*a^10*b*d*e^22 + 110
*(a^9*b^2*d^3 + 7*a^9*b^2*d)*e^20 + 33*(15*a^8*b^3*d^5 + 190*a^8*b^3*d^3 +
399*a^8*b^3*d)*e^18 + 264*(5*a^7*b^4*d^7 + 85*a^7*b^4*d^5 + 323*a^7*b^4*d^3
 + 323*a^7*b^4*d)*e^16 + 110*(21*a^6*b^5*d^9 + 420*a^6*b^5*d^7 + 2142*a^6*b
^5*d^5 + 3876*a^6*b^5*d^3 + 2261*a^6*b^5*d)*e^14 + 4*(693*a^5*b^6*d^11 + 15
015*a^5*b^6*d^9 + 90090*a^5*b^6*d^7 + 218790*a^5*b^6*d^5 + 230945*a^5*b^6*d
^3 + 88179*a^5*b^6*d)*e^12 + 110*(21*a^4*b^7*d^13 + 462*a^4*b^7*d^11 + 3003
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*a^4*b^7*d^9 + 8580*a^4*b^7*d^7 + 12155*a^4*b^7*d^5 + 8398*a^4*b^7*d^3 + 22
61*a^4*b^7*d)*e^10 + 264*(5*a^3*b^8*d^15 + 105*a^3*b^8*d^13 + 693*a^3*b^8*d
^11 + 2145*a^3*b^8*d^9 + 3575*a^3*b^8*d^7 + 3315*a^3*b^8*d^5 + 1615*a^3*b^8
*d^3 + 323*a^3*b^8*d)*e^8 + 33*(15*a^2*b^9*d^17 + 280*a^2*b^9*d^15 + 1764*a
^2*b^9*d^13 + 5544*a^2*b^9*d^11 + 10010*a^2*b^9*d^9 + 10920*a^2*b^9*d^7 + 7
140*a^2*b^9*d^5 + 2584*a^2*b^9*d^3 + 399*a^2*b^9*d)*e^6 + 110*(a*b^10*d^19
+ 15*a*b^10*d^17 + 84*a*b^10*d^15 + 252*a*b^10*d^13 + 462*a*b^10*d^11 + 546
*a*b^10*d^9 + 420*a*b^10*d^7 + 204*a*b^10*d^5 + 57*a*b^10*d^3 + 7*a*b^10*d)
*e^4 + 11*(b^11*d^21 + 10*b^11*d^19 + 45*b^11*d^17 + 120*b^11*d^15 + 210*b^
11*d^13 + 252*b^11*d^11 + 210*b^11*d^9 + 120*b^11*d^7 + 45*b^11*d^5 + 10*b^
11*d^3 + b^11*d)*e^2)*x)*sqrt(a)*sqrt(b) + 2*(a^11*b*d*e^23 + 11*(a^10*b^2*
d^3 + 21*a^10*b^2*d)*e^21 + 55*(a^9*b^3*d^5 + 38*a^9*b^3*d^3 + 133*a^9*b^3*
d)*e^19 + 33*(5*a^8*b^4*d^7 + 255*a^8*b^4*d^5 + 1615*a^8*b^4*d^3 + 2261*a^8
*b^4*d)*e^17 + 330*(a^7*b^5*d^9 + 60*a^7*b^5*d^7 + 510*a^7*b^5*d^5 + 1292*a
^7*b^5*d^3 + 969*a^7*b^5*d)*e^15 + 22*(21*a^6*b^6*d^11 + 1365*a^6*b^6*d^9 +
 13650*a^6*b^6*d^7 + 46410*a^6*b^6*d^5 + 62985*a^6*b^6*d^3 + 29393*a^6*b^6*
d)*e^13 + 22*(21*a^5*b^7*d^13 + 1386*a^5*b^7*d^11 + 15015*a^5*b^7*d^9 + 600
60*a^5*b^7*d^7 + 109395*a^5*b^7*d^5 + 92378*a^5*b^7*d^3 + 29393*a^5*b^7*d)*
e^11 + 330*(a^4*b^8*d^15 + 63*a^4*b^8*d^13 + 693*a^4*b^8*d^11 + 3003*a^4*b^
8*d^9 + 6435*a^4*b^8*d^7 + 7293*a^4*b^8*d^5 + 4199*a^4*b^8*d^3 + 969*a^4*b^
8*d)*e^9 + 33*(5*a^3*b^9*d^17 + 280*a^3*b^9*d^15 + 2940*a^3*b^9*d^13 + 1293
6*a^3*b^9*d^11 + 30030*a^3*b^9*d^9 + 40040*a^3*b^9*d^7 + 30940*a^3*b^9*d^5
+ 12920*a^3*b^9*d^3 + 2261*a^3*b^9*d)*e^7 + 55*(a^2*b^10*d^19 + 45*a^2*b^10
*d^17 + 420*a^2*b^10*d^15 + 1764*a^2*b^10*d^13 + 4158*a^2*b^10*d^11 + 6006*
a^2*b^10*d^9 + 5460*a^2*b^10*d^7 + 3060*a^2*b^10*d^5 + 969*a^2*b^10*d^3 + 1
33*a^2*b^10*d)*e^5 + 11*(a*b^11*d^21 + 30*a*b^11*d^19 + 225*a*b^11*d^17 + 8
40*a*b^11*d^15 + 1890*a*b^11*d^13 + 2772*a*b^11*d^11 + 2730*a*b^11*d^9 + 18
00*a*b^11*d^7 + 765*a*b^11*d^5 + 190*a*b^11*d^3 + 21*a*b^11*d)*e^3 + (b^12*
d^23 + 11*b^12*d^21 + 55*b^12*d^19 + 165*b^12*d^17 + 330*b^12*d^15 + 462*b^
12*d^13 + 462*b^12*d^11 + 330*b^12*d^9 + 165*b^12*d^7 + 55*b^12*d^5 + 11*b^
12*d^3 + b^12*d)*e)*x)/(b^12*d^24 + a^12*e^24 + 12*b^12*d^22 + 66*b^12*d^20
 + 220*b^12*d^18 + 495*b^12*d^16 + 792*b^12*d^14 + 924*b^12*d^12 + 12*(a^11
*b*d^2 + 23*a^11*b)*e^22 + 792*b^12*d^10 + 66*(a^10*b^2*d^4 + 42*a^10*b^2*d
^2 + 161*a^10*b^2)*e^20 + 495*b^12*d^8 + 44*(5*a^9*b^3*d^6 + 285*a^9*b^3*d^
4 + 1995*a^9*b^3*d^2 + 3059*a^9*b^3)*e^18 + 220*b^12*d^6 + 99*(5*a^8*b^4*d^
8 + 340*a^8*b^4*d^6 + 3230*a^8*b^4*d^4 + 9044*a^8*b^4*d^2 + 7429*a^8*b^4)*e
^16 + 66*b^12*d^4 + 264*(3*a^7*b^5*d^10 + 225*a^7*b^5*d^8 + 2550*a^7*b^5*d^
6 + 9690*a^7*b^5*d^4 + 14535*a^7*b^5*d^2 + 7429*a^7*b^5)*e^14 + 12*b^12*d^2
 + 4*(231*a^6*b^6*d^12 + 18018*a^6*b^6*d^10 + 225225*a^6*b^6*d^8 + 1021020*
a^6*b^6*d^6 + 2078505*a^6*b^6*d^4 + 1939938*a^6*b^6*d^2 + 676039*a^6*b^6)*e
^12 + b^12 + 264*(3*a^5*b^7*d^14 + 231*a^5*b^7*d^12 + 3003*a^5*b^7*d^10 + 1
5015*a^5*b^7*d^8 + 36465*a^5*b^7*d^6 + 46189*a^5*b^7*d^4 + 29393*a^5*b^7*d^
2 + 7429*a^5*b^7)*e^10 + 99*(5*a^4*b^8*d^16 + 360*a^4*b^8*d^14 + 4620*a^4*b
^8*d^12 + 24024*a^4*b^8*d^10 + 64350*a^4*b^8*d^8 + 97240*a^4*b^8*d^6 + 8398
0*a^4*b^8*d^4 + 38760*a^4*b^8*d^2 + 7429*a^4*b^8)*e^8 + 44*(5*a^3*b^9*d^18
+ 315*a^3*b^9*d^16 + 3780*a^3*b^9*d^14 + 19404*a^3*b^9*d^12 + 54054*a^3*b^9
*d^10 + 90090*a^3*b^9*d^8 + 92820*a^3*b^9*d^6 + 58140*a^3*b^9*d^4 + 20349*a
^3*b^9*d^2 + 3059*a^3*b^9)*e^6 + 66*(a^2*b^10*d^20 + 50*a^2*b^10*d^18 + 525
*a^2*b^10*d^16 + 2520*a^2*b^10*d^14 + 6930*a^2*b^10*d^12 + 12012*a^2*b^10*d
^10 + 13650*a^2*b^10*d^8 + 10200*a^2*b^10*d^6 + 4845*a^2*b^10*d^4 + 1330*a^
2*b^10*d^2 + 161*a^2*b^10)*e^4 + 12*(a*b^11*d^22 + 33*a*b^11*d^20 + 275*a*b
^11*d^18 + 1155*a*b^11*d^16 + 2970*a*b^11*d^14 + 5082*a*b^11*d^12 + 6006*a*
b^11*d^10 + 4950*a*b^11*d^8 + 2805*a*b^11*d^6 + 1045*a*b^11*d^4 + 231*a*b^1
1*d^2 + 23*a*b^11)*e^2 + 8*(3*a^11*e^23 + 11*(3*a^10*b*d^2 + 23*a^10*b)*e^2
1 + 33*(5*a^9*b^2*d^4 + 70*a^9*b^2*d^2 + 161*a^9*b^2)*e^19 + 99*(5*a^8*b^3*
d^6 + 95*a^8*b^3*d^4 + 399*a^8*b^3*d^2 + 437*a^8*b^3)*e^17 + 22*(45*a^7*b^4
*d^8 + 1020*a^7*b^4*d^6 + 5814*a^7*b^4*d^4 + 11628*a^7*b^4*d^2 + 7429*a^7*b
^4)*e^15 + 6*(231*a^6*b^5*d^10 + 5775*a^6*b^5*d^8 + 39270*a^6*b^5*d^6 + 106
590*a^6*b^5*d^4 + 124355*a^6*b^5*d^2 + 52003*a^6*b^5)*e^13 + 6*(231*a^5*b^6
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*d^12 + 6006*a^5*b^6*d^10 + 45045*a^5*b^6*d^8 + 145860*a^5*b^6*d^6 + 230945
*a^5*b^6*d^4 + 176358*a^5*b^6*d^2 + 52003*a^5*b^6)*e^11 + 22*(45*a^4*b^7*d^
14 + 1155*a^4*b^7*d^12 + 9009*a^4*b^7*d^10 + 32175*a^4*b^7*d^8 + 60775*a^4*
b^7*d^6 + 62985*a^4*b^7*d^4 + 33915*a^4*b^7*d^2 + 7429*a^4*b^7)*e^9 + 99*(5
*a^3*b^8*d^16 + 120*a^3*b^8*d^14 + 924*a^3*b^8*d^12 + 3432*a^3*b^8*d^10 + 7
150*a^3*b^8*d^8 + 8840*a^3*b^8*d^6 + 6460*a^3*b^8*d^4 + 2584*a^3*b^8*d^2 +
437*a^3*b^8)*e^7 + 33*(5*a^2*b^9*d^18 + 105*a^2*b^9*d^16 + 756*a^2*b^9*d^14
 + 2772*a^2*b^9*d^12 + 6006*a^2*b^9*d^10 + 8190*a^2*b^9*d^8 + 7140*a^2*b^9*
d^6 + 3876*a^2*b^9*d^4 + 1197*a^2*b^9*d^2 + 161*a^2*b^9)*e^5 + 11*(3*a*b^10
*d^20 + 50*a*b^10*d^18 + 315*a*b^10*d^16 + 1080*a*b^10*d^14 + 2310*a*b^10*d
^12 + 3276*a*b^10*d^10 + 3150*a*b^10*d^8 + 2040*a*b^10*d^6 + 855*a*b^10*d^4
 + 210*a*b^10*d^2 + 23*a*b^10)*e^3 + 3*(b^11*d^22 + 11*b^11*d^20 + 55*b^11*
d^18 + 165*b^11*d^16 + 330*b^11*d^14 + 462*b^11*d^12 + 462*b^11*d^10 + 330*
b^11*d^8 + 165*b^11*d^6 + 55*b^11*d^4 + 11*b^11*d^2 + b^11)*e)*sqrt(a)*sqrt
(b))) - log(b*x^2 + a)*log((b^12*d^24 + 12*b^12*d^22 + 66*b^12*d^20 + 220*b
^12*d^18 + 495*b^12*d^16 + 792*b^12*d^14 + 924*b^12*d^12 + (a^11*b*d^2 + a^
11*b)*e^22 + 792*b^12*d^10 + 11*(a^10*b^2*d^4 + 22*a^10*b^2*d^2 + 21*a^10*b
^2)*e^20 + 495*b^12*d^8 + 55*(a^9*b^3*d^6 + 39*a^9*b^3*d^4 + 171*a^9*b^3*d^
2 + 133*a^9*b^3)*e^18 + 220*b^12*d^6 + 33*(5*a^8*b^4*d^8 + 260*a^8*b^4*d^6
+ 1870*a^8*b^4*d^4 + 3876*a^8*b^4*d^2 + 2261*a^8*b^4)*e^16 + 66*b^12*d^4 +
330*(a^7*b^5*d^10 + 61*a^7*b^5*d^8 + 570*a^7*b^5*d^6 + 1802*a^7*b^5*d^4 + 2
261*a^7*b^5*d^2 + 969*a^7*b^5)*e^14 + 12*b^12*d^2 + 22*(21*a^6*b^6*d^12 + 1
386*a^6*b^6*d^10 + 15015*a^6*b^6*d^8 + 60060*a^6*b^6*d^6 + 109395*a^6*b^6*d
^4 + 92378*a^6*b^6*d^2 + 29393*a^6*b^6)*e^12 + b^12 + 22*(21*a^5*b^7*d^14 +
 1407*a^5*b^7*d^12 + 16401*a^5*b^7*d^10 + 75075*a^5*b^7*d^8 + 169455*a^5*b^
7*d^6 + 201773*a^5*b^7*d^4 + 121771*a^5*b^7*d^2 + 29393*a^5*b^7)*e^10 + 330
*(a^4*b^8*d^16 + 64*a^4*b^8*d^14 + 756*a^4*b^8*d^12 + 3696*a^4*b^8*d^10 + 9
438*a^4*b^8*d^8 + 13728*a^4*b^8*d^6 + 11492*a^4*b^8*d^4 + 5168*a^4*b^8*d^2
+ 969*a^4*b^8)*e^8 + 33*(5*a^3*b^9*d^18 + 285*a^3*b^9*d^16 + 3220*a^3*b^9*d
^14 + 15876*a^3*b^9*d^12 + 42966*a^3*b^9*d^10 + 70070*a^3*b^9*d^8 + 70980*a
^3*b^9*d^6 + 43860*a^3*b^9*d^4 + 15181*a^3*b^9*d^2 + 2261*a^3*b^9)*e^6 + 55
*(a^2*b^10*d^20 + 46*a^2*b^10*d^18 + 465*a^2*b^10*d^16 + 2184*a^2*b^10*d^14
 + 5922*a^2*b^10*d^12 + 10164*a^2*b^10*d^10 + 11466*a^2*b^10*d^8 + 8520*a^2
*b^10*d^6 + 4029*a^2*b^10*d^4 + 1102*a^2*b^10*d^2 + 133*a^2*b^10)*e^4 + 11*
(a*b^11*d^22 + 31*a*b^11*d^20 + 255*a*b^11*d^18 + 1065*a*b^11*d^16 + 2730*a
*b^11*d^14 + 4662*a*b^11*d^12 + 5502*a*b^11*d^10 + 4530*a*b^11*d^8 + 2565*a
*b^11*d^6 + 955*a*b^11*d^4 + 211*a*b^11*d^2 + 21*a*b^11)*e^2 + (a^11*b*e^24
 + 11*(a^10*b^2*d^2 + 21*a^10*b^2)*e^22 + 55*(a^9*b^3*d^4 + 38*a^9*b^3*d^2
+ 133*a^9*b^3)*e^20 + 33*(5*a^8*b^4*d^6 + 255*a^8*b^4*d^4 + 1615*a^8*b^4*d^
2 + 2261*a^8*b^4)*e^18 + 330*(a^7*b^5*d^8 + 60*a^7*b^5*d^6 + 510*a^7*b^5*d^
4 + 1292*a^7*b^5*d^2 + 969*a^7*b^5)*e^16 + 22*(21*a^6*b^6*d^10 + 1365*a^6*b
^6*d^8 + 13650*a^6*b^6*d^6 + 46410*a^6*b^6*d^4 + 62985*a^6*b^6*d^2 + 29393*
a^6*b^6)*e^14 + 22*(21*a^5*b^7*d^12 + 1386*a^5*b^7*d^10 + 15015*a^5*b^7*d^8
 + 60060*a^5*b^7*d^6 + 109395*a^5*b^7*d^4 + 92378*a^5*b^7*d^2 + 29393*a^5*b
^7)*e^12 + 330*(a^4*b^8*d^14 + 63*a^4*b^8*d^12 + 693*a^4*b^8*d^10 + 3003*a^
4*b^8*d^8 + 6435*a^4*b^8*d^6 + 7293*a^4*b^8*d^4 + 4199*a^4*b^8*d^2 + 969*a^
4*b^8)*e^10 + 33*(5*a^3*b^9*d^16 + 280*a^3*b^9*d^14 + 2940*a^3*b^9*d^12 + 1
2936*a^3*b^9*d^10 + 30030*a^3*b^9*d^8 + 40040*a^3*b^9*d^6 + 30940*a^3*b^9*d
^4 + 12920*a^3*b^9*d^2 + 2261*a^3*b^9)*e^8 + 55*(a^2*b^10*d^18 + 45*a^2*b^1
0*d^16 + 420*a^2*b^10*d^14 + 1764*a^2*b^10*d^12 + 4158*a^2*b^10*d^10 + 6006
*a^2*b^10*d^8 + 5460*a^2*b^10*d^6 + 3060*a^2*b^10*d^4 + 969*a^2*b^10*d^2 +
133*a^2*b^10)*e^6 + 11*(a*b^11*d^20 + 30*a*b^11*d^18 + 225*a*b^11*d^16 + 84
0*a*b^11*d^14 + 1890*a*b^11*d^12 + 2772*a*b^11*d^10 + 2730*a*b^11*d^8 + 180
0*a*b^11*d^6 + 765*a*b^11*d^4 + 190*a*b^11*d^2 + 21*a*b^11)*e^4 + (b^12*d^2
2 + 11*b^12*d^20 + 55*b^12*d^18 + 165*b^12*d^16 + 330*b^12*d^14 + 462*b^12*
d^12 + 462*b^12*d^10 + 330*b^12*d^8 + 165*b^12*d^6 + 55*b^12*d^4 + 11*b^12*
d^2 + b^12)*e^2)*x^2 - 2*(11*(a^10*b*d^2 + a^10*b)*e^21 + 110*(a^9*b^2*d^4
+ 8*a^9*b^2*d^2 + 7*a^9*b^2)*e^19 + 33*(15*a^8*b^3*d^6 + 205*a^8*b^3*d^4 +
589*a^8*b^3*d^2 + 399*a^8*b^3)*e^17 + 264*(5*a^7*b^4*d^8 + 90*a^7*b^4*d^6 +
```

408*a^7*b^4*d^4 + 646*a^7*b^4*d^2 + 323*a^7*b^4)*e^15 + 110*(21*a^6*b^5*d^
10 + 441*a^6*b^5*d^8 + 2562*a^6*b^5*d^6 + 6018*a^6*b^5*d^4 + 6137*a^6*b^5*d
^2 + 2261*a^6*b^5)*e^13 + 4*(693*a^5*b^6*d^12 + 15708*a^5*b^6*d^10 + 105105
*a^5*b^6*d^8 + 308880*a^5*b^6*d^6 + 449735*a^5*b^6*d^4 + 319124*a^5*b^6*d^2
+ 88179*a^5*b^6)*e^11 + 110*(21*a^4*b^7*d^14 + 483*a^4*b^7*d^12 + 3465*a^4
*b^7*d^10 + 11583*a^4*b^7*d^8 + 20735*a^4*b^7*d^6 + 20553*a^4*b^7*d^4 + 106
59*a^4*b^7*d^2 + 2261*a^4*b^7)*e^9 + 264*(5*a^3*b^8*d^16 + 110*a^3*b^8*d^14
+ 798*a^3*b^8*d^12 + 2838*a^3*b^8*d^10 + 5720*a^3*b^8*d^8 + 6890*a^3*b^8*d
^6 + 4930*a^3*b^8*d^4 + 1938*a^3*b^8*d^2 + 323*a^3*b^8)*e^7 + 33*(15*a^2*b^
9*d^18 + 295*a^2*b^9*d^16 + 2044*a^2*b^9*d^14 + 7308*a^2*b^9*d^12 + 15554*a
^2*b^9*d^10 + 20930*a^2*b^9*d^8 + 18060*a^2*b^9*d^6 + 9724*a^2*b^9*d^4 + 29
83*a^2*b^9*d^2 + 399*a^2*b^9)*e^5 + 110*(a*b^10*d^20 + 16*a*b^10*d^18 + 99*
a*b^10*d^16 + 336*a*b^10*d^14 + 714*a*b^10*d^12 + 1008*a*b^10*d^10 + 966*a*
b^10*d^8 + 624*a*b^10*d^6 + 261*a*b^10*d^4 + 64*a*b^10*d^2 + 7*a*b^10)*e^3
+ (11*a^10*b*e^23 + 110*(a^9*b^2*d^2 + 7*a^9*b^2)*e^21 + 33*(15*a^8*b^3*d^4
+ 190*a^8*b^3*d^2 + 399*a^8*b^3)*e^19 + 264*(5*a^7*b^4*d^6 + 85*a^7*b^4*d^
4 + 323*a^7*b^4*d^2 + 323*a^7*b^4)*e^17 + 110*(21*a^6*b^5*d^8 + 420*a^6*b^5
*d^6 + 2142*a^6*b^5*d^4 + 3876*a^6*b^5*d^2 + 2261*a^6*b^5)*e^15 + 4*(693*a^
5*b^6*d^10 + 15015*a^5*b^6*d^8 + 90090*a^5*b^6*d^6 + 218790*a^5*b^6*d^4 + 2
30945*a^5*b^6*d^2 + 88179*a^5*b^6)*e^13 + 110*(21*a^4*b^7*d^12 + 462*a^4*b^
7*d^10 + 3003*a^4*b^7*d^8 + 8580*a^4*b^7*d^6 + 12155*a^4*b^7*d^4 + 8398*a^4
*b^7*d^2 + 2261*a^4*b^7)*e^11 + 264*(5*a^3*b^8*d^14 + 105*a^3*b^8*d^12 + 69
3*a^3*b^8*d^10 + 2145*a^3*b^8*d^8 + 3575*a^3*b^8*d^6 + 3315*a^3*b^8*d^4 + 1
615*a^3*b^8*d^2 + 323*a^3*b^8)*e^9 + 33*(15*a^2*b^9*d^16 + 280*a^2*b^9*d^14
+ 1764*a^2*b^9*d^12 + 5544*a^2*b^9*d^10 + 10010*a^2*b^9*d^8 + 10920*a^2*b^
9*d^6 + 7140*a^2*b^9*d^4 + 2584*a^2*b^9*d^2 + 399*a^2*b^9)*e^7 + 110*(a*b^1
0*d^18 + 15*a*b^10*d^16 + 84*a*b^10*d^14 + 252*a*b^10*d^12 + 462*a*b^10*d^1
0 + 546*a*b^10*d^8 + 420*a*b^10*d^6 + 204*a*b^10*d^4 + 57*a*b^10*d^2 + 7*a*
b^10)*e^5 + 11*(b^11*d^20 + 10*b^11*d^18 + 45*b^11*d^16 + 120*b^11*d^14 + 2
10*b^11*d^12 + 252*b^11*d^10 + 210*b^11*d^8 + 120*b^11*d^6 + 45*b^11*d^4 +
10*b^11*d^2 + b^11)*e^3)*x^2 + 11*(b^11*d^22 + 11*b^11*d^20 + 55*b^11*d^18
+ 165*b^11*d^16 + 330*b^11*d^14 + 462*b^11*d^12 + 462*b^11*d^10 + 330*b^11*
d^8 + 165*b^11*d^6 + 55*b^11*d^4 + 11*b^11*d^2 + b^11)*e + 2*(11*a^10*b*d*e
^22 + 110*(a^9*b^2*d^3 + 7*a^9*b^2*d)*e^20 + 33*(15*a^8*b^3*d^5 + 190*a^8*b
^3*d^3 + 399*a^8*b^3*d)*e^18 + 264*(5*a^7*b^4*d^7 + 85*a^7*b^4*d^5 + 323*a^
7*b^4*d^3 + 323*a^7*b^4*d)*e^16 + 110*(21*a^6*b^5*d^9 + 420*a^6*b^5*d^7 + 2
142*a^6*b^5*d^5 + 3876*a^6*b^5*d^3 + 2261*a^6*b^5*d)*e^14 + 4*(693*a^5*b^6*
d^11 + 15015*a^5*b^6*d^9 + 90090*a^5*b^6*d^7 + 218790*a^5*b^6*d^5 + 230945*
a^5*b^6*d^3 + 88179*a^5*b^6*d)*e^12 + 110*(21*a^4*b^7*d^13 + 462*a^4*b^7*d^
11 + 3003*a^4*b^7*d^9 + 8580*a^4*b^7*d^7 + 12155*a^4*b^7*d^5 + 8398*a^4*b^7
*d^3 + 2261*a^4*b^7*d)*e^10 + 264*(5*a^3*b^8*d^15 + 105*a^3*b^8*d^13 + 693*
a^3*b^8*d^11 + 2145*a^3*b^8*d^9 + 3575*a^3*b^8*d^7 + 3315*a^3*b^8*d^5 + 161
5*a^3*b^8*d^3 + 323*a^3*b^8*d)*e^8 + 33*(15*a^2*b^9*d^17 + 280*a^2*b^9*d^15
+ 1764*a^2*b^9*d^13 + 5544*a^2*b^9*d^11 + 10010*a^2*b^9*d^9 + 10920*a^2*b^
9*d^7 + 7140*a^2*b^9*d^5 + 2584*a^2*b^9*d^3 + 399*a^2*b^9*d)*e^6 + 110*(a*b
^10*d^19 + 15*a*b^10*d^17 + 84*a*b^10*d^15 + 252*a*b^10*d^13 + 462*a*b^10*d
^11 + 546*a*b^10*d^9 + 420*a*b^10*d^7 + 204*a*b^10*d^5 + 57*a*b^10*d^3 + 7*
a*b^10*d)*e^4 + 11*(b^11*d^21 + 10*b^11*d^19 + 45*b^11*d^17 + 120*b^11*d^15
+ 210*b^11*d^13 + 252*b^11*d^11 + 210*b^11*d^9 + 120*b^11*d^7 + 45*b^11*d^
5 + 10*b^11*d^3 + b^11*d)*e^2)*x)*sqrt(a)*sqrt(b) + 2*(a^11*b*d*e^23 + 11*(
a^10*b^2*d^3 + 21*a^10*b^2*d)*e^21 + 55*(a^9*b^3*d^5 + 38*a^9*b^3*d^3 + 133
*a^9*b^3*d)*e^19 + 33*(5*a^8*b^4*d^7 + 255*a^8*b^4*d^5 + 1615*a^8*b^4*d^3 +
2261*a^8*b^4*d)*e^17 + 330*(a^7*b^5*d^9 + 60*a^7*b^5*d^7 + 510*a^7*b^5*d^5
+ 1292*a^7*b^5*d^3 + 969*a^7*b^5*d)*e^15 + 22*(21*a^6*b^6*d^11 + 1365*a^6*
b^6*d^9 + 13650*a^6*b^6*d^7 + 46410*a^6*b^6*d^5 + 62985*a^6*b^6*d^3 + 29393
*a^6*b^6*d)*e^13 + 22*(21*a^5*b^7*d^13 + 1386*a^5*b^7*d^11 + 15015*a^5*b^7*
d^9 + 60060*a^5*b^7*d^7 + 109395*a^5*b^7*d^5 + 92378*a^5*b^7*d^3 + 29393*a^
5*b^7*d)*e^11 + 330*(a^4*b^8*d^15 + 63*a^4*b^8*d^13 + 693*a^4*b^8*d^11 + 30
03*a^4*b^8*d^9 + 6435*a^4*b^8*d^7 + 7293*a^4*b^8*d^5 + 4199*a^4*b^8*d^3 + 9

69*a^4*b^8*d)*e^9 + 33*(5*a^3*b^9*d^17 + 280*a^3*b^9*d^15 + 2940*a^3*b^9*d^
13 + 12936*a^3*b^9*d^11 + 30030*a^3*b^9*d^9 + 40040*a^3*b^9*d^7 + 30940*a^3
*b^9*d^5 + 12920*a^3*b^9*d^3 + 2261*a^3*b^9*d)*e^7 + 55*(a^2*b^10*d^19 + 45
*a^2*b^10*d^17 + 420*a^2*b^10*d^15 + 1764*a^2*b^10*d^13 + 4158*a^2*b^10*d^1
1 + 6006*a^2*b^10*d^9 + 5460*a^2*b^10*d^7 + 3060*a^2*b^10*d^5 + 969*a^2*b^1
0*d^3 + 133*a^2*b^10*d)*e^5 + 11*(a*b^11*d^21 + 30*a*b^11*d^19 + 225*a*b^11
*d^17 + 840*a*b^11*d^15 + 1890*a*b^11*d^13 + 2772*a*b^11*d^11 + 2730*a*b^11
*d^9 + 1800*a*b^11*d^7 + 765*a*b^11*d^5 + 190*a*b^11*d^3 + 21*a*b^11*d)*e^3
 + (b^12*d^23 + 11*b^12*d^21 + 55*b^12*d^19 + 165*b^12*d^17 + 330*b^12*d^15
 + 462*b^12*d^13 + 462*b^12*d^11 + 330*b^12*d^9 + 165*b^12*d^7 + 55*b^12*d^
5 + 11*b^12*d^3 + b^12*d)*e)*x)/(b^12*d^24 + a^12*e^24 + 12*b^12*d^22 + 66*
b^12*d^20 + 220*b^12*d^18 + 495*b^12*d^16 + 792*b^12*d^14 + 924*b^12*d^12 +
 12*(a^11*b*d^2 + 23*a^11*b)*e^22 + 792*b^12*d^10 + 66*(a^10*b^2*d^4 + 42*a
^10*b^2*d^2 + 161*a^10*b^2)*e^20 + 495*b^12*d^8 + 44*(5*a^9*b^3*d^6 + 285*a
^9*b^3*d^4 + 1995*a^9*b^3*d^2 + 3059*a^9*b^3)*e^18 + 220*b^12*d^6 + 99*(5*a
^8*b^4*d^8 + 340*a^8*b^4*d^6 + 3230*a^8*b^4*d^4 + 9044*a^8*b^4*d^2 + 7429*a
^8*b^4)*e^16 + 66*b^12*d^4 + 264*(3*a^7*b^5*d^10 + 225*a^7*b^5*d^8 + 2550*a
^7*b^5*d^6 + 9690*a^7*b^5*d^4 + 14535*a^7*b^5*d^2 + 7429*a^7*b^5)*e^14 + 12
*b^12*d^2 + 4*(231*a^6*b^6*d^12 + 18018*a^6*b^6*d^10 + 225225*a^6*b^6*d^8 +
 1021020*a^6*b^6*d^6 + 2078505*a^6*b^6*d^4 + 1939938*a^6*b^6*d^2 + 676039*a
^6*b^6)*e^12 + b^12 + 264*(3*a^5*b^7*d^14 + 231*a^5*b^7*d^12 + 3003*a^5*b^7
*d^10 + 15015*a^5*b^7*d^8 + 36465*a^5*b^7*d^6 + 46189*a^5*b^7*d^4 + 29393*a
^5*b^7*d^2 + 7429*a^5*b^7)*e^10 + 99*(5*a^4*b^8*d^16 + 360*a^4*b^8*d^14 + 4
620*a^4*b^8*d^12 + 24024*a^4*b^8*d^10 + 64350*a^4*b^8*d^8 + 97240*a^4*b^8*d
^6 + 83980*a^4*b^8*d^4 + 38760*a^4*b^8*d^2 + 7429*a^4*b^8)*e^8 + 44*(5*a^3*
b^9*d^18 + 315*a^3*b^9*d^16 + 3780*a^3*b^9*d^14 + 19404*a^3*b^9*d^12 + 5405
4*a^3*b^9*d^10 + 90090*a^3*b^9*d^8 + 92820*a^3*b^9*d^6 + 58140*a^3*b^9*d^4
+ 20349*a^3*b^9*d^2 + 3059*a^3*b^9)*e^6 + 66*(a^2*b^10*d^20 + 50*a^2*b^10*d
^18 + 525*a^2*b^10*d^16 + 2520*a^2*b^10*d^14 + 6930*a^2*b^10*d^12 + 12012*a
^2*b^10*d^10 + 13650*a^2*b^10*d^8 + 10200*a^2*b^10*d^6 + 4845*a^2*b^10*d^4
+ 1330*a^2*b^10*d^2 + 161*a^2*b^10)*e^4 + 12*(a*b^11*d^22 + 33*a*b^11*d^20
+ 275*a*b^11*d^18 + 1155*a*b^11*d^16 + 2970*a*b^11*d^14 + 5082*a*b^11*d^12
+ 6006*a*b^11*d^10 + 4950*a*b^11*d^8 + 2805*a*b^11*d^6 + 1045*a*b^11*d^4 +
231*a*b^11*d^2 + 23*a*b^11)*e^2 - 8*(3*a^11*e^23 + 11*(3*a^10*b*d^2 + 23*a^
10*b)*e^21 + 33*(5*a^9*b^2*d^4 + 70*a^9*b^2*d^2 + 161*a^9*b^2)*e^19 + 99*(5
*a^8*b^3*d^6 + 95*a^8*b^3*d^4 + 399*a^8*b^3*d^2 + 437*a^8*b^3)*e^17 + 22*(4
5*a^7*b^4*d^8 + 1020*a^7*b^4*d^6 + 5814*a^7*b^4*d^4 + 11628*a^7*b^4*d^2 + 7
429*a^7*b^4)*e^15 + 6*(231*a^6*b^5*d^10 + 5775*a^6*b^5*d^8 + 39270*a^6*b^5*
d^6 + 106590*a^6*b^5*d^4 + 124355*a^6*b^5*d^2 + 52003*a^6*b^5)*e^13 + 6*(23
1*a^5*b^6*d^12 + 6006*a^5*b^6*d^10 + 45045*a^5*b^6*d^8 + 145860*a^5*b^6*d^6
 + 230945*a^5*b^6*d^4 + 176358*a^5*b^6*d^2 + 52003*a^5*b^6)*e^11 + 22*(45*a
^4*b^7*d^14 + 1155*a^4*b^7*d^12 + 9009*a^4*b^7*d^10 + 32175*a^4*b^7*d^8 + 6
0775*a^4*b^7*d^6 + 62985*a^4*b^7*d^4 + 33915*a^4*b^7*d^2 + 7429*a^4*b^7)*e^
9 + 99*(5*a^3*b^8*d^16 + 120*a^3*b^8*d^14 + 924*a^3*b^8*d^12 + 3432*a^3*b^8
*d^10 + 7150*a^3*b^8*d^8 + 8840*a^3*b^8*d^6 + 6460*a^3*b^8*d^4 + 2584*a^3*b
^8*d^2 + 437*a^3*b^8)*e^7 + 33*(5*a^2*b^9*d^18 + 105*a^2*b^9*d^16 + 756*a^2
*b^9*d^14 + 2772*a^2*b^9*d^12 + 6006*a^2*b^9*d^10 + 8190*a^2*b^9*d^8 + 7140
*a^2*b^9*d^6 + 3876*a^2*b^9*d^4 + 1197*a^2*b^9*d^2 + 161*a^2*b^9)*e^5 + 11*
(3*a*b^10*d^20 + 50*a*b^10*d^18 + 315*a*b^10*d^16 + 1080*a*b^10*d^14 + 2310
*a*b^10*d^12 + 3276*a*b^10*d^10 + 3150*a*b^10*d^8 + 2040*a*b^10*d^6 + 855*a
*b^10*d^4 + 210*a*b^10*d^2 + 23*a*b^10)*e^3 + 3*(b^11*d^22 + 11*b^11*d^20 +
 55*b^11*d^18 + 165*b^11*d^16 + 330*b^11*d^14 + 462*b^11*d^12 + 462*b^11*d^
10 + 330*b^11*d^8 + 165*b^11*d^6 + 55*b^11*d^4 + 11*b^11*d^2 + b^11)*e)*sqr
t(a)*sqrt(b))) + 2*dilog(-(a*e^2 + (b*d + I*b)*e*x + (I*e^2*x + (-I*d + 1)*
e)*sqrt(a)*sqrt(b))/(b*d^2 - 2*sqrt(a)*sqrt(b)*(-I*d + 1)*e - a*e^2 + 2*I*b
*d - b)) - 2*dilog(-(a*e^2 + (b*d + I*b)*e*x - (I*e^2*x + (-I*d + 1)*e)*sqr
t(a)*sqrt(b))/(b*d^2 + 2*sqrt(a)*sqrt(b)*(-I*d + 1)*e - a*e^2 + 2*I*b*d - b
)) - 2*dilog(-(a*e^2 + (b*d - I*b)*e*x + (I*e^2*x + (-I*d - 1)*e)*sqrt(a)*s
qrt(b))/(b*d^2 - 2*sqrt(a)*sqrt(b)*(-I*d - 1)*e - a*e^2 - 2*I*b*d - b)) + 2

```
*dilog(-(a*e^2 + (b*d - I*b)*e*x - (I*e^2*x + (-I*d - 1)*e)*sqrt(a)*sqrt(b)
)/(b*d^2 + 2*sqrt(a)*sqrt(b)*(-I*d - 1)*e - a*e^2 - 2*I*b*d - b)))/e)/sqrt(
a*b) + arctan(e*x + d)*arctan(b*x/sqrt(a*b))/sqrt(a*b) - arctan(b*x/sqrt(a*
b))*arctan((e^2*x + d*e)/e)/sqrt(a*b)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}\left(d+e\,x\right)}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(d + e*x)/(a + b*x^2),x)
```

```
[Out] int(atan(d + e*x)/(a + b*x^2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(e*x+d)/(b*x**2+a),x)
```

```
[Out] Timed out
```

3.62 $\int \frac{\tan^{-1}(d+ex)}{a+bx+cx^2}\,dx$

Optimal. Leaf size=367

$$-\frac{i\text{Li}_2\left(\frac{2\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e-2c(d+ex)\right)}{\left(-2dc+2ic+be-\sqrt{b^2-4ac}\,e\right)(1-i(d+ex))}+1\right)}{2\sqrt{b^2-4ac}}+\frac{i\text{Li}_2\left(\frac{2\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e-2c(d+ex)\right)}{\left(2c(i-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(1-i(d+ex))}+1\right)}{2\sqrt{b^2-4ac}}+\frac{\tan^{-1}(d+ex)\log\left(\frac{2e(\cdots}{(1-i(d+ex))\cdots}\right)}{\sqrt{b^2-4\cdots}}$$

```
[Out] arctan(e*x+d)*ln(2*e*(b+2*c*x-(-4*a*c+b^2)^(1/2))/(1-I*(e*x+d))/(2*c*(I-d)+
e*(b-(-4*a*c+b^2)^(1/2))))/(-4*a*c+b^2)^(1/2)-arctan(e*x+d)*ln(2*e*(b+2*c*x
+(-4*a*c+b^2)^(1/2))/(1-I*(e*x+d))/(2*c*(I-d)+e*(b+(-4*a*c+b^2)^(1/2))))/(-
4*a*c+b^2)^(1/2)-1/2*I*polylog(2,1+2*(2*c*d-2*c*(e*x+d)-e*(b-(-4*a*c+b^2)^(
1/2)))/(1-I*(e*x+d))/(2*I*c-2*c*d+b*e-e*(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(
1/2)+1/2*I*polylog(2,1+2*(2*c*d-2*c*(e*x+d)-e*(b+(-4*a*c+b^2)^(1/2)))/(1-I*
(e*x+d))/(2*c*(I-d)+e*(b+(-4*a*c+b^2)^(1/2))))/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.67, antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {618, 206, 6728, 5047, 4856, 2402, 2315, 2447}

$$-\frac{i\text{PolyLog}\left(2,1+\frac{2\left(-e\left(b-\sqrt{b^2-4ac}\right)-2c(d+ex)+2cd\right)}{(1-i(d+ex))\left(-e\sqrt{b^2-4ac}+be-2cd+2ic\right)}\right)}{2\sqrt{b^2-4ac}}+\frac{i\text{PolyLog}\left(2,1+\frac{2\left(-e\left(\sqrt{b^2-4ac}+b\right)-2c(d+ex)+2cd\right)}{(1-i(d+ex))\left(e\left(\sqrt{b^2-4ac}+b\right)+2c(-d+i)\right)}\right)}{2\sqrt{b^2-4ac}}+\frac{\tan^{-1}(d+\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[d + e*x]/(a + b*x + c*x^2),x]
```

```
[Out] (ArcTan[d + e*x]*Log[(2*e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*(I - d) +
(b - Sqrt[b^2 - 4*a*c])*e)*(1 - I*(d + e*x)))])/Sqrt[b^2 - 4*a*c] - (ArcTan
[d + e*x]*Log[(2*e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*(I - d) + (b + Sq
rt[b^2 - 4*a*c])*e)*(1 - I*(d + e*x)))])/Sqrt[b^2 - 4*a*c] - ((I/2)*PolyLog
[2, 1 + (2*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e - 2*c*(d + e*x)))/(((2*I)*c -
 2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e)*(1 - I*(d + e*x)))])/Sqrt[b^2 - 4*a*c]
+ ((I/2)*PolyLog[2, 1 + (2*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e - 2*c*(d + e*
x)))/((2*c*(I - d) + (b + Sqrt[b^2 - 4*a*c])*e)*(1 - I*(d + e*x)))])/Sqrt[b
^2 - 4*a*c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}(d+ex)}{a+bx+cx^2}\,dx = \int \left(\frac{2c\tan^{-1}(d+ex)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx\right)} - \frac{2c\tan^{-1}(d+ex)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}+2cx\right)} \right) dx$$

$$= \frac{(2c)\int \frac{\tan^{-1}(d+ex)}{b-\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{\tan^{-1}(d+ex)}{b+\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{(2c)\,\mathrm{Subst}\left(\int \frac{\tan^{-1}(x)}{\frac{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}{e}+\frac{2cx}{e}}\,dx, x, d+ex\right)}{\sqrt{b^2-4ac}\,e} - \frac{(2c)\,\mathrm{Subst}\left(\int \frac{\tan^{-1}(x)}{\frac{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}{e}+\frac{2cx}{e}}\,dx, x, d\right.}{\sqrt{b^2-4ac}\,e}$$

$$= \frac{\tan^{-1}(d+ex)\log\left(\frac{2e\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(2ic-2cd+be-\sqrt{b^2-4ac}\,e\right)(1-i(d+ex))}\right)}{\sqrt{b^2-4ac}} - \frac{\tan^{-1}(d+ex)\log\left(\frac{2e\left(b+\sqrt{b^2-4ac}+2\right.}{\left(2c(i-d)+\left(b+\sqrt{b^2-4ac}\right)e\right.}\right.}{\sqrt{b^2-4ac}}$$

$$= \frac{\tan^{-1}(d+ex)\log\left(\frac{2e\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(2ic-2cd+be-\sqrt{b^2-4ac}\,e\right)(1-i(d+ex))}\right)}{\sqrt{b^2-4ac}} - \frac{\tan^{-1}(d+ex)\log\left(\frac{2e\left(b+\sqrt{b^2-4ac}+2\right.}{\left(2c(i-d)+\left(b+\sqrt{b^2-4ac}\right)e\right.}\right.}{\sqrt{b^2-4ac}}$$

Mathematica [A] time = 0.46, size = 443, normalized size = 1.21

$$i\left(-\mathrm{Li}_2\left(\frac{2c(d+ex-i)}{2c(d-i)+\left(\sqrt{b^2-4ac}-b\right)e}\right)+\mathrm{Li}_2\left(\frac{2c(d+ex-i)}{2c(d-i)-\left(b+\sqrt{b^2-4ac}\right)e}\right)+\mathrm{Li}_2\left(\frac{2c(d+ex+i)}{2c(d+i)+\left(\sqrt{b^2-4ac}-b\right)e}\right)-\mathrm{Li}_2\left(\frac{2c(d+ex+i)}{2c(d+i)-\left(b+\sqrt{b^2-4ac}\right)e}\right)+\mathrm{l}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[d + e*x]/(a + b*x + c*x^2),x]
```

```
[Out] ((I/2)*(Log[(e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*c*(I + d) + (-b + Sqrt[
b^2 - 4*a*c])*e)]*Log[1 - I*(d + e*x)] - Log[(e*(b + Sqrt[b^2 - 4*a*c] + 2*
c*x))/(-2*c*(I + d) + (b + Sqrt[b^2 - 4*a*c])*e)]*Log[1 - I*(d + e*x)] - Lo
g[(e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*c*(-I + d) + (-b + Sqrt[b^2 - 4*a
*c])*e)]*Log[1 + I*(d + e*x)] + Log[(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2
*c*(-I + d) + (b + Sqrt[b^2 - 4*a*c])*e)]*Log[1 + I*(d + e*x)] - PolyLog[2,
 (2*c*(-I + d + e*x))/(2*c*(-I + d) + (-b + Sqrt[b^2 - 4*a*c])*e)] + PolyLo
g[2, (2*c*(-I + d + e*x))/(2*c*(-I + d) - (b + Sqrt[b^2 - 4*a*c])*e)] + Pol
yLog[2, (2*c*(I + d + e*x))/(2*c*(I + d) + (-b + Sqrt[b^2 - 4*a*c])*e)] - P
olyLog[2, (2*c*(I + d + e*x))/(2*c*(I + d) - (b + Sqrt[b^2 - 4*a*c])*e)]))/
Sqrt[b^2 - 4*a*c]
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\arctan\left(ex+d\right)}{cx^2+bx+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x+d)/(c*x^2+b*x+a),x, algorithm="fricas")
```

[Out] integral(arctan(e*x + d)/(c*x^2 + b*x + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(e*x+d)/(c*x^2+b*x+a),x, algorithm="giac")

[Out] sage0*x

maple [B] time = 1.62, size = 4743, normalized size = 12.92

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctan(e*x+d)/(c*x^2+b*x+a),x)

[Out] 1/8*e*(e^2*(4*a*c-b^2))^(1/2)/c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2,(-I*b*e+2*I*d*c+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2-(e^2*(4*a*c-b^2))^(1/2)-c))*b^2+1/2/e*(e^2*(4*a*c-b^2))^(1/2)*c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2,(-I*b*e+2*I*d*c+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2-(e^2*(4*a*c-b^2))^(1/2)-c))*d^2-1/4*I*e*(e^2*(4*a*c-b^2))^(1/2)/c/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*ln(1-(-I*b*e+2*I*d*c+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2+(e^2*(4*a*c-b^2))^(1/2)-c))*arctan(e*x+d)+1/4*I*e*(e^2*(4*a*c-b^2))^(1/2)/c/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*ln(1-(-I*b*e+2*I*d*c+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2-(e^2*(4*a*c-b^2))^(1/2)-c))*arctan(e*x+d)-1/2/e*(e^2*(4*a*c-b^2))^(1/2)*c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2,(-I*b*e+2*I*d*c+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2+(e^2*(4*a*c-b^2))^(1/2)-c))*d^2-1/e*(e^2*(4*a*c-b^2))^(1/2)*c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*arctan(e*x+d)^2*d^2+1/e*(e^2*(4*a*c-b^2))^(1/2)*c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*arctan(e*x+d)^2*d^2-1/8*e*(e^2*(4*a*c-b^2))^(1/2)/c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2,(-I*b*e+2*I*d*c+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2+(e^2*(4*a*c-b^2))^(1/2)-c))*b^2+1/4*e*(e^2*(4*a*c-b^2))^(1/2)/c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*arctan(e*x+d)^2*b^2-1/4*e*(e^2*(4*a*c-b^2))^(1/2)/c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*arctan(e*x+d)^2*b^2+1/2*e/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2,(-I*b*e+2*I*d*c+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2+(e^2*(4*a*c-b^2))^(1/2)-c))+1/2*e/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2,(-I*b*e+2*I*d*c+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2-(e^2*(4*a*c-b^2))^(1/2)-c))+e/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*arctan(e*x+d)^2+e/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*arctan(e*x+d)^2-I*(e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*ln(1-(-I*b*e+2*I*d*c+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2-(e^2*(4*a*c-b^2))^(1/2)-c))*arctan(e*x+d)*b*d-I/e*(e^2*(4*a*c-b^2))^(1/2)*c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*ln(1-(-I*b*e+2*I*d*c+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2+(e^2*(4*a*c-b^2))^(1/2)-c))*arctan(e*x+d)-1/8*e*(e^2*(4*a*c-b^2))^(1/2)/c/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2,(-I*b*e+2*I*d*c+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2+(e^2*(4*a*c-b^2))^(1/2)-c))+I*e/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*ln(1-(-I*b*e+2*I*d*c+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2-(e^2*(4*a*c-b^2))^(1/2)-c))*arctan(e*x+d)+1/8*e*(e^2*(4*a*c-b^2))^(1/2)/c/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2,(-I*b*e+2

```
*I*d*c+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d
^2-(e^2*(4*a*c-b^2))^(1/2)-c))-1/4*e*(e^2*(4*a*c-b^2))^(1/2)/c/(a*e^2-b*e*d
+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*arctan(e*x+d)^2-I/e*(e^2*(4*a*c-b^2))^(1/
2)*c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*ln(1-(-I*b*e
+2*I*d*c+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c
*d^2+(e^2*(4*a*c-b^2))^(1/2)-c))*arctan(e*x+d)*d^2+I/e*(e^2*(4*a*c-b^2))^(1
/2)*c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*ln(1-(-I*b*
e+2*I*d*c+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-
c*d^2-(e^2*(4*a*c-b^2))^(1/2)-c))*arctan(e*x+d)*d^2-1/4*I*e*(e^2*(4*a*c-b^2
))^(1/2)/c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*ln(1-(
-I*b*e+2*I*d*c+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b
*e*d-c*d^2+(e^2*(4*a*c-b^2))^(1/2)-c))*arctan(e*x+d)*b^2+1/4*I*e*(e^2*(4*a*
c-b^2))^(1/2)/c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*l
n(1-(-I*b*e+2*I*d*c+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*
e^2+b*e*d-c*d^2-(e^2*(4*a*c-b^2))^(1/2)-c))*arctan(e*x+d)*b^2+I*e/(a*e^2-b*
e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*ln(1-(-I*b*e+2*I*d*c+a*e^2-b*e*d+c*d^2
-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2+(e^2*(4*a*c-b^2))^(1/
2)-c))*arctan(e*x+d)+1/e*(e^2*(4*a*c-b^2))^(1/2)*c/(4*a*c-b^2)/(a*e^2-b*e*d
+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*arctan(e*x+d)^2+1/2/e*(e^2*(4*a*c-b^2))^(
1/2)*c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2,
(-I*b*e+2*I*d*c+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+
b*e*d-c*d^2-(e^2*(4*a*c-b^2))^(1/2)-c))-1/e*(e^2*(4*a*c-b^2))^(1/2)*c/(4*a*
c-b^2)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*arctan(e*x+d)^2-1/2/e*
(e^2*(4*a*c-b^2))^(1/2)*c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^
(1/2)+c)*polylog(2,(-I*b*e+2*I*d*c+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e
*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2+(e^2*(4*a*c-b^2))^(1/2)-c))+(e^2*(4*a*c-b^2)
)^(1/2)/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*arctan(e*
x+d)^2*b*d-(e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a
*c-b^2))^(1/2)+c)*arctan(e*x+d)^2*b*d+1/2*(e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^
2)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2,(-I*b*e+2*I*d*c+
a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2+(e^2
*(4*a*c-b^2))^(1/2)-c))*b*d-1/2*(e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)/(a*e^2-
b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2,(-I*b*e+2*I*d*c+a*e^2-b*e*
d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2-(e^2*(4*a*c-b^
2))^(1/2)-c))*b*d+1/4*e*(e^2*(4*a*c-b^2))^(1/2)/c/(a*e^2-b*e*d+c*d^2+(e^2*(
4*a*c-b^2))^(1/2)+c)*arctan(e*x+d)^2+I*(e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)/
(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*ln(1-(-I*b*e+2*I*d*c+a*e^2-b*
e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2+(e^2*(4*a*c-
b^2))^(1/2)-c))*arctan(e*x+d)*b*d+I/e*(e^2*(4*a*c-b^2))^(1/2)*c/(4*a*c-b^2)
/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*ln(1-(-I*b*e+2*I*d*c+a*e^2-b
*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2-(e^2*(4*a*c
-b^2))^(1/2)-c))*arctan(e*x+d)
```

maxima [F(-2)]   time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x+d)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}\left(d+e\,x\right)}{c\,x^2+b\,x+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(d + e*x)/(a + b*x + c*x^2),x)
```

```
[Out] int(atan(d + e*x)/(a + b*x + c*x^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(e*x+d)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

3.63 $\int \frac{\tan^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=132

$$\frac{i\text{Li}_2\left(-\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{b}-\frac{i\text{Li}_2\left(\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{b}-\frac{2i\tan^{-1}(a+bx)\tan^{-1}\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}$$

```
[Out] -2*I*arctan(b*x+a)*arctan((1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b+I*poly
log(2,-I*(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b-I*polylog(2,I*(1+I*(b*x
+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b
```

Rubi [A] time = 0.10, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5055, 4886}

$$\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}-\frac{i\text{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}-\frac{2i\tan^{-1}(a+bx)\tan^{-1}\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] ((-2*I)*ArcTan[a + b*x]*ArcTan[Sqrt[1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)]]
)/b + (I*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/b
- (I*PolyLog[2, (I*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/b
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5055

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((A_.) + (B_.)*(x_) + (
C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)^
q*(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C, p
, q}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\tan^{-1}(x)}{\sqrt{1+x^2}}\,dx,x,a+bx\right)}{b}\\ &= -\frac{2i\tan^{-1}(a+bx)\tan^{-1}\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}+\frac{i\text{Li}_2\left(-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}-\frac{i\text{Li}_2\left(\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.12, size = 97, normalized size = 0.73

$$\frac{i\text{Li}_2\left(-ie^{i\tan^{-1}(a+bx)}\right)-i\text{Li}_2\left(ie^{i\tan^{-1}(a+bx)}\right)+\tan^{-1}(a+bx)\left(\log\left(1-ie^{i\tan^{-1}(a+bx)}\right)-\log\left(1+ie^{i\tan^{-1}(a+bx)}\right)\right)}{b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a + b*x]/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] (ArcTan[a + b*x]*(Log[1 - I*E^(I*ArcTan[a + b*x])] - Log[1 + I*E^(I*ArcTan[
a + b*x])]) + I*PolyLog[2, (-I)*E^(I*ArcTan[a + b*x])] - I*PolyLog[2, I*E^(
I*ArcTan[a + b*x])])/b
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\arctan\left(bx+a\right)}{\sqrt{b^2x^2+2\,abx+a^2+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(arctan(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.68, size = 143, normalized size = 1.08

$$-\frac{\arctan\left(bx+a\right)\ln\left(1+\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^2}}\right)}{b}+\frac{\arctan\left(bx+a\right)\ln\left(1-\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^2}}\right)}{b}+\frac{i\,\text{dilog}\left(1+\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^2}}\right)}{b}-\frac{i\,\text{dilog}\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x)
```

```
[Out] -1/b*arctan(b*x+a)*ln(1+I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))+1/b*arctan(b*x
+a)*ln(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))+I/b*dilog(1+I*(1+I*(b*x+a))/(
1+(b*x+a)^2)^(1/2))-I/b*dilog(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(bx+a\right)}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(arctan(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}\left(a+b\,x\right)}{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
```

```
[Out] int(atan(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}(a+bx)}{\sqrt{a^2+2abx+b^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
[Out] Integral(atan(a + b*x)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1), x)
```

3.64 $\int \frac{\tan^{-1}(a+bx)}{\sqrt{(1+a^2)c+2abcx+b^2cx^2}}\,dx$

Optimal. Leaf size=216

$$\frac{i\sqrt{(a+bx)^2+1}\,\mathrm{Li}_2\left(-\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c(a+bx)^2+c}}-\frac{i\sqrt{(a+bx)^2+1}\,\mathrm{Li}_2\left(\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c(a+bx)^2+c}}-\frac{2i\sqrt{(a+bx)^2+1}\,\tan^{-1}(a+bx)\tan^{-1}\left(\right.}{b\sqrt{c(a+bx)^2+c}}$$

```
[Out] -2*I*arctan(b*x+a)*arctan((1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))*(1+(b*x+
a)^2)^(1/2)/b/(c+c*(b*x+a)^2)^(1/2)+I*polylog(2,-I*(1+I*(b*x+a))^(1/2)/(1-I
*(b*x+a))^(1/2))*(1+(b*x+a)^2)^(1/2)/b/(c+c*(b*x+a)^2)^(1/2)-I*polylog(2,I*
(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))*(1+(b*x+a)^2)^(1/2)/b/(c+c*(b*x+a)
^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5055, 4890, 4886}

$$\frac{i\sqrt{(a+bx)^2+1}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c(a+bx)^2+c}}-\frac{i\sqrt{(a+bx)^2+1}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c(a+bx)^2+c}}-\frac{2i\sqrt{(a+bx)^2+1}\,\tan^{-1}}{b\sqrt{c(a}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2],x]
```

```
[Out] ((-2*I)*Sqrt[1 + (a + b*x)^2]*ArcTan[a + b*x]*ArcTan[Sqrt[1 + I*(a + b*x)]/
Sqrt[1 - I*(a + b*x)]])/(b*Sqrt[c + c*(a + b*x)^2]) + (I*Sqrt[1 + (a + b*x)
^2]*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/(b*Sqrt
[c + c*(a + b*x)^2]) - (I*Sqrt[1 + (a + b*x)^2]*PolyLog[2, (I*Sqrt[1 + I*(a
 + b*x)])/Sqrt[1 - I*(a + b*x)]])/(b*Sqrt[c + c*(a + b*x)^2])
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5055

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((A_.) + (B_.)*(x_) + (
C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)^
q*(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C, p
, q}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{\sqrt{(1+a^2)c+2abcx+b^2cx^2}}\,dx = \frac{\text{Subst}\left(\int \frac{\tan^{-1}(x)}{\sqrt{c+cx^2}}\,dx, x, a+bx\right)}{b}$$

$$= \frac{\sqrt{1+(a+bx)^2}\,\text{Subst}\left(\int \frac{\tan^{-1}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b\sqrt{c+c(a+bx)^2}}$$

$$= -\frac{2i\sqrt{1+(a+bx)^2}\,\tan^{-1}(a+bx)\tan^{-1}\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}} + \frac{i\sqrt{1+(a+bx)^2}\,\text{Li}_2}{b\sqrt{c+c(a+}}$$

Mathematica [A] time = 0.07, size = 125, normalized size = 0.58

$$\frac{\sqrt{(a+bx)^2+1}\left(i\text{Li}_2\left(-ie^{i\tan^{-1}(a+bx)}\right)-i\text{Li}_2\left(ie^{i\tan^{-1}(a+bx)}\right)+\tan^{-1}(a+bx)\left(\log\left(1-ie^{i\tan^{-1}(a+bx)}\right)-\log\left(1+ie^{i\tan}\right.\right.\right.}{b\sqrt{c\left((a+bx)^2+1\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a + b*x]/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2],x]
```

```
[Out] (Sqrt[1 + (a + b*x)^2]*(ArcTan[a + b*x]*(Log[1 - I*E^(I*ArcTan[a + b*x])] -
 Log[1 + I*E^(I*ArcTan[a + b*x])]) + I*PolyLog[2, (-I)*E^(I*ArcTan[a + b*x]
)] - I*PolyLog[2, I*E^(I*ArcTan[a + b*x])]))/(b*Sqrt[c*(1 + (a + b*x)^2)])
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\arctan(bx+a)}{\sqrt{b^2cx^2+2\,abcx+\left(a^2+1\right)c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+c*x^2*b^2)^(1/2),x, algorithm=
"fricas")
```

```
[Out] integral(arctan(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+c*x^2*b^2)^(1/2),x, algorithm=
"giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.93, size = 176, normalized size = 0.81

$$\frac{i\left(i\arctan(bx+a)\ln\left(1+\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^2}}\right)-i\arctan(bx+a)\ln\left(1-\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^2}}\right)+\text{dilog}\left(1+\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^2}}\right)-\text{dilog}\left(1\right.\right.}{\sqrt{b^2x^2+2abx+a^2+1}\;bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x)
```

[Out] I*(I*arctan(b*x+a)*ln(1+I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))-I*arctan(b*x+a)*ln(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))+dilog(1+I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))-dilog(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)))*(c*(-I+a+b*x)*(I+a+b*x))^(1/2)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b/c

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(bx+a\right)}{\sqrt{b^{2}cx^{2}+2\,abcx+\left(a^{2}+1\right)c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+c*x^2*b^2)^(1/2),x, algorithm="maxima")

[Out] integrate(arctan(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}\left(a+b\,x\right)}{\sqrt{c\,b^2\,x^2+2\,a\,c\,b\,x+c\,\left(a^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(atan(a + b*x)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/2),x)

[Out] int(atan(a + b*x)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}{\left(a+bx \right)}}{\sqrt{c\left(a^{2}+2abx+b^{2}x^{2}+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atan(b*x+a)/((a**2+1)*c+2*a*b*c*x+c*x**2*b**2)**(1/2),x)

[Out] Integral(atan(a + b*x)/sqrt(c*(a**2 + 2*a*b*x + b**2*x**2 + 1)), x)

3.65 $\int \frac{\tan^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\tan^{-1}(a+bx)}{\sqrt[3]{(a+bx)^2+1}},x\right)$$

```
[Out] Unintegrable(arctan(b*x+a)/(1+(b*x+a)^2)^(1/3),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tan^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a + b*x]/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3),x]
```

```
[Out] Defer[Subst][Defer[Int][ArcTan[x]/(1 + x^2)^(1/3), x], x, a + b*x]/b
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx=\frac{\text{Subst}\left(\int \frac{\tan^{-1}(x)}{\sqrt[3]{1+x^2}}\,dx,x,a+bx\right)}{b}$$

Mathematica [A] time = 0.42, size = 163, normalized size = 7.09

$$\frac{\frac{5\sqrt[3]{2}\sqrt{\pi}\Gamma\left(\frac{5}{3}\right){}_3F_2\left(1,\frac{4}{3},\frac{4}{3};\frac{11}{6},\frac{7}{3};\frac{1}{(a+bx)^2+1}\right)}{(a+bx)^2+1}+6\Gamma\left(\frac{11}{6}\right)\Gamma\left(\frac{7}{3}\right)\left(\frac{4(a+bx)\,{}_2F_1\left(1,\frac{4}{3};\frac{11}{6};\frac{1}{(a+bx)^2+1}\right)\tan^{-1}(a+bx)}{(a+bx)^2+1}+10(a+bx)\tan^{-1}(a+bx)+\right.}{20b\Gamma\left(\frac{11}{6}\right)\Gamma\left(\frac{7}{3}\right)\sqrt[3]{a^2+2abx+b^2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a + b*x]/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3),x]
```

```
[Out] (6*Gamma[11/6]*Gamma[7/3]*(15 + 10*(a + b*x)*ArcTan[a + b*x] + (4*(a + b*x)
*ArcTan[a + b*x]*Hypergeometric2F1[1, 4/3, 11/6, (1 + (a + b*x)^2)^(-1)])/(
1 + (a + b*x)^2)) + (5*2^(1/3)*Sqrt[Pi]*Gamma[5/3]*HypergeometricPFQ[{1, 4/
3, 4/3}, {11/6, 7/3}, (1 + (a + b*x)^2)^(-1)])/(1 + (a + b*x)^2))/(20*b*(1
+ a^2 + 2*a*b*x + b^2*x^2)^(1/3)*Gamma[11/6]*Gamma[7/3])
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\arctan(bx+a)}{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorithm="fricas"
)
```

```
[Out] integral(arctan(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 1.47, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(bx+a\right)}{\left(b^{2}x^{2}+2abx+a^{2}+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
[Out] int(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(bx+a\right)}{\left(b^{2}x^{2}+2\,abx+a^{2}+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorithm="maxima"
)
[Out] integrate(arctan(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{atan}\left(a+b\,x\right)}{{\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/3),x)
[Out] int(atan(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}{\left(a+bx\right)}}{\sqrt[3]{a^{2}+2abx+b^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/3),x)
[Out] Integral(atan(a + b*x)/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(1/3), x)
```

3.66 $\int \frac{\tan^{-1}(a+bx)}{\sqrt[3]{(1+a^2)c+2abcx+b^2cx^2}}\,dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left(\frac{\tan^{-1}(a+bx)}{\sqrt[3]{c(a+bx)^2+c}},x\right)$$

[Out] Unintegrable(arctan(b*x+a)/(c+c*(b*x+a)^2)^(1/3),x)

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tan^{-1}(a+bx)}{\sqrt[3]{\left(1+a^2\right)c+2abcx+b^2cx^2}}\,dx$$

Verification is Not applicable to the result.

[In] Int[ArcTan[a + b*x]/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2)^(1/3),x]

[Out] Defer[Subst][Defer[Int][ArcTan[x]/(c + c*x^2)^(1/3), x], x, a + b*x]/b

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{\sqrt[3]{\left(1+a^2\right)c+2abcx+b^2cx^2}}\,dx=\frac{\mathrm{Subst}\left(\int \frac{\tan^{-1}(x)}{\sqrt[3]{c+cx^2}}\,dx,x,a+bx\right)}{b}$$

Mathematica [A] time = 0.10, size = 165, normalized size = 6.60

$$\frac{\frac{5\sqrt[3]{2}\sqrt{\pi}\,\Gamma\left(\frac{5}{3}\right){}_3F_2\left(1,\frac{4}{3},\frac{4}{3};\frac{11}{6},\frac{7}{3};\frac{1}{(a+bx)^2+1}\right)}{(a+bx)^2+1}+6\Gamma\left(\frac{11}{6}\right)\Gamma\left(\frac{7}{3}\right)\left(\frac{4(a+bx)\,{}_2F_1\left(1,\frac{4}{3};\frac{11}{6};\frac{1}{(a+bx)^2+1}\right)\tan^{-1}(a+bx)}{(a+bx)^2+1}+10(a+bx)\tan^{-1}(a+bx)+\right.}{20b\Gamma\left(\frac{11}{6}\right)\Gamma\left(\frac{7}{3}\right)\sqrt[3]{c\left(a^2+2abx+b^2x^2+1\right)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcTan[a + b*x]/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2)^(1/3),x]

[Out] (6*Gamma[11/6]*Gamma[7/3]*(15 + 10*(a + b*x)*ArcTan[a + b*x] + (4*(a + b*x)*ArcTan[a + b*x]*Hypergeometric2F1[1, 4/3, 11/6, (1 + (a + b*x)^2)^(-1)])/(1 + (a + b*x)^2)) + (5*2^(1/3)*Sqrt[Pi]*Gamma[5/3]*HypergeometricPFQ[{1, 4/3, 4/3}, {11/6, 7/3}, (1 + (a + b*x)^2)^(-1)])/(1 + (a + b*x)^2))/(20*b*(c*(1 + a^2 + 2*a*b*x + b^2*x^2))^(1/3)*Gamma[11/6]*Gamma[7/3])

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\arctan(bx+a)}{\left(b^2cx^2+2\,abcx+\left(a^2+1\right)c\right)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+c*x^2*b^2)^(1/3),x, algorithm="fricas")

[Out] integral(arctan(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c)^(1/3), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+c*x^2*b^2)^(1/3),x, algorithm=
"giac")
```

```
[Out] sage0*x
```

maple [A] time = 1.38, size = 0, normalized size = 0.00

$$\int \frac{\arctan(bx+a)}{\left(\left(a^2+1\right)c+2abcx+b^2c\,x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

```
[Out] int(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan(bx+a)}{\left(b^2cx^2+2\,abcx+\left(a^2+1\right)c\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+c*x^2*b^2)^(1/3),x, algorithm=
"maxima")
```

```
[Out] integrate(arctan(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c)^(1/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{atan}\left(a+b\,x\right)}{{\left(c\,b^2\,x^2+2\,a\,c\,b\,x+c\,\left(a^2+1\right)\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/3),x)
```

```
[Out] int(atan(a + b*x)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}(a+bx)}{\sqrt[3]{c\left(a^2+2abx+b^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/((a**2+1)*c+2*a*b*c*x+c*x**2*b**2)**(1/3),x)
```

```
[Out] Integral(atan(a + b*x)/(c*(a**2 + 2*a*b*x + b**2*x**2 + 1))**(1/3), x)
```

3.67 $\int \frac{(a+bx)^2 \tan^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}} \, dx$

Optimal. Leaf size=187

$$-\frac{i\text{Li}_2\left(-\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{2b}+\frac{i\text{Li}_2\left(\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{2b}-\frac{\sqrt{(a+bx)^2+1}}{2b}+\frac{i\tan^{-1}\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)\tan^{-1}(a+bx)}{b}+\frac{(a+bx)\sqrt{(a+bx)^2}}{2}$$

```
[Out] I*arctan(b*x+a)*arctan((1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b-1/2*I*pol
ylog(2,-I*(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b+1/2*I*polylog(2,I*(1+I
*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b-1/2*(1+(b*x+a)^2)^(1/2)/b+1/2*(b*x+a
)*arctan(b*x+a)*(1+(b*x+a)^2)^(1/2)/b
```

Rubi [A] time = 0.22, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {5057, 4952, 261, 4886}

$$-\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b}+\frac{i\text{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b}-\frac{\sqrt{(a+bx)^2+1}}{2b}+\frac{i\tan^{-1}\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)\tan^{-1}(a+bx)}{b}+(a$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*ArcTan[a + b*x])/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] -Sqrt[1 + (a + b*x)^2]/(2*b) + ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcTan[a +
b*x])/(2*b) + (I*ArcTan[a + b*x]*ArcTan[Sqrt[1 + I*(a + b*x)]/Sqrt[1 - I*(a
 + b*x)]])/b - ((I/2)*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a
 + b*x)]])/b + ((I/2)*PolyLog[2, (I*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a +
b*x)]])/b
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 4952

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcTan[c*x])^p)/(c^2*d*m), x] + (-Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*(a
 + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(f^2*(m - 1))/(c^
2*m), Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^2], x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5057

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst
[Int[((d*e - c*f)/d + (f*x)/d)^m*(C/d^2 + (C*x^2)/d^2)^q*(a + b*ArcTan[x])^
p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &&
```

```
EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\int \frac{(a+bx)^2\tan^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\tan^{-1}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b}$$

$$= \frac{(a+bx)\sqrt{1+(a+bx)^2}\,\tan^{-1}(a+bx)}{2b} - \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{2b} - \text{Sub}$$

$$= -\frac{\sqrt{1+(a+bx)^2}}{2b} + \frac{(a+bx)\sqrt{1+(a+bx)^2}\,\tan^{-1}(a+bx)}{2b} + \frac{i\tan^{-1}(a+bx)\,\text{t}}{}$$

Mathematica [A] time = 0.72, size = 145, normalized size = 0.78

$$\frac{-i\text{Li}_2\left(-ie^{i\tan^{-1}(a+bx)}\right)+i\text{Li}_2\left(ie^{i\tan^{-1}(a+bx)}\right)-\sqrt{(a+bx)^2+1}+(a+bx)\sqrt{(a+bx)^2+1}\,\tan^{-1}(a+bx)+\tan^{-1}(}{2b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x)^2*ArcTan[a + b*x])/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x
]
```

```
[Out] (-Sqrt[1 + (a + b*x)^2] + (a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcTan[a + b*x] -
 ArcTan[a + b*x]*Log[1 - I*E^(I*ArcTan[a + b*x])] + ArcTan[a + b*x]*Log[1 +
 I*E^(I*ArcTan[a + b*x])] - I*PolyLog[2, (-I)*E^(I*ArcTan[a + b*x])] + I*Po
lyLog[2, I*E^(I*ArcTan[a + b*x])])/(2*b)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2x^2+2\,abx+a^2\right)\arctan\left(bx+a\right)}{\sqrt{b^2x^2+2\,abx+a^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorith
m="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*arctan(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorith
m="giac")
```

```
[Out] Timed out
```

maple [A] time = 3.16, size = 187, normalized size = 1.00

$$\frac{\left(\arctan\left(bx+a\right)xb+\arctan\left(bx+a\right)a-1\right)\sqrt{b^2x^2+2abx+a^2+1}}{2b}+\frac{\arctan\left(bx+a\right)\ln\left(1+\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^2}}\right)}{2b}-\frac{\text{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x)
```

```
[Out] 1/2*(arctan(b*x+a)*x*b+arctan(b*x+a)*a-1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b+1
/2/b*arctan(b*x+a)*ln(1+I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))-1/2/b*arctan(b
*x+a)*ln(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))-1/2*I/b*dilog(1+I*(1+I*(b*x
+a))/(1+(b*x+a)^2)^(1/2))+1/2*I/b*dilog(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/
2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^2 \arctan(bx+a)}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate((b*x + a)^2*arctan(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}\left(a+b\,x\right)\,{\left(a+b\,x\right)}^2}{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a + b*x)*(a + b*x)^2)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
```

```
[Out] int((atan(a + b*x)*(a + b*x)^2)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2 \operatorname{atan}(a+bx)}{\sqrt{a^2+2abx+b^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*atan(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Integral((a + b*x)**2*atan(a + b*x)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1), x
)
```

3.68 $\int \frac{(a+bx)^2 \tan^{-1}(a+bx)}{\sqrt{(1+a^2)c+2abcx+b^2cx^2}}\,dx$

Optimal. Leaf size=281

$$-\frac{i\sqrt{(a+bx)^2+1}\,\text{Li}_2\left(-\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{2b\sqrt{c(a+bx)^2+c}}+\frac{i\sqrt{(a+bx)^2+1}\,\text{Li}_2\left(\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{2b\sqrt{c(a+bx)^2+c}}-\frac{\sqrt{c(a+bx)^2+c}}{2bc}+\frac{i\sqrt{(a+bx)^2+1}\,\tan}{b\sqrt{c}}$$

```
[Out] I*arctan(b*x+a)*arctan((1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))*(1+(b*x+a)^
2)^(1/2)/b/(c+c*(b*x+a)^2)^(1/2)-1/2*I*polylog(2,-I*(1+I*(b*x+a))^(1/2)/(1-
I*(b*x+a))^(1/2))*(1+(b*x+a)^2)^(1/2)/b/(c+c*(b*x+a)^2)^(1/2)+1/2*I*polylog
(2,I*(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))*(1+(b*x+a)^2)^(1/2)/b/(c+c*(b
*x+a)^2)^(1/2)-1/2*(c+c*(b*x+a)^2)^(1/2)/b/c+1/2*(b*x+a)*arctan(b*x+a)*(c+c
*(b*x+a)^2)^(1/2)/b/c
```

Rubi [A] time = 0.33, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5057, 4952, 261, 4890, 4886}

$$-\frac{i\sqrt{(a+bx)^2+1}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b\sqrt{c(a+bx)^2+c}}+\frac{i\sqrt{(a+bx)^2+1}\,\text{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b\sqrt{c(a+bx)^2+c}}-\frac{\sqrt{c(a+bx)^2+c}}{2bc}+\frac{i\sqrt{(}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*ArcTan[a + b*x])/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2]
,x]
```

```
[Out] -Sqrt[c + c*(a + b*x)^2]/(2*b*c) + ((a + b*x)*Sqrt[c + c*(a + b*x)^2]*ArcTa
n[a + b*x])/(2*b*c) + (I*Sqrt[1 + (a + b*x)^2]*ArcTan[a + b*x]*ArcTan[Sqrt[
1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)]])/(b*Sqrt[c + c*(a + b*x)^2]) - ((I/
2)*Sqrt[1 + (a + b*x)^2]*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I
*(a + b*x)]])/(b*Sqrt[c + c*(a + b*x)^2]) + ((I/2)*Sqrt[1 + (a + b*x)^2]*Po
lyLog[2, (I*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/(b*Sqrt[c + c*(a
 + b*x)^2])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 4952

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcTan[c*x])^p)/(c^2*d*m), x] + (-Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*(a
 + b*ArcTan[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] - Dist[(f^2*(m - 1))/(c^
2*m), Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^2], x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5057

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst
[Int[((d*e - c*f)/d + (f*x)/d)^m*(C/d^2 + (C*x^2)/d^2)^q*(a + b*ArcTan[x])^
p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &&
 EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\int \frac{(a+bx)^2\tan^{-1}(a+bx)}{\sqrt{\left(1+a^2\right)c+2abcx+b^2cx^2}}\,dx=\frac{\text{Subst}\left(\int \frac{x^2\tan^{-1}(x)}{\sqrt{c+cx^2}}\,dx,x,a+bx\right)}{b}$$

$$=\frac{(a+bx)\sqrt{c+c(a+bx)^2}\,\tan^{-1}(a+bx)}{2bc}-\frac{\text{Subst}\left(\int \frac{x}{\sqrt{c+cx^2}}\,dx,x,a+bx\right)}{2b}-$$

$$=-\frac{\sqrt{c+c(a+bx)^2}}{2bc}+\frac{(a+bx)\sqrt{c+c(a+bx)^2}\,\tan^{-1}(a+bx)}{2bc}-\frac{\sqrt{1+(a+b}}{}$$

$$=-\frac{\sqrt{c+c(a+bx)^2}}{2bc}+\frac{(a+bx)\sqrt{c+c(a+bx)^2}\,\tan^{-1}(a+bx)}{2bc}+\frac{i\sqrt{1+(a+b}}{}$$

Mathematica [A] time = 0.16, size = 189, normalized size = 0.67

$$\frac{\sqrt{a^2+2abx+b^2x^2+1}\left(-i\text{Li}_2\left(-ie^{i\tan^{-1}(a+bx)}\right)+i\text{Li}_2\left(ie^{i\tan^{-1}(a+bx)}\right)-\sqrt{(a+bx)^2+1}+(a+bx)\sqrt{(a+bx)^2+1}\,\text{t}\right.}{2b\sqrt{c\left(a^2+2abx+b^2x^2\right.}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x)^2*ArcTan[a + b*x])/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*
c*x^2],x]
```

```
[Out] (Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(-Sqrt[1 + (a + b*x)^2] + (a + b*x)*Sqrt
[1 + (a + b*x)^2]*ArcTan[a + b*x] - ArcTan[a + b*x]*Log[1 - I*E^(I*ArcTan[a
 + b*x])] + ArcTan[a + b*x]*Log[1 + I*E^(I*ArcTan[a + b*x])] - I*PolyLog[2,
 (-I)*E^(I*ArcTan[a + b*x])] + I*PolyLog[2, I*E^(I*ArcTan[a + b*x])]))/(2*b
*Sqrt[c*(1 + a^2 + 2*a*b*x + b^2*x^2)])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2x^2+2\,abx+a^2\right)\arctan\left(bx+a\right)}{\sqrt{b^2cx^2+2\,abcx+\left(a^2+1\right)c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+c*x^2*b^2)^(1/2),x,
algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*arctan(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c
*x + (a^2 + 1)*c), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+c*x^2*b^2)^(1/2),x,
algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 4.53, size = 222, normalized size = 0.79

$$\frac{\left(\arctan\left(bx+a\right)xb+\arctan\left(bx+a\right)a-1\right)\sqrt{c\left(bx+a-i\right)\left(bx+a+i\right)}}{2bc}-\frac{i\left(i\arctan\left(bx+a\right)\ln\left(1+\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^2}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x)
```

```
[Out] 1/2*(arctan(b*x+a)*x*b+arctan(b*x+a)*a-1)*(c*(-I+a+b*x)*(I+a+b*x))^(1/2)/b/
c-1/2*I*(I*arctan(b*x+a)*ln(1+I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))-I*arctan
(b*x+a)*ln(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))+dilog(1+I*(1+I*(b*x+a))/(
1+(b*x+a)^2)^(1/2))-dilog(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)))*(c*(-I+a+
b*x)*(I+a+b*x))^(1/2)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^2 \arctan\left(bx+a\right)}{\sqrt{b^2cx^2+2\,abcx+\left(a^2+1\right)c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+c*x^2*b^2)^(1/2),x,
algorithm="maxima")
```

```
[Out] integrate((b*x + a)^2*arctan(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1
)*c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}\left(a+b\,x\right)\,{\left(a+b\,x\right)}^2}{\sqrt{c\,b^2\,x^2+2\,a\,c\,b\,x+c\,\left(a^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a + b*x)*(a + b*x)^2)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/2)
,x)
```

```
[Out] int((atan(a + b*x)*(a + b*x)^2)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/2)
, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2 \operatorname{atan}{\left(a+bx \right)}}{\sqrt{c\left(a^2+2abx+b^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*atan(b*x+a)/((a**2+1)*c+2*a*b*c*x+c*x**2*b**2)**(1/2),
x)
```

```
[Out] Integral((a + b*x)**2*atan(a + b*x)/sqrt(c*(a**2 + 2*a*b*x + b**2*x**2 + 1)
), x)
```

3.69 $\int \frac{(a+bx)^2 \tan^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}} \, dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{(a+bx)^2\tan^{-1}(a+bx)}{\sqrt[3]{(a+bx)^2+1}},x\right)$$

```
[Out] Unintegrable((b*x+a)^2*arctan(b*x+a)/(1+(b*x+a)^2)^(1/3),x)
```

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+bx)^2\tan^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((a + b*x)^2*ArcTan[a + b*x])/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3),x]
```

```
[Out] Defer[Subst][Defer[Int][(x^2*ArcTan[x])/(1 + x^2)^(1/3), x], x, a + b*x]/b
```

Rubi steps

$$\int \frac{(a+bx)^2\tan^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\tan^{-1}(x)}{\sqrt[3]{1+x^2}}\,dx,x,a+bx\right)}{b}$$

Mathematica [A] time = 1.58, size = 181, normalized size = 6.03

$$-\frac{3\left((a+bx)^2+1\right)^{2/3}\left(\frac{5\sqrt[3]{2}\sqrt{\pi}\Gamma\left(\frac{5}{3}\right){}_3F_2\left(1,\frac{4}{3},\frac{4}{3};\frac{11}{6},\frac{7}{3};\frac{1}{(a+bx)^2+1}\right)}{\left((a+bx)^2+1\right)^2}+\Gamma\left(\frac{11}{6}\right)\Gamma\left(\frac{7}{3}\right)\left(\frac{24(a+bx)\,{}_2F_1\left(1,\frac{4}{3};\frac{11}{6};\frac{1}{(a+bx)^2+1}\right)\tan^{-1}(a+bx)}{\left((a+bx)^2+1\right)^2}+\frac{}{(a+}\right.\right.}{140b\Gamma\left(\frac{11}{6}\right)\Gamma\left(\frac{7}{3}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x)^2*ArcTan[a + b*x])/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3)
,x]
```

```
[Out] (-3*(1 + (a + b*x)^2)^(2/3)*((5*2^(1/3)*Sqrt[Pi]*Gamma[5/3]*HypergeometricP
FQ[{1, 4/3, 4/3}, {11/6, 7/3}, (1 + (a + b*x)^2)^(-1)])/(1 + (a + b*x)^2)^2
 + Gamma[11/6]*Gamma[7/3]*(15 + 90/(1 + (a + b*x)^2) + (24*(a + b*x)*ArcTan
[a + b*x]*Hypergeometric2F1[1, 4/3, 11/6, (1 + (a + b*x)^2)^(-1)])/(1 + (a
+ b*x)^2)^2 + 5*ArcTan[a + b*x]*(-4*(a + b*x) + 6*Sin[2*ArcTan[a + b*x]])))
)/(140*b*Gamma[11/6]*Gamma[7/3])
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2x^2+2\,abx+a^2\right)\arctan\left(bx+a\right)}{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorith
m="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*arctan(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2
 + 1)^(1/3), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorith
m="giac")
```

```
[Out] Timed out
```

maple [A] time = 4.68, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{2} \arctan (bx+a)}{\left(b^{2} x^{2}+2 a b x+a^{2}+1\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

```
[Out] int((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{2} \arctan (bx+a)}{\left(b^{2} x^{2}+2\, a b x+a^{2}+1\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorith
m="maxima")
```

```
[Out] integrate((b*x + a)^2*arctan(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3),
x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\mathrm{atan}\left(a+b\,x\right)\,{\left(a+b\,x\right)}^2}{{\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a + b*x)*(a + b*x)^2)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/3),x)
```

```
[Out] int((atan(a + b*x)*(a + b*x)^2)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{2} \operatorname{atan}{(a+bx)}}{\sqrt[3]{a^{2}+2abx+b^{2}x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*atan(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/3),x)
```

```
[Out] Integral((a + b*x)**2*atan(a + b*x)/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(1/3)
, x)
```

3.70 $\int \frac{(a+bx)^2 \tan^{-1}(a+bx)}{\sqrt[3]{\left(1+a^2\right)c+2abcx+b^2cx^2}} \, dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{(a+bx)^2 \tan^{-1}(a+bx)}{\sqrt[3]{c(a+bx)^2+c}},x\right)$$

```
[Out] Unintegrable((b*x+a)^2*arctan(b*x+a)/(c+c*(b*x+a)^2)^(1/3),x)
```

Rubi [A] time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+bx)^2 \tan^{-1}(a+bx)}{\sqrt[3]{\left(1+a^2\right)c+2abcx+b^2cx^2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[((a + b*x)^2*ArcTan[a + b*x])/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2)^(1/
3),x]
```

```
[Out] Defer[Subst][Defer[Int][(x^2*ArcTan[x])/(c + c*x^2)^(1/3), x], x, a + b*x]/
b
```

Rubi steps

$$\int \frac{(a+bx)^2 \tan^{-1}(a+bx)}{\sqrt[3]{\left(1+a^2\right)c+2abcx+b^2cx^2}} \, dx = \frac{\text{Subst}\left(\int \frac{x^2 \tan^{-1}(x)}{\sqrt[3]{c+cx^2}} \, dx,x,a+bx\right)}{b}$$

Mathematica [A] time = 0.80, size = 225, normalized size = 7.03

$$-\frac{3\sqrt[3]{a^2+2abx+b^2x^2+1}\,\left((a+bx)^2+1\right)^{2/3}\left(\frac{5\sqrt[3]{2}\,\sqrt{\pi}\,\Gamma\left(\frac{5}{3}\right)\,{}_3F_2\left(1,\frac{4}{3},\frac{4}{3};\frac{11}{6},\frac{7}{3};\frac{1}{(a+bx)^2+1}\right)}{\left((a+bx)^2+1\right)^2}+\Gamma\left(\frac{11}{6}\right)\Gamma\left(\frac{7}{3}\right)\left(\frac{24(a+bx)\,{}_2F_1\left(1,\frac{4}{3};\frac{11}{6};\ldots\right.}{\left((a+b\ldots\right.}\right.\right.}{140b\Gamma\left(\frac{11}{6}\right)\Gamma\left(\frac{7}{3}\right)\sqrt[3]{c\left(a^2+2ab\ldots\right.}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x)^2*ArcTan[a + b*x])/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^
2)^(1/3),x]
```

```
[Out] (-3*(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3)*(1 + (a + b*x)^2)^(2/3)*((5*2^(1/3)
*Sqrt[Pi]*Gamma[5/3]*HypergeometricPFQ[{1, 4/3, 4/3}, {11/6, 7/3}, (1 + (a
+ b*x)^2)^(-1)])/(1 + (a + b*x)^2)^2 + Gamma[11/6]*Gamma[7/3]*(15 + 90/(1 +
 (a + b*x)^2) + (24*(a + b*x)*ArcTan[a + b*x]*Hypergeometric2F1[1, 4/3, 11/
6, (1 + (a + b*x)^2)^(-1)])/(1 + (a + b*x)^2)^2 + 5*ArcTan[a + b*x]*(-4*(a
+ b*x) + 6*Sin[2*ArcTan[a + b*x]])))))/(140*b*(c*(1 + a^2 + 2*a*b*x + b^2*x^
2))^(1/3)*Gamma[11/6]*Gamma[7/3])
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2x^2+2\,abx+a^2\right)\arctan\left(bx+a\right)}{\left(b^2cx^2+2\,abcx+\left(a^2+1\right)c\right)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+c*x^2*b^2)^(1/3),x,
algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*arctan(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x +
 (a^2 + 1)*c)^(1/3), x)
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+c*x^2*b^2)^(1/3),x,
algorithm="giac")
```

```
[Out] Timed out
```

maple [**A**] time = 4.70, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{2} \arctan (bx+a)}{\left(\left(a^{2}+1\right) c+2abcx+b^{2} c\, x^{2}\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

```
[Out] int((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

maxima [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{2} \arctan (bx+a)}{\left(b^{2} cx^{2}+2\, abcx+\left(a^{2}+1\right)c\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+c*x^2*b^2)^(1/3),x,
algorithm="maxima")
```

```
[Out] integrate((b*x + a)^2*arctan(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c)
^(1/3), x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\mathrm{atan}\left(a+b\,x\right)\,{\left(a+b\,x\right)}^2}{{\left(c\,b^2\,x^2+2\,a\,c\,b\,x+c\,\left(a^2+1\right)\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a + b*x)*(a + b*x)^2)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/3)
,x)
```

```
[Out] int((atan(a + b*x)*(a + b*x)^2)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/3)
, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*atan(b*x+a)/((a**2+1)*c+2*a*b*c*x+c*x**2*b**2)**(1/3),
x)
```

```
[Out] Timed out
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
      return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```